IYGB GCE

Mathematics FP2

Advanced Level

Practice Paper O

Difficulty Rating: 3.5333/1.6210

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Find a general solution of the following differential equation.

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+13y=13x^2-x+22. \qquad (7)$$

Question 2

$$f(r)=\frac{1}{r(r+2)}, \quad r\in\mathbb{N}.$$

a) Express $f(r)$ in partial fractions. **(1)**

b) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} f(r)=\frac{n(An+B)}{4(n+1)(n+2)},$$

where A and B are constants to be found. **(7)**

Question 3

$$f(x)=(1-x)^2\ln(1-x), \; -1\le x<1.$$

Find the Maclaurin expansion of $f(x)$ up and including the term in x^3. **(6)**

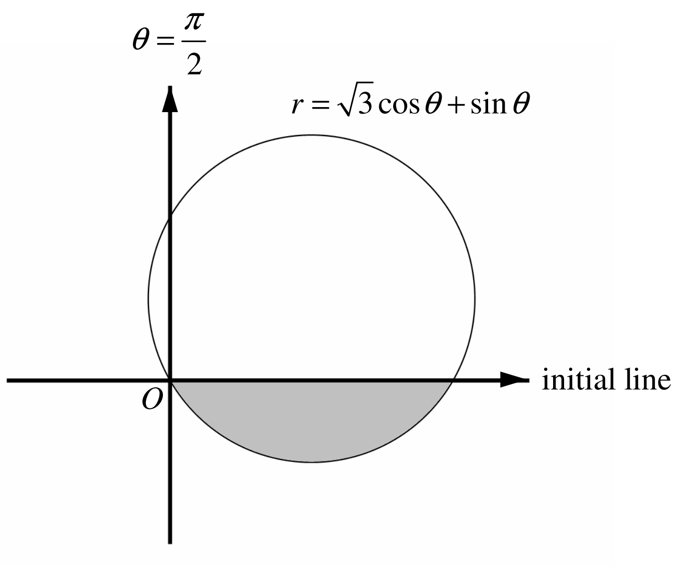
Question 4

Show that

$$\left(\sqrt{5}-2\right)\ln\left(\sqrt{5}-2\right)+\left(\sqrt{5}+2\right)\ln\left(\sqrt{5}+2\right),$$

can be written in the form $a \operatorname{arsinh} b$, where a and b are integers to be found. **(6)**

Question 5



The diagram above shows the curve with polar equation

$$r = \sqrt{3}\cos\theta + \sin\theta\,,\ -\frac{\pi}{3} \le \theta < \frac{2\pi}{3}.$$

By using a method involving integration in polar coordinates, show that the area of the shaded region is

$$\frac{1}{12}\left(4\pi - 3\sqrt{3}\right). \qquad \textbf{(7)}$$

Question 6

Find in exact simplified form the value of

$$\int_0^{\ln 2} \frac{e^x}{\cosh x}\ dx\,. \qquad \textbf{(6)}$$

Question 7

a) Use De Moivre's theorem to show that

$$\sin 5\theta \equiv 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta \, . \qquad (5)$$

b) By considering the solutions of the equation $\sin 5\theta = 0$, find in trigonometric form the four solutions of the equation

$$16x^4 - 20x^2 + 5 = 0 \, . \qquad (5)$$

c) Hence show clearly that

$$\sin^2\left(\frac{\pi}{5}\right) = \frac{5-\sqrt{5}}{8} \, . \qquad (5)$$

Question 8

$$y = \arccos x, \; -1 \le x \le 1, \; 0 \le y \le \pi \, .$$

a) By writing $y = \arccos x$ as $x = \cos y$, show that

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}} \, . \qquad (3)$$

The curve C has equation

$$y = \arccos x - \tfrac{1}{2}\ln\left(1-x^2\right), \; x > 0 \, .$$

b) Show that the y coordinate of the stationary point of C is

$$\frac{1}{4}(\pi + \ln 4) \, . \qquad (8)$$

Question 9

$$\left(1-x^2\right)\frac{dy}{dx}+y=\left(1-x^2\right)\left(1-x\right)^{\frac{1}{2}}\ ,\ -1<x<1.$$

Given that $y=\frac{\sqrt{2}}{2}$ at $x=\frac{1}{2}$, show that the solution of the above differential equation can be written as

$$y=\frac{2}{3}\sqrt{\left(1-x^2\right)\left(1+x\right)}.$$

(9)