IYGB GCE

Mathematics FP1

Advanced Level

Practice Paper N

Difficulty Rating: 3.5467/1.6304

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

I.Y.G.B. www.madasmaths.com

Question 1

A transformation in three dimensional space is defined by the following 3×3 matrix, where x is a scalar constant.

$$\mathbf{C} = \begin{pmatrix} 2 & -2 & 4 \\ 5 & x-2 & 2 \\ -1 & 3 & x \end{pmatrix}.$$

Show that $\mathbf{C}$ is non singular for all values of x. **(5)**

Question 2

$$|z-1-\mathrm{i}| = 4 \ , \ z \in \mathbb{C} .$$

a) Sketch, in a standard Argand diagram, the locus of the points that satisfy the above equation. **(2)**

b) Find the minimum and maximum value of $|z|$ for points that lie on this locus. **(3)**

Question 3

Prove by induction that

$$\sum_{r=1}^{n} \frac{1}{4r^2-1} = \frac{n}{2n+1}, \ \ n \geq 1, \ n \in \mathbb{N} .$$ **(7)**

Question 4

The complex numbers z and w are defined as

$$z = 3 + \mathrm{i} \quad \text{and} \quad w = 1 + 2\mathrm{i}.$$

Determine the possible values of the real constant λ if

$$\left| \frac{z}{w} + \lambda \right| = \sqrt{\lambda + 2}. \qquad (7)$$

Question 5

The 2×2 matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 2 & a \\ 3 & b \end{pmatrix},$$

where a and b are scalar constants.

a) If the point with coordinates $(1,1)$ is mapped by $\mathbf{A}$ onto the point with coordinates $(1,3)$, determine the value of a and the value of b. **(2)**

b) Show that

$$\mathbf{A}^2 = 2\mathbf{A} - 3\mathbf{I}. \qquad (4)$$

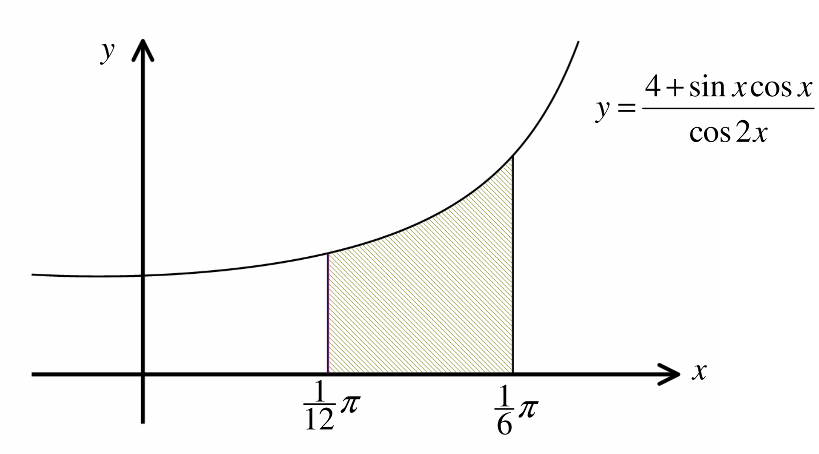
The inverse of $\mathbf{A}$ is denoted by $\mathbf{A}^{-1}$ and $\mathbf{I}$ is the 2×2 identity matrix.

c) Use part **(b)** to show further that …

i. … $\mathbf{A}^3 = \mathbf{A} - 6\mathbf{I}$.

ii. … $\mathbf{A}^{-1} = \frac{1}{3}(2\mathbf{I} - \mathbf{A})$ **(5)**

Question 6



The figure above shows part of the graph of the curve with equation

$$y = \frac{4+\sin x\cos x}{\cos 2x}.$$

The finite area bounded by the curve, the x axis and the straight lines with equations $x = \frac{1}{12}\pi$ and $x = \frac{1}{6}\pi$, shown shaded in the figure, is fully revolved about the x axis, forming a solid, S.

Calculate the volume of S, correct to 3 significant figures. **(8)**

Question 7

The roots of the cubic equation

$$16x^3 - 8x^2 + 4x - 1 = 0 \quad x \in \mathbb{R},$$

are denoted in the usual notation by α, β and γ.

Find a cubic equation, with integer coefficients, whose roots are

$$\tfrac{4}{3}(\alpha - 1), \ \tfrac{4}{3}(\beta - 1) \ \text{ and } \ \tfrac{4}{3}(\gamma - 1).$$ **(8)**

Question 8

Show by a detailed method that

$$\sum_{r=0}^{n}\left[2r\left(2r^2-3r-1\right)+n+1\right] = \left(n^2-1\right)^2. \qquad \textbf{(8)}$$

Question 9

Relative to a fixed origin O, the straight lines l_1 and l_2 have the following respective vector equations

$$\mathbf{r}_1=\begin{pmatrix}8\\q\\-3\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\p\end{pmatrix} \quad \text{and} \quad \mathbf{r}_2=\begin{pmatrix}3\\-4\\-5\end{pmatrix}+\mu\begin{pmatrix}1\\1\\1\end{pmatrix},$$

where λ and μ are scalar parameters, and p and q are scalar constants.

a) Given that l_1 and l_2 are perpendicular, determine the value of p. **(2)**

The point D is the intersection of l_1 and l_2

b) Find the value of q and the coordinates of D. **(5)**

Another straight line l_3 intersects **both** l_1 and l_2 , and is also perpendicular to **both** l_1 and l_2.

c) Find a vector equation for l_3. **(5)**

You may not use the vector (cross) product in this part

The points $A(8,1,-3)$, $B(8,1,0)$ and $C(8,-1,-1)$ lie on l_1, l_2 and l_3, respectively.

d) Show that the volume of the triangle based pyramid with vertices at A, B, C and D is 1 cubic unit. **(4)**
