Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

A level Further Mathematics

Decision Mathematics 1
Practice Paper 1

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **6** questions in this question paper. The total mark for this paper is **75**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

2.5 0.9 3.1 1.4 1.5 2.0 1.9 1.2 0.3 0.4 3.9

The numbers in the list are the lengths, in metres, of eleven pieces of wood. They are to be cut from planks of wood of length 5 metres. You should ignore wastage due to cutting.

(a) Calculate a lower bound for the number of planks needed. You must make your method clear.

(2)

(b) Use the first-fit bin packing algorithm to determine how these pieces could be cut from 5 metre planks.

(3)

(c) Carry out a quick sort to produce a list of the lengths in descending order. You should show the result of each pass and identify your pivots clearly.

(4)

(d) Use the first-fit decreasing bin packing algorithm to determine how these pieces could be cut from 5 metre planks.

(2)

(Total 11 marks)

Mark scheme for Question 1

Examiner comment

2. The tableau below is the initial tableau for a linear programming problem in x, y and z. The objective is to maximise the profit, P.

Basic variable	x	y	z	r	s	t	Value
r	2	–4	1	1	0	0	15
s	4	2	–8	0	1	0	20
t	1	–1	4	0	0	1	8
P	–3	2	7	0	0	0	0

(a) Perform **one** iteration of the Simplex algorithm to obtain a new tableau, T. State the row operations you use.

(5)

(b) Write down the profit equation given by T and state the current values of the slack variables.

(2)

(Total 7 marks)

Mark scheme for Question 2

Examiner comment

3.

	A	B	C	D	E	F	G
A	–	x	41	43	38	21	30
B	x	–	27	38	19	29	51
C	41	27	–	24	37	35	40
D	43	38	24	–	44	52	25
E	38	19	37	44	–	20	28
F	21	29	35	52	20	–	49
G	30	51	40	25	28	49	–

The network represented by the table shows the least distances, in km, between seven theatres, A, B, C, D, E, F and G.

Jasmine needs to visit each theatre at least once starting and finishing at A. She wishes to minimise the total distance she travels. The least distance between A and B, is x km, where $21 < x < 27$.

(a) Using Prim's algorithm, starting at A, obtain a minimum spanning tree for the network. You should list the arcs in the order in which you consider them.

(2)

(b) Use your answer to (*a*) to determine an initial upper bound for the length of Jasmine's route.

(1)

(c) Use the nearest neighbour algorithm, starting at A, to find a second upper bound for the length of the route.

(2)

The nearest neighbour algorithm starting at F gives a route of F – E – B – A – G – D – C – F.

(d) State which of these two nearest neighbour routes gives the better upper bound. Give a reason for your answer.

(2)

Starting by deleting A, and all of its arcs, a lower bound of 159 km for the length of the route is found.

(e) Find x, making your method clear.

(3)

(f) Write down the smallest interval that you can be confident contains the optimal length of Jasmine's route. Give your answer as an inequality.

(2)

(Total 12 marks)

[Mark scheme for Question 3](#)

[Examiner comment](#)

4.

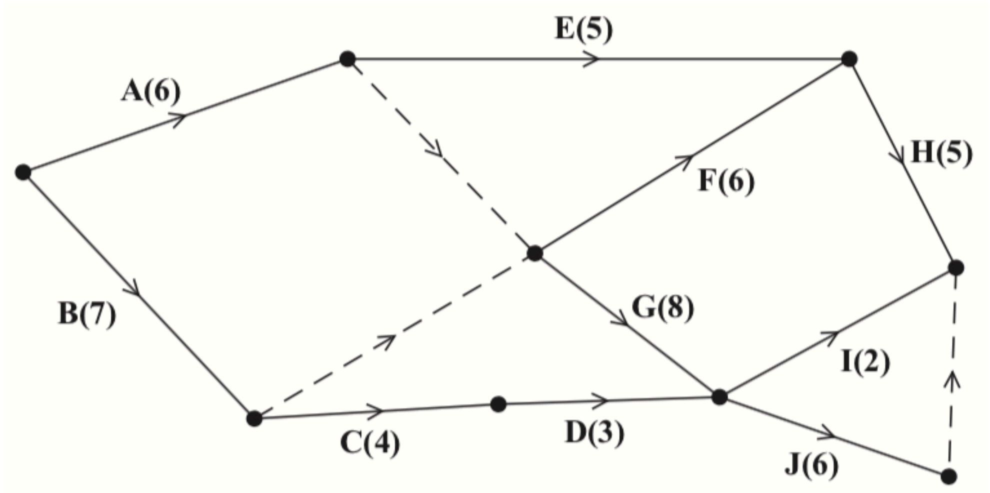


Figure 1

A project is modelled by the activity network shown in Figure 1. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the corresponding activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete the precedence table in the answer book.

(2)

(b) Complete Diagram 1 in the answer book to show the early event times and late event times.

(3)

(c) State the minimum project completion time and list the critical activities.

(2)

(d) Calculate the maximum number of hours by which activity E could be delayed without affecting the shortest possible completion time of the project. You must make the numbers used in your calculation clear.

(1)

(e) Calculate a lower bound for the number of workers needed to complete the project in The minimum time. You must show your working.

(1)

The project is to be completed in the minimum time using as few workers as possible.

(f) Schedule the activities using Grid 1 in the answer book.

(3)

Before the project begins it becomes apparent that activity E will require an additional 6 hours to complete. The project is still to be completed in the shortest possible time and the time to complete all other activities is unchanged.

(g) State the new minimum project completion time and list the new critical activities.

(2)

(Total 14 marks)

[Mark scheme for Question 4]

[Examiner comment]

Leave blank

4.

Activity	Immediately preceding activities
A	
B	
C	
D	
E	

Activity	Immediately preceding activities
F	
G	
H	
I	
J	

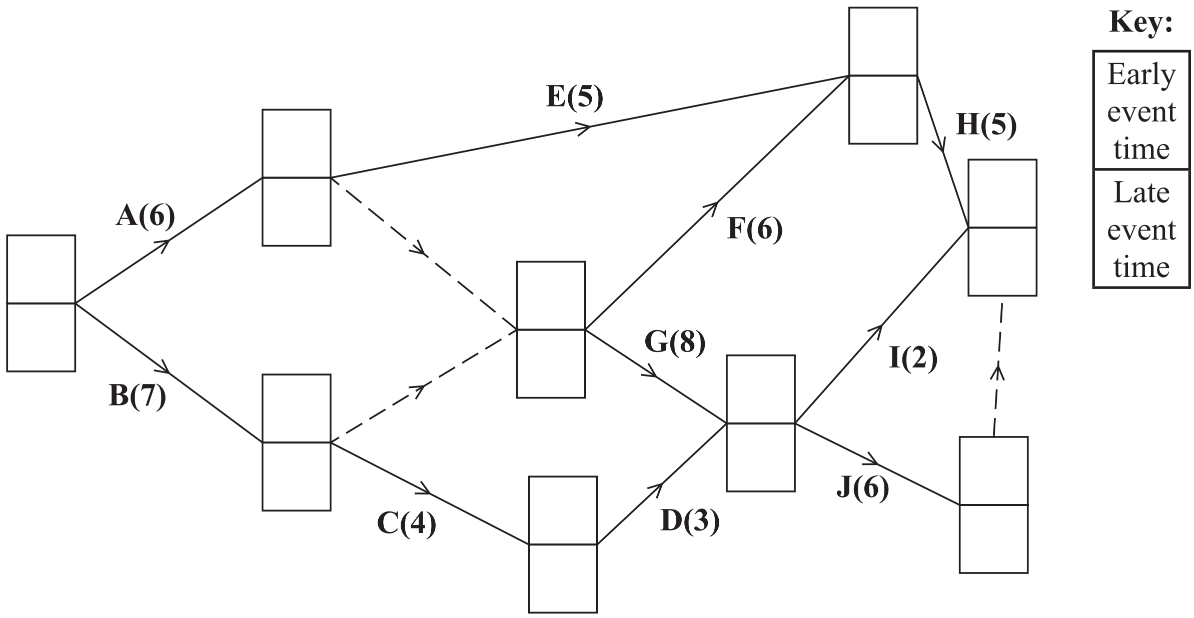
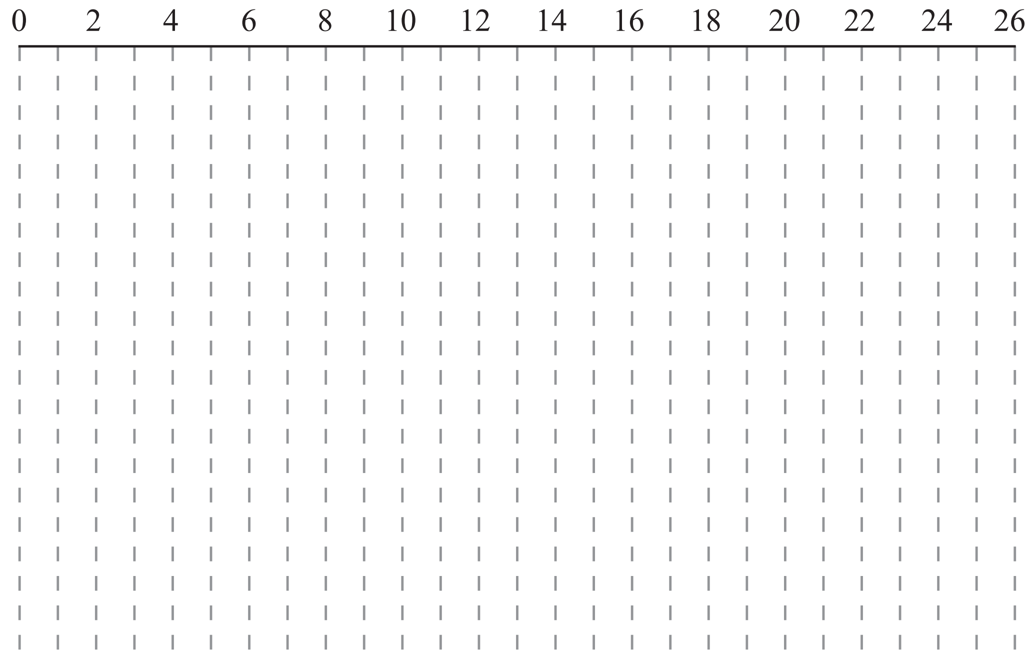


Diagram 1

Leave blank

Question 4 continued



Grid 1

Q5

(Total 14 marks)

P 5 1 5 1 2 A 0 1 5 2 0

Turn over

5. A café sells two types of scone, plain and fruit.

The café manager knows that each week she should order

- at least 400 scones in total
- at most 350 fruit scones

In addition, for every 3 fruit scones ordered, at most 5 plain scones should be ordered.

Each plain scone costs £0.11 and is sold at a profit of £0.75.
Each fruit scone costs £0.14 and is sold at a profit of £1.

The manager has £77 to spend each week on scones. The manager wants to maximise her profit and it can be assumed that all scones ordered will be sold.

Let x represent the number of plain scones and let y represent the number of fruit scones that are sold.

(a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(6)

(b) Represent these constraints on Diagram 1 in the answer book.
Hence determine the feasible region and label it R.

(4)

(c) Use the objective line method to find the optimal vertex, V, of the feasible region.
You must make your objective line clear and label the optimal vertex V.

(3)

(d) Calculate the exact coordinates of V.

(2)

(e) State the number of each type of scone that the manager should order and calculate The maximum profit.

(2)

(Total 17 marks)

[Mark scheme for Question 5](#)

[Examiner comment](#)

Leave blank

Question 5 continued

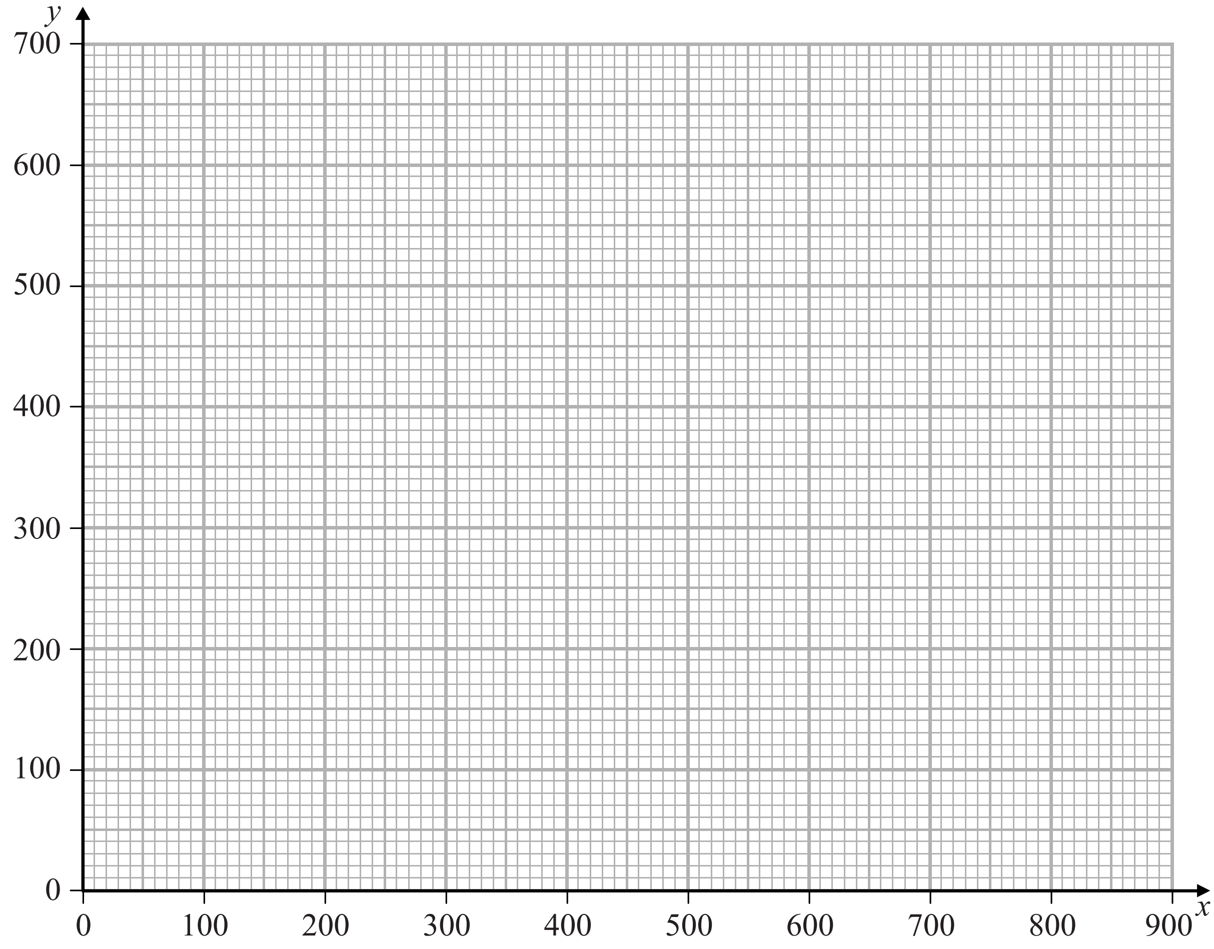


Diagram 1

P 5 1 5 7 1 A 0 1 7 2 0

Turn over

6.

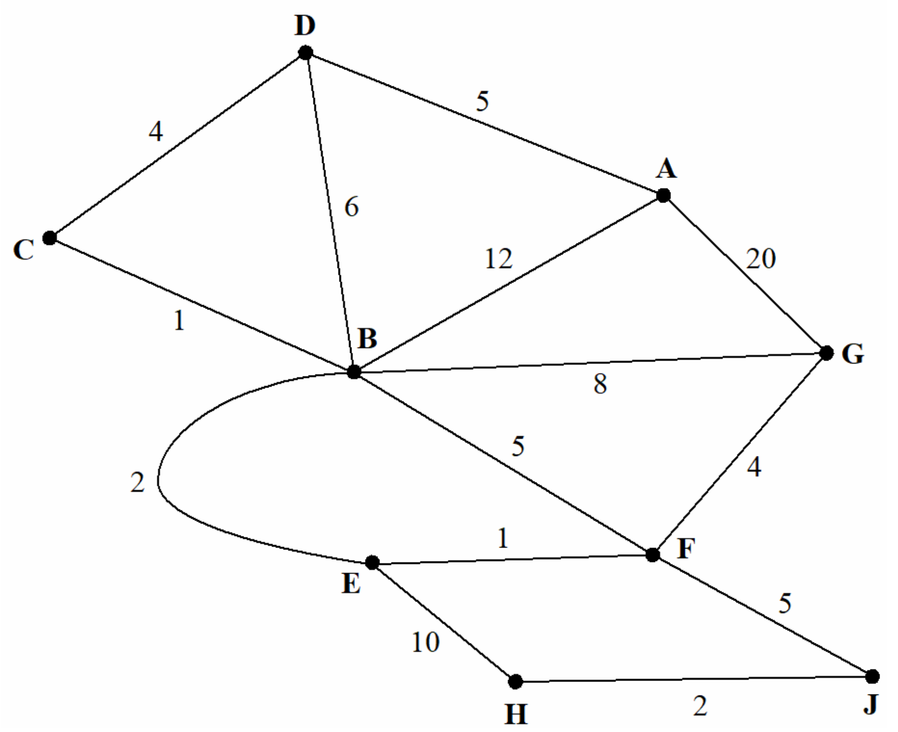


Figure 2

[*The total weight of the network is 85*]

Figure 2 represents a network of roads. The number on each edge represents the length, in miles, of the corresponding road. Robyn wishes to travel from A to H. She wishes to minimise the distance she travels.

(a) Use Dijkstra's algorithm to find the shortest path from A to H. State the shortest path and its length.

(5)*

On a particular day, Robyn needs to check each road. She must travel along each road at least once. Robyn must start and finish at vertex A.

(b) Use the route inspection algorithm to find the length of the shortest inspection route. State the edges that should be repeated. You should make your method and working clear.

(5)

The roads BD and BE become damaged and cannot be used. Robyn needs to travel along all the remaining roads to check that there is no damage to any of them. The inspection route must still start and finish at vertex A.

(c) (i) State the edges that should be repeated.

(ii) State a possible route and calculate its length. You must make your method and working clear.

(4)

(Total 14 marks)

[Mark scheme for Question 6]

[Examiner comment]

TOTAL FOR PAPER: 75 MARKS

A level Further Mathematics – Decision Mathematics 1 – Practice Paper 01 – Examiner report –

Mark scheme for Question 1 Examiner comment (Return to Question 1)

Question	Scheme	Marks
1(a)	$\frac{19.1}{5}$ == 3.82 so lower bound is 4 bins	M1A1
		(2)
(b)	Bin 1: **2.5 0.9 1.4** Bin 2: **3.1 1.5** 0.3 Bin 3: 2.0 1.9 0.4 Bin 4: 1.2 Bin 5:3.9	M1A1 A1
		(3)
(c)	e.g middle right	M1
	Pivot(s) 2.5 0.9 3.1 1.4 1.5 2.0 1.9 1.2 0.3 0.4 3.9 — 2.0 2.5 3.1 3.9 2.0 0.9 1.4 1.5 1.9 1.2 0.3 0.4 — 3.1 1.9 3.9 3.1 2.5 2.0 1.9 0.9 1.4 1.5 1.2 0.3 0.4 — (3.9) (2.5) 1.2 3.9 3.1 2.5 2.0 1.9 1.4 1.5 1.2 0.9 0.3 0.4 — 1.5 0.3 3.9 3.1 2.5 2.0 1.9 1.5 1.4 1.2 0.9 0.4 0.3 — (1.4) 0.4 3.9 3.1 2.5 2.0 1.9 1.5 1.4 1.2 0.9 0.4 0.3	M1A1 A1ft A1
		(4)
(d)	Bin 1: **3.9** 0.9 Bin 2: **3.1 1.9** Bin 3: **2.5 2.0** 0.4 Bin 4: **1.5** 1.4 1.2 0.3	M1A1
		(2)
		(11 marks)

Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2(a)	(see table below)	M1 A1 M1 A1ft A1
		(5)
(b)	$P+\frac{7}{2}y+z+\frac{3}{4}s=15$ $r=5, s=0, t=3$	B1ft B1
		(2)
		(7 marks)

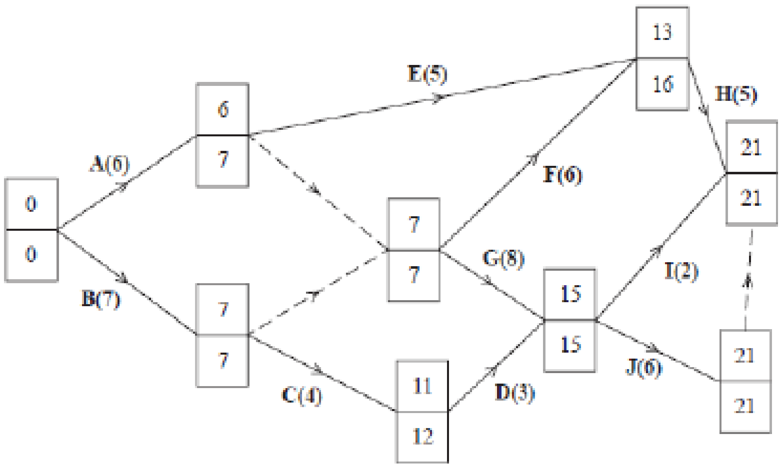
b.v	x	y	z	r	s	t	Value	Row ops
r	0	−5	5	1	$-\frac{1}{2}$	0	5	R_1-2R_2
x	1	$\frac{1}{2}$	−2	0	$\frac{1}{4}$	0	5	$R_2\div 4$
t	0	$-\frac{3}{2}$	6	0	$-\frac{1}{4}$	1	3	R_3-R_2
P	0	$\frac{7}{2}$	1	0	$\frac{3}{4}$	0	15	R_4+3R_2

Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

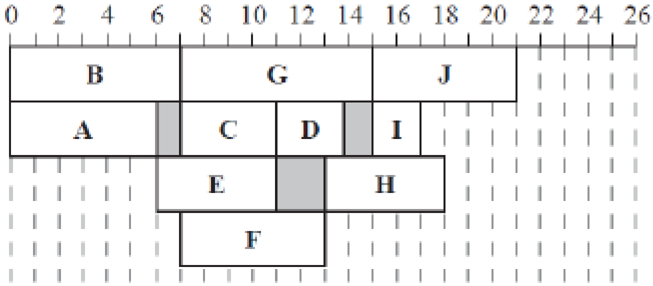
Question	Scheme	Marks
3(a)	Prim: AF, EF, BE, BC, CD, DG	M1A1
		(2)
(b)	$2 \times 136 = 272$ (km)	B1
		(1)
(c)	A F E B C D G A 21 20 19 27 24 25 30 = 166 (km)	B1 B1
		(2)
(d)	Starting at F route length is $153 + x$	B1
	With $x > 21$, $153 + x$ is greater than 166 so the better upper bound is the one starting at A	dB1
		(2)
(e)	Length of RMST = 115	B1
	$115 + 21 + x = 159 \therefore x = 23$ (km)	M1A1
		(3)
(f)	$159 \leqslant$ optimal $\leqslant 166$ [accept $159 <$ optimal $\leqslant 166$]	B2,1,0
		(2)
		(12 marks)

Mark scheme for Question 4

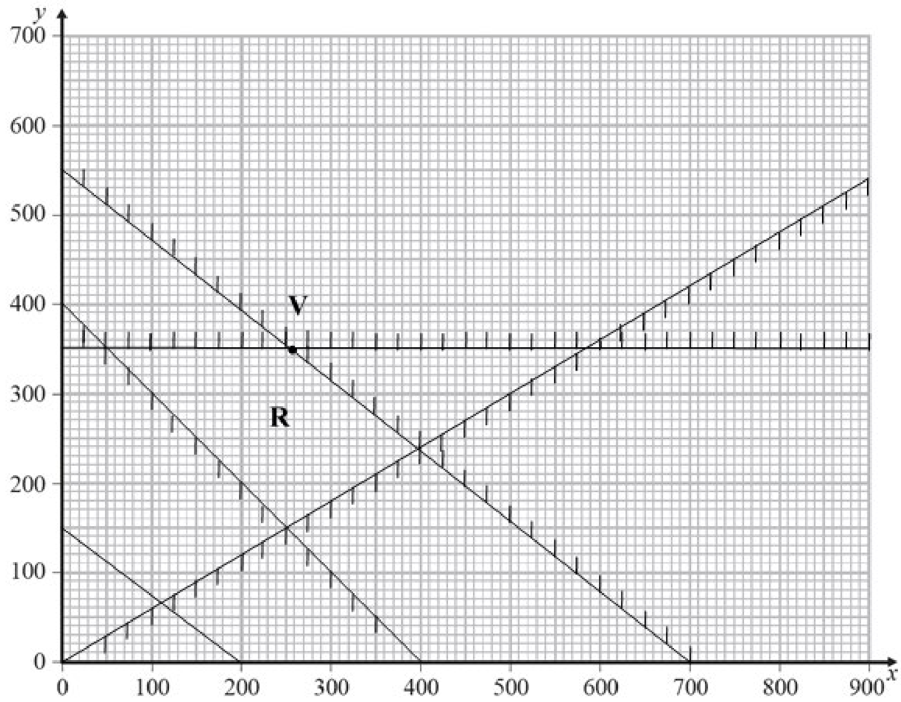
(Examiner comment) (Return to Question 4)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4(a)</td><td>
<table>
<tr><th>Activity</th><th>Immediately preceding activities</th><th>Activity</th><th>Immediately preceding activities</th></tr>
<tr><td>A</td><td>-</td><td>F</td><td>A, B</td></tr>
<tr><td>B</td><td>-</td><td>G</td><td>A, B</td></tr>
<tr><td>C</td><td>B</td><td>H</td><td>E ,F</td></tr>
<tr><td>D</td><td>C</td><td>I</td><td>D, G</td></tr>
<tr><td>E</td><td>A</td><td>J</td><td>D, G</td></tr>
</table>
</td><td>B2,1,0</td></tr>
<tr><td></td><td></td><td>(2)</td></tr>
<tr><td>(b)</td><td>


</td><td>M1
A1
A1</td></tr>
<tr><td></td><td></td><td>(3)</td></tr>
<tr><td>(c)</td><td>Minimum project completion time is 21 (hours)</td><td>B1ft</td></tr>
<tr><td></td><td>Critical activities: B, G, J</td><td>B1</td></tr>
<tr><td></td><td></td><td>(2)</td></tr>
<tr><td>(d)</td><td>E could be delayed by 16 – 5 − 6 = 5 (hours)</td><td>B1</td></tr>
<tr><td></td><td></td><td>(1)</td></tr>
<tr><td>(e)</td><td>Lower bound = $\frac{52}{21}$ = 2.476… so 3 workers required</td><td>B1</td></tr>
<tr><td></td><td></td><td>(1)</td></tr>
</table>

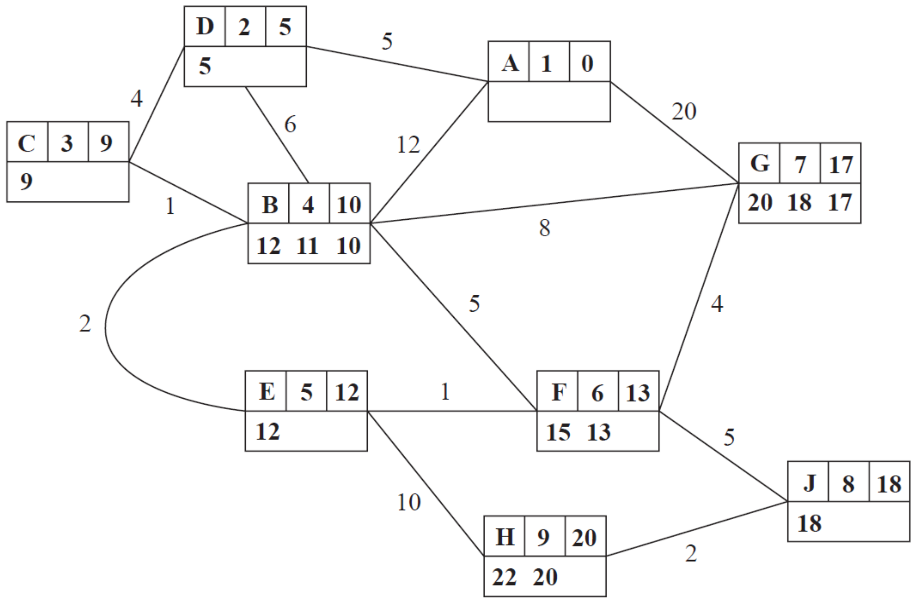
Question	Scheme	Marks
4(f)	e.g. 	M1 A1 A1
		(3)
(g)	Activities A, E and H are now critical The minimum project completion time is now 22 (hours)	B1 B1
		(2)
		(14 marks)

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme	Marks
5(a)	Maximise $0.75x + y$	B1
	Subject to $x + y \geqslant 400$	B1
	$y \leqslant 350$	B1
	$5y \geqslant 3x$	M1A1
	$11x + 14y \leqslant 7700$	B1
		(6)
(b)		B1 B1 B1 B1
		(4)
(c)	Drawing an objective line accept reciprocal gradient Correct objective line V correctly labelled	M1 A1 A1
		(3)
(d)	$V\left(\frac{2800}{11}, 350\right)$	M1A1
		(2)
(e)	(The manager should buy) 254 plain (scones) and 350 fruit (scones)	B1
	Profit is (£) 540.50	B1

5(e) *continued*		**(2)**
		(17 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(a)		M1 A1(DCBE) A1 (FG) A1ft (JH)
	Shortest path A to H: ADCBEFJH	A1
	Length of shortest path: 20 (miles)	A1ft
	***Part (a) has been reduced from 6 marks to 5 marks in line with the new A level**	
		(5)*
(b)	AD + E(F)G = 5 + 5 = 10*	M1
	A(DCB)E + D(CBEF)G = 12 + 12 = 24	A1
	A(DCBEF)G + D(CB)E = 17 + 7 = 24	A1
	Repeat edges: AD, EF, FG	A1
	Length = 85 + 10 = 95 (miles)	A1ft
		(5)
(c)(i)	AD, CD, BC, BG	B1
(ii)	Route: e.g. ADABCDCBFEHJFGBGA	B1
	Length = 85 + 18 – 6 – 2 = 95 (miles)	M1A1
		(4)
		(14 marks)

A level Further Mathematics – Decision Mathematics 1 – Practice Paper 01 – Examiner report –

Examiner comment for Question 1 (Mark scheme) (Return to Question 1)

1. Part (a) was generally very successfully attempted. The vast majority of students carried out a correct calculation and rounded their value up to give the correct lower bound. It was rare to see '19.1' (the total of all the numbers) divided by 11 (the number of pieces of wood).

Examiners reported that a significant number of students struggled in applying the first-fit bin packing algorithm in part (b). This was mainly down to not applying the algorithm correctly. First fit is just that; students must decide if the current item under consideration will fit in their first bin rather than the most recent bin used. In this part a number of students placed the 1.4 in the second bin (and not the first bin) and others did not place the 0.3 in the second bin.

Many correct solutions were seen in part (c), but a number of students did not choose their pivots consistently, switching between middle-left and middle-right pivots during the course of the quick sort algorithm. A number of students either lost an item or changed an item during the sort, and in a small number of cases only one pivot was chosen per iteration. Some students did not complete a fifth pass in which the 0.4 was used as a pivot as it was probably (incorrectly) assumed that as the list was in the 'correct' order after a fourth pass the sort was complete. Common errors included the items 2.5, 3.1 and 3.9 being interchanged in the first pass and/or the 1.4 and 1.5 not being interchanged in the fourth pass; students should be reminded that items should remain in the order from the previous pass as they move into sub-lists. There were only a few instances where students selected the first or last items as the pivot. Pivots were usually chosen consistently although the spacing and notation on some solutions made these difficult for examiners to follow. Some students over complicated the process by insisting on using a different 'symbol' to indicate the pivots for each pass. Those students who sorted into ascending order usually remembered to reverse their list at the end to gain full credit although a number of students left their list in ascending order.

The first-fit decreasing in part (d) was well carried out with only a small minority failing to attempt this part. There were a large number of wholly correct answers. A small number performed first-fit increasing therefore scoring no marks. A small minority of students lost both marks by placing the 1.9 in the 4th rather than 2nd bin (so failing to apply the algorithm at its first real test). Some students wrote totals in the bin rather than the next value. A variety of different layouts were used but in nearly all cases were easy to read and decipher.

Examiner comment for Question 2 **(Mark scheme)** **(Return to Question 2)**

2. This question on the simplex algorithm was well answered by the majority of candidates. The overwhelming majority chose both the correct column and indeed the correct row to pivot on. Examiners commented on seeing very few candidates who either pivoted on an incorrect column or an incorrect row within the x-column. The majority of candidates changed the basic variable correctly and almost every candidate gave the correct row operations. The source of most errors appeared to be numerical inaccuracy which led to either the loss of one or sometimes two marks in part (a).

Part (b) was more challenging for some candidates, although the majority scored both marks. It is surprising that a significant number of candidates are still unable to write down the profit equation correctly. There were 'equations' with two equals signs, the coefficients of y and z and s were of the incorrect sign and sometimes the 15 was omitted. It was clear that some candidates were unsure what was being asked when asked to write down the value of the slack variables and some candidates only gave two of the three variables (usually $s = 0$ was absent). Some candidates gave the values of all the variables including P and some incorrectly read off from the bottom row of the table.

Examiner comment for Question 3 **(Mark scheme)** **(Return to Question 3)**

3. This question seem to differentiate nicely between candidates. It was surprising that even some of the more standard and straightforward parts of the question caused difficulty for some candidates.

Part (a) was usually done correctly although a minority of candidates used the nearest neighbour algorithm rather than Prim's as evidenced by a return to vertex A. Some candidates tried to involve x despite x being within a range that made it irrelevant in this part. A small minority of candidates did not use Prim and other candidates did not show any obvious method but simply drew a MST. Some candidates who clearly were using Prim lost marks for only listing the nodes in order rather than listing the arcs as requested.

Part (b) was usually done correctly by those candidates who had obtained the correct minimum spanning tree in part (a) although some stated the weight of the tree plus the weight of arc AG.

Part (c) was completed correctly by the majority of candidates who gave their nearest neighbour route either as a list of nodes or arcs. The most common error was to omit the return to vertex A. Often these candidates were able to recover in the calculation for the upper bound by adding in the weight of the extra arc at this point.

Part (d) was less standard but nonetheless it was relatively well attempted and most candidates produced a good numerical reason together with a clear conclusion. The approaches taken by candidates seemed fairly evenly split between those who considered $x + 153$ together with the minimum value of x and those who considered the range of values that the nearest neighbour route from F could take. It is worth noting however that a surprising number of candidates were unable to accurately sum the arcs for the nearest neighbour route from F. Examiners noted that only a handful of candidates thought that the better upper bound was the larger of the two numbers.

A large proportion of candidates answered part (e) well and many provided clear diagrams alongside their work to illustrate the addition of the two least arcs incident to A. It was clear however that a minority of candidates were unsure what to do here. Some candidates made errors in their RMST and others added $2x$ to their RMST rather than 21 and x.

Part (f) was generally done well by most candidates. Many obtained both marks or scored one mark on the follow through from an earlier stated upper bound in part (d). Others lost the final mark for use of a strict inequality for the upper limit. It was rare but nonetheless worrying to see statements along the lines of $178 < x < 166$.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. In part (a) most students scored both marks for correctly completing the precedence table, with only a few eccentric attempts, some involving event numbers rather than activity letters. Part (b) was answered extremely well with many students correctly complete the diagram with the early event and late event times. When errors did occur they mostly occurred at the ends of activities B, C and/or E.

Examiners noted that parts (c), (d) and (e) were generally answered correctly provided the corresponding diagram in (b) was correct.

Most students did attempt to produce a schedule in (f). However a significant proportion of students tried to construct a schedule with only 3 workers (possibly due to their answer for (e)), therefore scoring only one mark and completely disregarding the significance of the Immediately Preceding Activities (IPA). Of those students who did have 4 workers in their schedule, a pleasing number were correct, though errors were sometimes seen in either the duration, time interval or IPA for one or more activities. It was pleasing to note that cascade charts were rarely seen.

Many students correctly calculated the revised timing and new critical activities in (g), although sometimes B was still stated as a critical activity.

Examiner comment for Question 5 (Mark scheme) (Return to Question 5)

5. In (a) the objective function was often found correctly but the absence of the word 'maximise' meant that the first mark could not be awarded. The first two constraints (the requirements that there must be at least 400 scones and that there must be at most 350 fruit scones) were usually correct. The next constraint based on the fact that there needed to be at most 5 plain scones for every 3 fruit scones was almost always correct with the most common incorrect answer being $3y \geqslant 5x$. The final constraints regarding the fact that the manager had £77 to spend was either dealt with very well or not attempted at all. Simplified inequalities were not always seen and, on occasion, coefficients were left as fractions rather than integers.

Most students were able to draw the required lines correctly in (b) although some were unable to draw lines sufficiently accurately (some drew lines without a ruler) or sufficiently long enough. As stated in previous reports the following general principle should always be adopted by students.

- Lines should always be drawn which cover the entire graph paper supplied in the answer book and therefore,
- lines with negative gradient should always be drawn from axis to axis.

The rationale behind this is that until all the lines are drawn (and shaded accordingly) it is unclear which lines (or parts of lines) will define the boundary of the feasible region. If students only draw the line segments that they believe define the boundary of the feasible region then examiners are unaware of the order in which the lines were drawn and therefore it is unclear to examiners why some parts of the lines have been omitted. In general the lines and were drawn correctly. Furthermore, a significant number of students were unable to select (or even label) the correct feasible region.

In (c), the majority of students drew the correct objective line, however, a line with reciprocal gradient was sometimes seen or, in a number of cases, no objective line was drawn (and therefore no marks could be awarded in this part). Some used obscure constant values to plot the objective line and some students did not label the optimal vertex clearly.

Most students in (d) correctly stated the exact coordinate of V as requested.

In (e) many students did not state in context the number of scones that the manager should buy and the corresponding cost was often given incorrectly as either £540 or £541 rather than the correct £540.50.

Examiner comment for Question 6 (Mark scheme) (Return to Question 6)

6. Part (a) was usually very well done with most students applying Dijkstra's algorithm correctly. The boxes at each node in part (a) were usually completed correctly. When errors were made it was either an order of labelling error (some students repeated the same labelling at two different nodes) or working values were either missing, not in the correct order or simply incorrect (usually these errors occurred at nodes B, G and/or H). The path was usually given correctly and most students realised that whatever their final value was at H, this was therefore the value that they should give for the length of their path. As noted in previous reports because the working values are so important in judging the candidate's proficiency at applying the algorithm it would be wise to avoid methods of presentation that require values to be crossed out.

The vast majority of students did not realise the connection between part (a), in which the shortest distances from vertex A to any other vertex had been found and part (b). Therefore many students went on to make at least one error in the totals for the pairings in part (b). Most students stated the repeated arcs correctly although there were a few who simply stated "AD, EG". Very few students failed to give three distinct pairings and corresponding totals in this part.

Part (c)(i) was well answered with the majority of students correctly stating that arcs AD, CD, BC and BG needed to be repeated. Both the route and its corresponding length in part (c)(ii) were answered well (although in a number of cases the route was left blank). Most of the students remembering to subtract the weight of the arcs BD and BE from their total.