IYGB GCE

Mathematics FP1

Advanced Level

Practice Paper Q

Difficulty Rating: 3.3467/1.5075

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The following complex numbers are given.

$$z_1 = 2 - 2\mathrm{i}, \quad z_2 = \sqrt{3} + \mathrm{i} \quad \text{and} \quad z_3 = a + b\mathrm{i} \quad \text{where } a \in \mathbb{R},\ b \in \mathbb{R}.$$

a) If $|z_1 z_3| = 16$, find the modulus z_3. **(3)**

b) Given further that $\arg\left(\frac{z_3}{z_2}\right) = \frac{7\pi}{12}$, determine the argument of z_3. **(3)**

c) Find the values of a and b, and hence show $\frac{z_3}{z_1} = -2$. **(4)**

Question 2

The roots of the quadratic equation

$$x^2 + 2x + 3 = 0$$

are denoted, in the usual notation, as α and β.

Find the quadratic equation, with integer coefficients, whose roots are

$$\alpha - \frac{1}{\beta^2} \quad \text{and} \quad \beta - \frac{1}{\alpha^2}.$$ **(8)**

Question 3

Prove by induction that for all natural numbers n,

$$4^n + 6n - 1$$

is divisible by 9. **(9)**

Question 4

Solve the equation

$$2z^4 - 14z^3 + 33z^2 - 26z + 10 = 0, \ z \in \mathbb{C}$$

given that one of its roots is $3 + \mathrm{i}$. **(7)**

Question 5

The 3×3 matrices $\mathbf{A}$ and $\mathbf{B}$ are given below.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

a) Describe geometrically the transformations given by each of the two matrices. **(4)**

The matrix $\mathbf{C}$ is defined as the transformation defined by the matrix $\mathbf{A}$, followed by the transformation defined by the matrix $\mathbf{B}$.

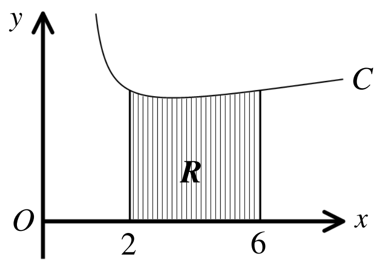
b) Describe geometrically the transformation represented by $\mathbf{C}$. **(5)**

Question 6

Find, in fully factorized form, an expression for the following sum.

$$\sum_{r=n}^{2n} \left(r^3 - 2r\right).$$ **(8)**

Question 7



The figure above shows part of the curve C with equation

$$y = \frac{x+1}{\sqrt{x-1}}, \quad x \geq 1.$$

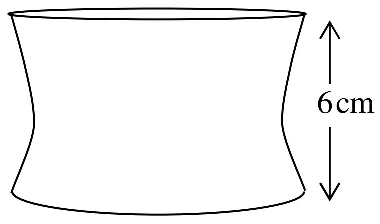
The shaded region R is bounded by the curve, the x axis and the straight lines with equations $x = 2$ and $x = 6$. The region R is rotated by $360°$ about the x axis to form a solid of revolution.

a) Show that the volume of the solid is

$$\pi\left(28 + 4\ln 5\right). \qquad \textbf{(8)}$$

The solid of part **(a)** is used to model the wooden leg of a sofa.

The shape of the leg is geometrically similar to the solid of part **(a)**.



b) Given the height of the leg is 6 cm, determine the volume of the wooden leg to the nearest cubic centimetre.

(3)

Question 8

The position vector $\mathbf{r}$ of a variable point traces the plane Π with equation

$$\mathbf{r} = (4+\lambda+5\mu)\mathbf{i} + (8+2\lambda-4\mu)\mathbf{j} + (-5+\lambda+7\mu)\mathbf{k},$$

where λ and μ are parameters.

a) Express the equation of Π in the form

$$\mathbf{r}\cdot\mathbf{n} = c,$$

where $\mathbf{n}$ and c is a vector and scalar constant, respectively. **(6)**

The point $P(12,-1,44)$ is reflected about Π onto the point P'.

b) Determine the coordinates of P'. **(7)**
