Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | | | | | |

Candidate Number | | | | |

A level Further Mathematics

Decision Mathematics 1
Practice Paper 2

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **7** questions in this question paper. The total mark for this paper is **75**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Draw the activity network described in the precedence table below, using activity on arc and exactly three dummies.

Activity	**Immediately preceding activities**
A	–
B	–
C	A
D	A
E	B
F	B
G	A, E, F
H	F
I	C
J	D, G
K	D, G

(Total 5 marks)

Mark scheme for Question 1

Examiner comment

2.

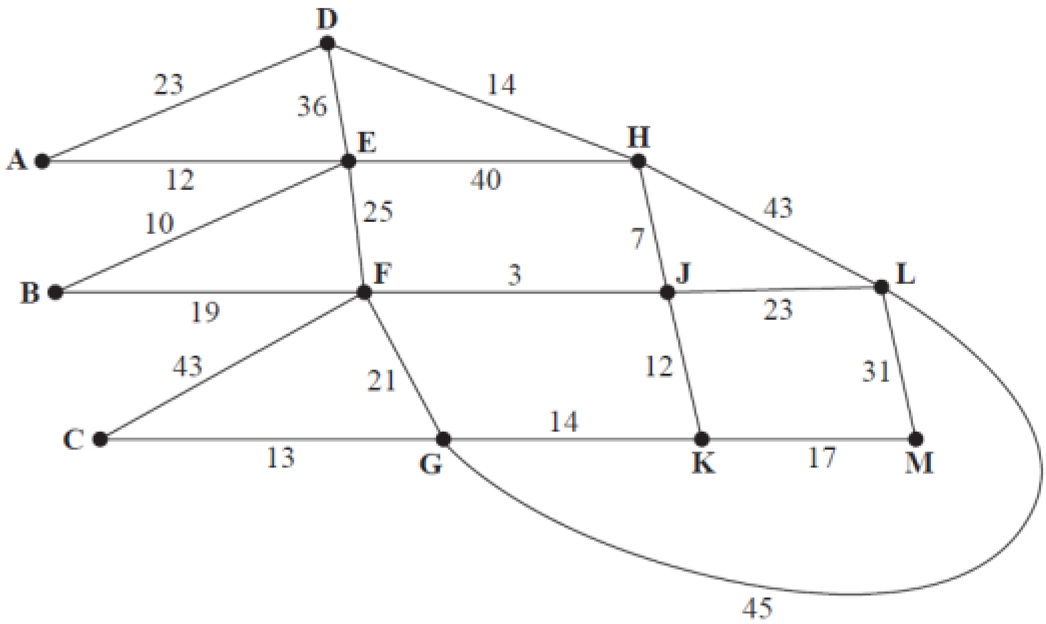


Figure 1

[The total weight of the network is 451]

Figure 1 models a network of tracks in a forest that need to be inspected by a park ranger. The number on each arc is the length, in km, of that section of the forest track.

Each track must be traversed at least once and the length of the inspection route must be minimised. The inspection route taken by the ranger must start and end at vertex A.

(a) Use the route inspection algorithm to find the length of a shortest inspection route. State the arcs that should be repeated. You should make your method and working clear.

(5)

(b) State the number of times that vertex J would appear in the inspection route.

(1)

The landowner decides to build two huts, one hut at vertex K and the other hut at a different vertex. In future, the ranger will be able to start his inspection route at one hut and finish at the other. The inspection route must still traverse each track at least once.

(c) Determine where the other hut should be built so that the length of the route is minimised. You must give reasons for your answer and state a possible route and its length.

(4)

(Total 10 marks)

Mark scheme for Question 2

Examiner comment

3.

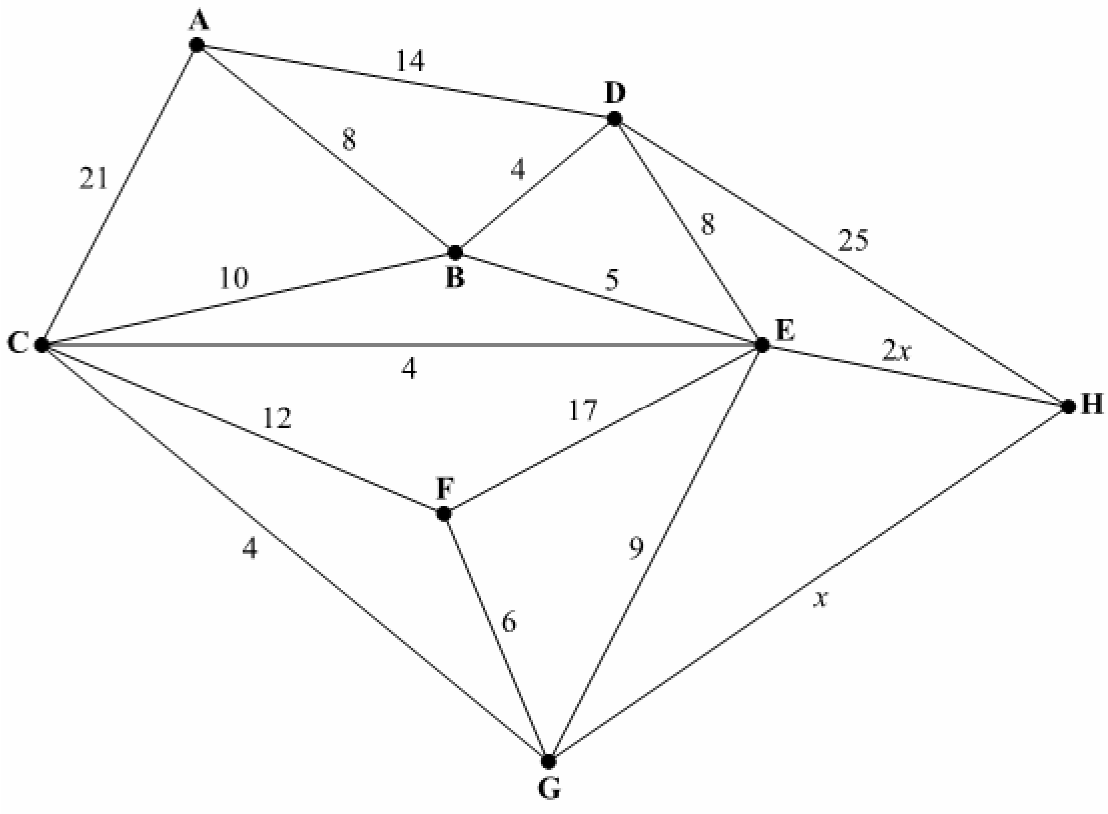


Figure 2

Figure 2 represents a network of roads. The number on each arc represents the time taken, in minutes, to drive along the corresponding road.

Stieg wishes to minimise the time spent driving from his home at A, to his office at H. The amount of traffic on two of the roads leading into H varies each day, and so the length of time taken to drive along these roads is expressed in terms of x, where $x > 7$.

(a) Use Dijkstra's algorithm to find the possible routes that minimise the driving time from A to H. State the length of each route, leaving your answer in terms of x where necessary.

(7)

On a particular day, the quickest route from A to H via G is 2 minutes quicker than the quickest route from A to H via E.

(b) Calculate the value of x. You must make your method and working clear.

(2)

(Total 9 marks)

Mark scheme for Question 3

Examiner comment

4.

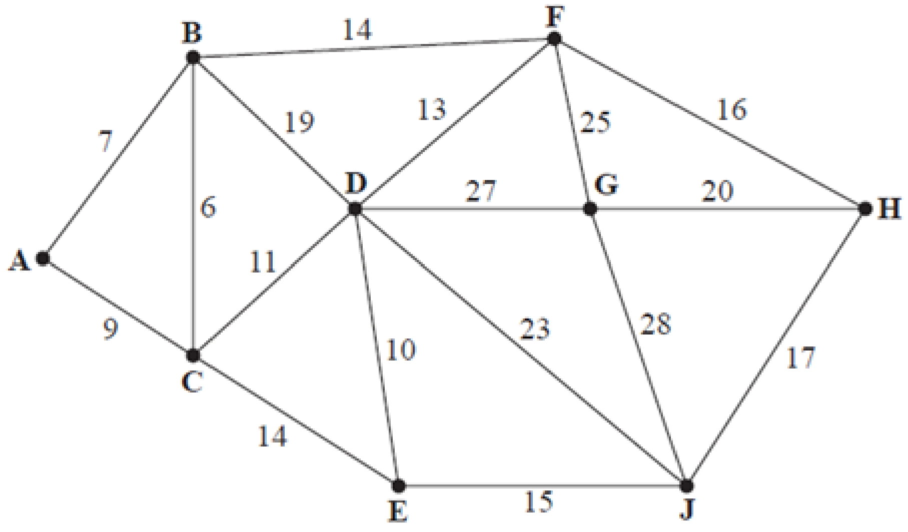


Figure 3

The numbers on the 17 arcs in the network shown in Figure 3 represent the distances, in km, between nine nodes, **A**, **B**, **C**, **D**, **E**, **F**, **G**, **H** and **J**.

(a) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(b) Starting at **G**, use Prim's algorithm to find a minimum spanning tree. You must clearly state the order in which you select the arcs of your tree.

(3)

(c) Find the weight of the minimum spanning tree.

(1)

A connected graph V has n nodes. The sum of the degrees of all the nodes in V is m. The graph T is a minimum spanning tree of V.

(d) (i) Write down, in terms of m, the number of arcs in V.

(ii) Write down, in terms of n, the number of arcs in T.

(iii) Hence, write down an inequality, in terms of m and n, comparing the number of arcs in T and V.

(3)

(Total 10 marks)

Mark scheme for Question 4

Examiner comment

5.

24 14 8 x 19 25 6 17 9

The numbers in the list represent the exact weights, in kilograms, of 9 suitcases. One suitcase is weighed inaccurately and the only information known about the unknown weight, x kg, of this suitcase is that $19 < x \leq 23$. The suitcases are to be transported in containers that can hold a maximum of 50 kilograms.

(a) Use the first-fit bin packing algorithm, on the list provided, to allocate the suitcases to containers.

(3)

(b) Using the list provided, carry out a quick sort to produce a list of the weights in **descending** order. Show the result of each pass and identify your pivots clearly.

(4)

(c) Apply the first-fit decreasing bin packing algorithm to the ordered list to determine the 2 possible allocations of suitcases to containers.

(4)

After the first-fit decreasing bin packing algorithm has been applied to the ordered list, one of the containers is full.

(d) Calculate the possible integer values of x. You must show your working.

(2)

(Total 13 marks)

Mark scheme for Question 5

Examiner comment

6.

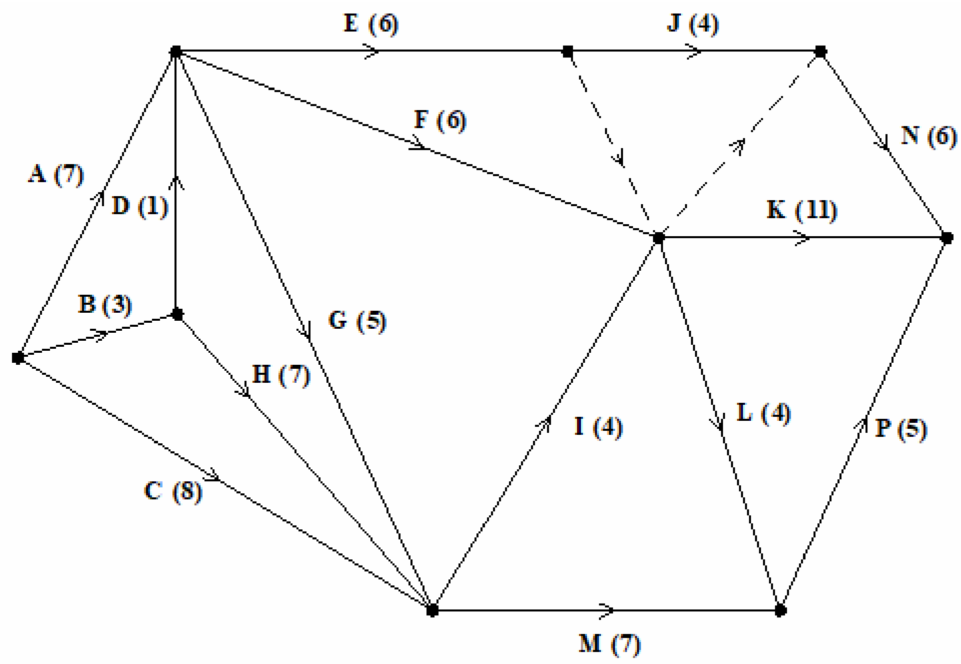


Figure 5

A project is modelled by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the corresponding activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and late event times.
(4)

(b) Explain what is meant by a critical path.
(2)

(c) List the critical path for this network.
(1)

(d) For each of the situations below, state the effect that the delay would have on the project completion date.

(i) A 4-day delay during activity J.
(ii) A 4-day delay during activity M.
(2)

The delays mentioned in (*d*) do not occur.

(e) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.
(2)

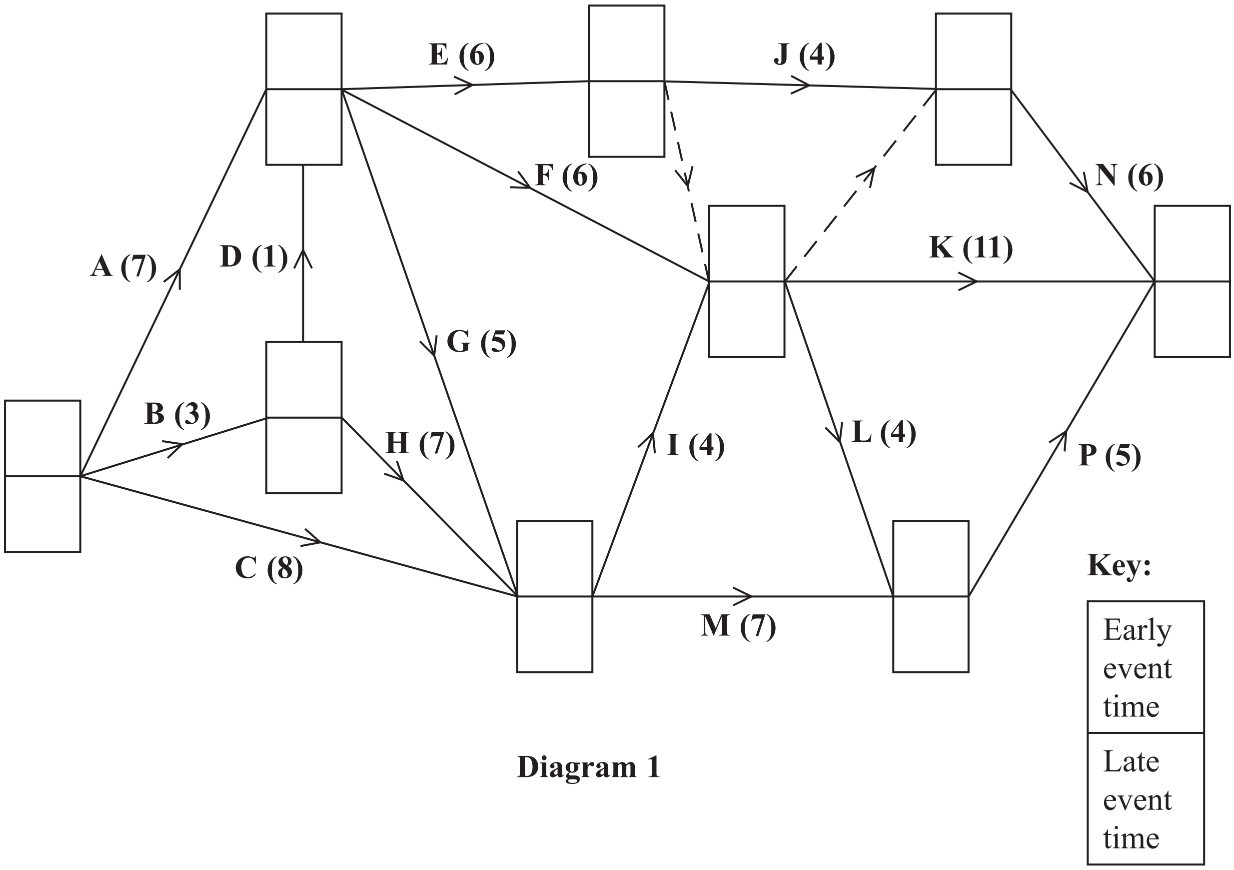
(f) Schedule the activities using the minimum number of workers so that the project is completed in the minimum time.
(3)

(Total 14 marks)

Mark scheme for Question 6

Examiner comment

6. (a)



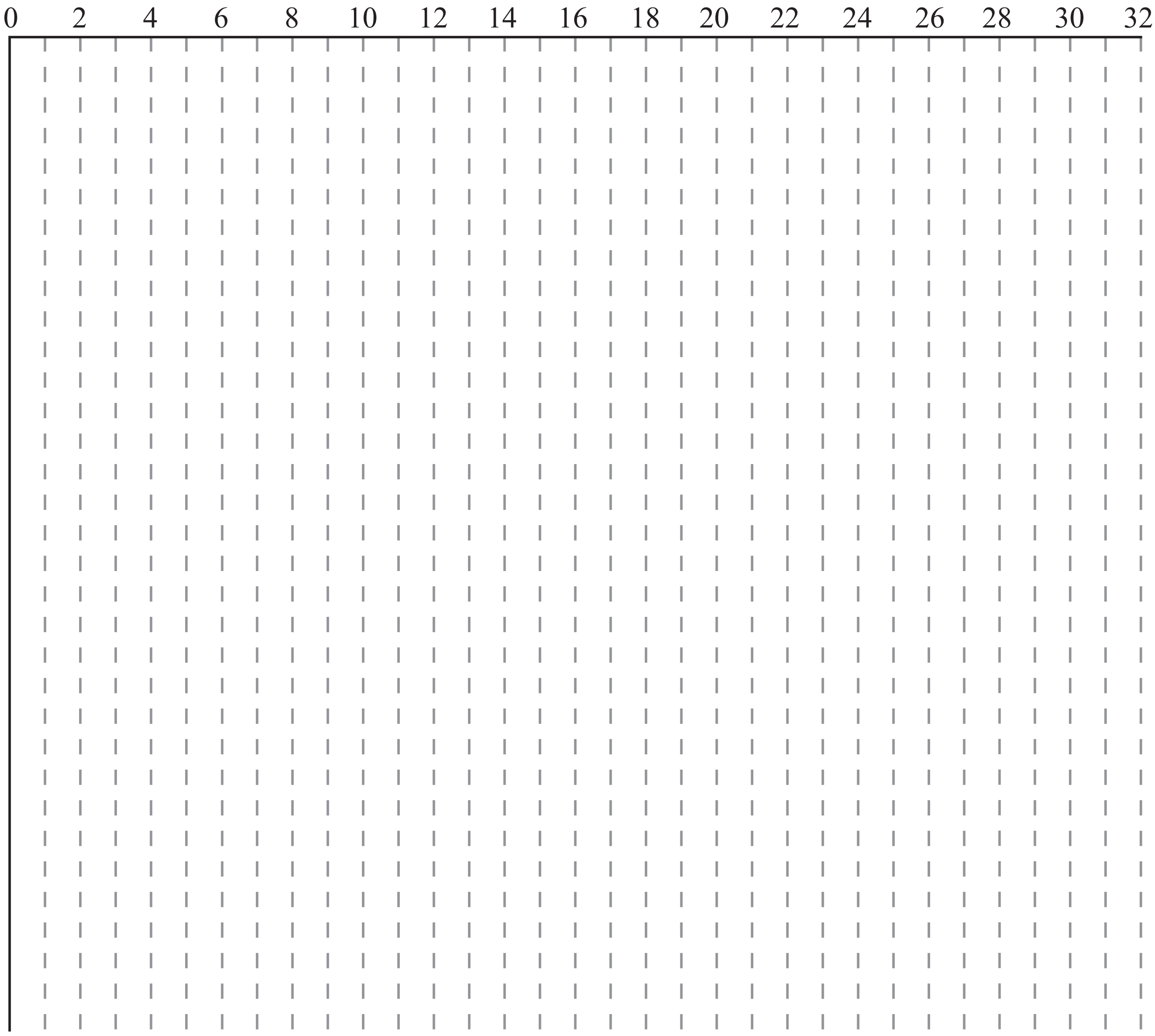
A (7)
B (3)
C (8)
D (1)
E (6)
F (6)
G (5)
H (7)
I (4)
J (4)
K (11)
L (4)
M (7)
N (6)
P (5)
Key:
Early event time
Late event time

Diagram 1

Leave blank

Question 6 continued

(f)



Q7

(Total 14 marks)

7. Charlie needs to buy storage containers.

There are two different types of storage container available, standard and deluxe.

Standard containers cost £20 and deluxe containers cost £65. Let x be the number of standard containers and y be the number of deluxe containers.

The maximum budget available is £520.

(a) Write down an inequality, in terms of x and y, to model this constraint.

(1)

Three further constraints are:

$x \geq 2$

$-x + 24y \geq 24$

$7x + 8y \leq 112$

(b) Add lines and shading to Diagram 1 in the answer book to represent all four constraints. Hence determine the feasible region and label it R.

(4)

The capacity of a deluxe container is 50% greater than the capacity of a standard container. Charlie wishes to maximise the total capacity.

(c) State an objective function, in terms of x and y.

(1)

(d) Use the objective line method to find the optimal vertex, V, of the feasible region. You must make your objective line clear and label the optimal vertex V.

(3)

(e) Calculate the exact coordinates of vertex V.

(2)

(f) Determine the number of each type of container that Charlie should buy. You must make your method clear and calculate the cost of purchasing the storage containers.

(3)

(Total 14 marks)

[Mark scheme for Question 7](#)

[Examiner comment](#)

TOTAL FOR PAPER: 75 MARKS

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7.

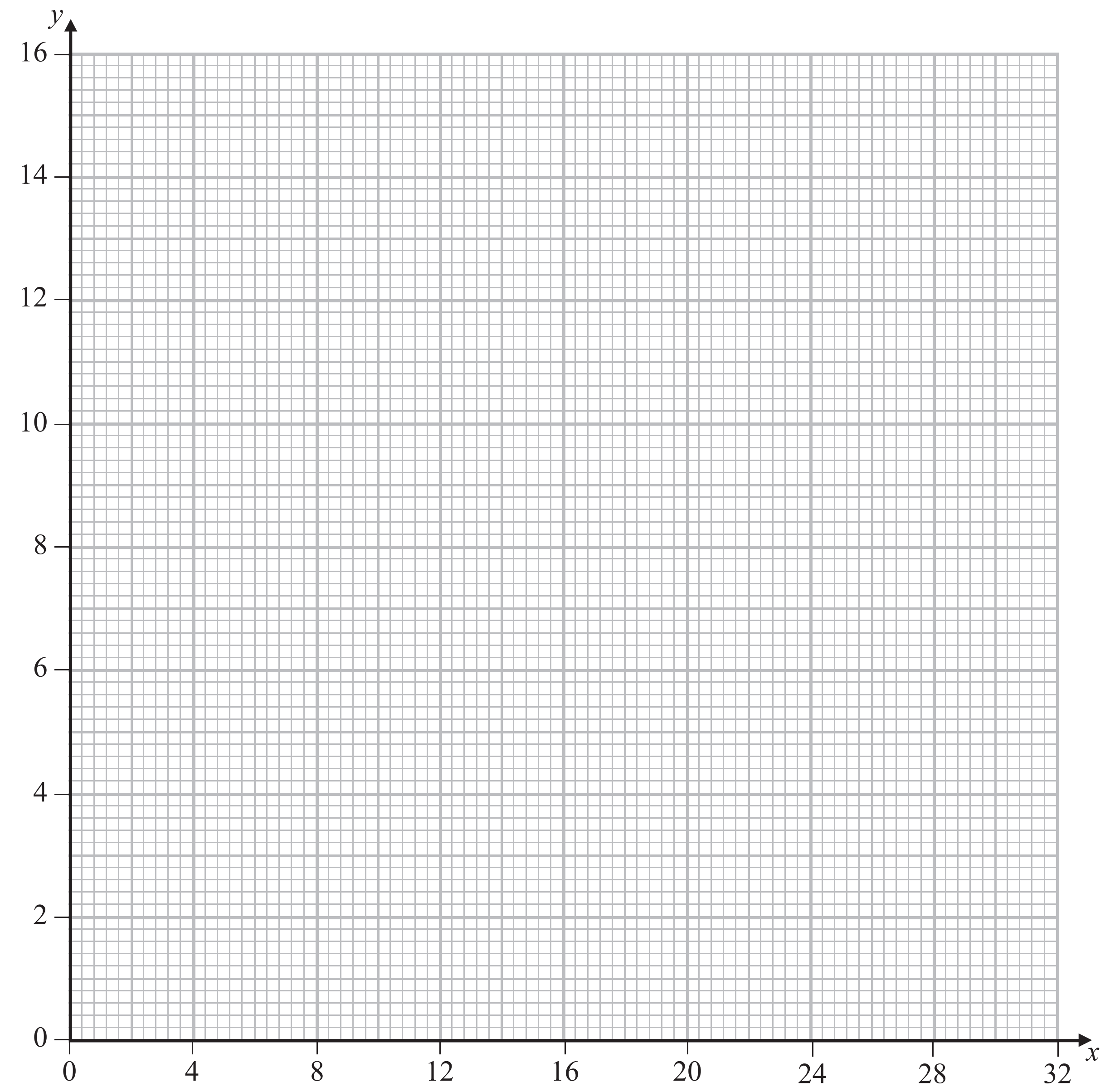
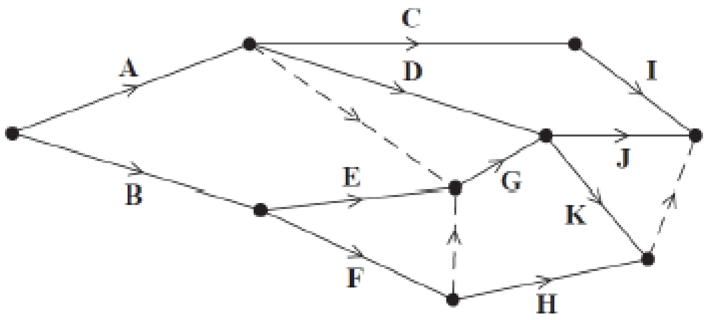


Diagram 1

A level Further Mathematics – Decision Mathematics 1 – Practice Paper 02 – Mark scheme –

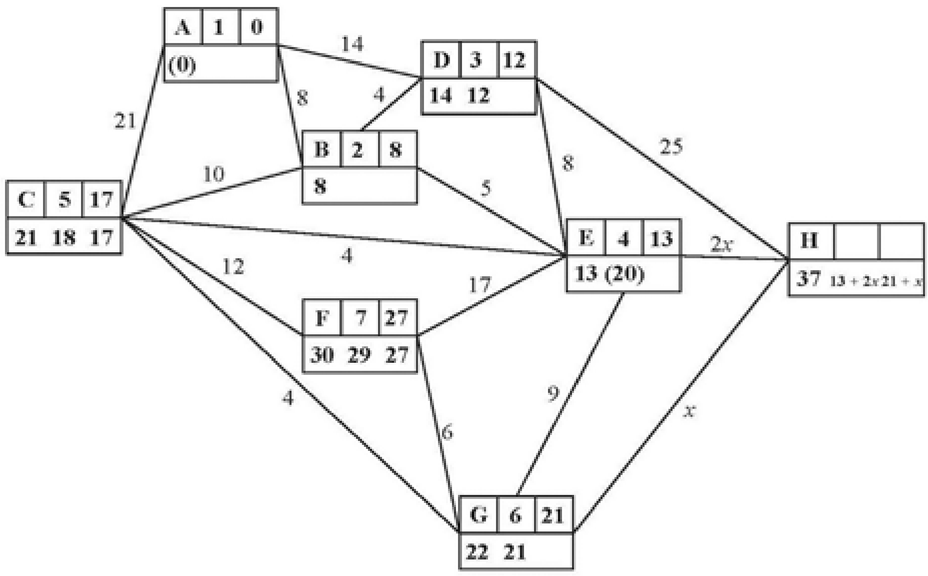
Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

Question	Scheme	Marks
1(a)		
	7 activities, 1 start and 2 dummies	M1
	ABCDEF	A1
	GH + first two dummies	A1
	IJK	A1
	cso	A1
		(5)
		(5 marks)

Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2(a)	D(A)E + F(J)K = 35 + 15 = 50*	**M1**
	D(HJ)F + E(FJ)K = 24 + 40 = 64	**A1**
	D(HJ)K + EF = 33 + 25 = 58	**A1**
	Arcs DA, AE, FJ, JK will be traversed twice	**A1**
	Route length = 451 + 50 = 501 (km)	**A1ft**
		(5)
(b)	Vertex J would appear 3 times in the shortest inspection route	**B1**
		(1)
(c)	We only have to repeat one pair of odd vertices which does not include vertex K (DE = 35, DF = 24, EF = 25)	**dM1**
	DF is the smallest of the three so repeat DF (DH, HJ, JF) and therefore the other hut should be built at E	**A1**
	Route e.g. EADEHDHJFBEFCGFJHLGKJLMK	**A1**
	The length of the route is 475 (km)	**A1ft**
		(4)
		(10 marks)

Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>3(a)</td><td>
</td><td>M1
A1 (ABDE)
A1 (CG)
A1ft</td></tr>
<tr><td></td><td>Shortest path from A to H via D: ABDH length: 37</td><td>A1</td></tr>
<tr><td></td><td>Shortest path from A to H via G: ABECGH length: $21 + x$</td><td>A1</td></tr>
<tr><td></td><td>Shortest path from A to H via E: ABEH length: $13 + 2x$</td><td>A1</td></tr>
<tr><td></td><td></td><td>(7)</td></tr>
<tr><td>(b)</td><td>$21 + x + 2 = 13 + 2x$</td><td>M1</td></tr>
<tr><td></td><td>$x = 10$</td><td>A1</td></tr>
<tr><td></td><td></td><td>(2)</td></tr>
<tr><td></td><td></td><td>(9 marks)</td></tr>
</table>

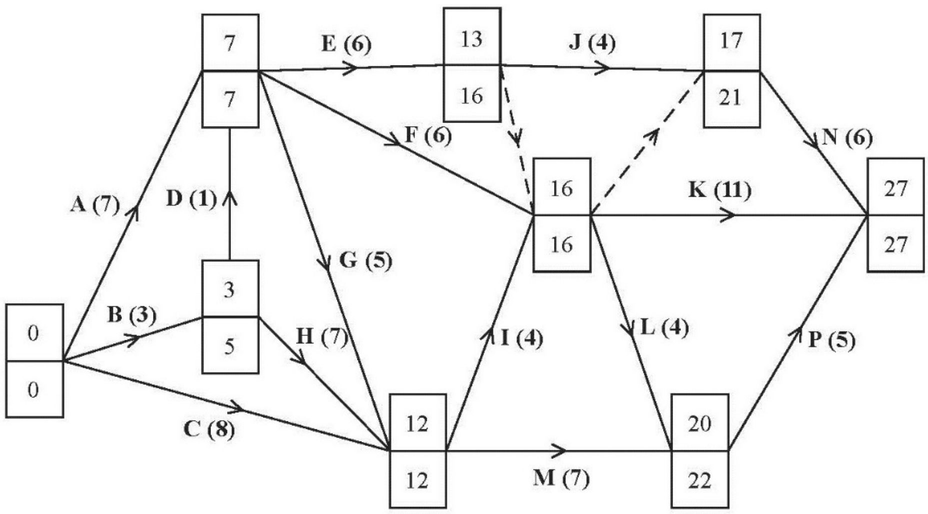
Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

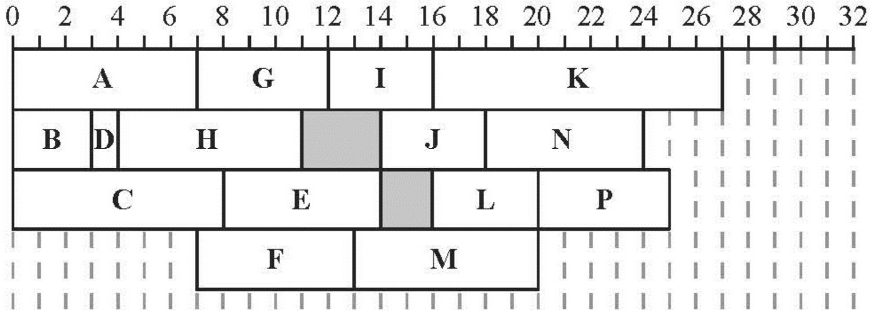
Question	Scheme	Marks
4(a)	Kruskal: BC, AB, (not AC), DE, CD, DF, (not $\begin{matrix}\text{BF}\\\text{CE}\end{matrix}$), EJ, FH, (not HJ), (not BD), GH	M1 A1 A1
		(3)
(b)	Prim: GH, FH, DF, DE; CD, BC; AB, EJ	M1 A1 A1
		(3)
(c)	98 (km)	B1
		(1)
(d)(i)	$\frac{m}{2}$	B1
(ii)	$n - 1$	B1
(iii)	m $\geqslant 2(n-1)$ (oe)	B1
		(3)
		(10 marks)

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

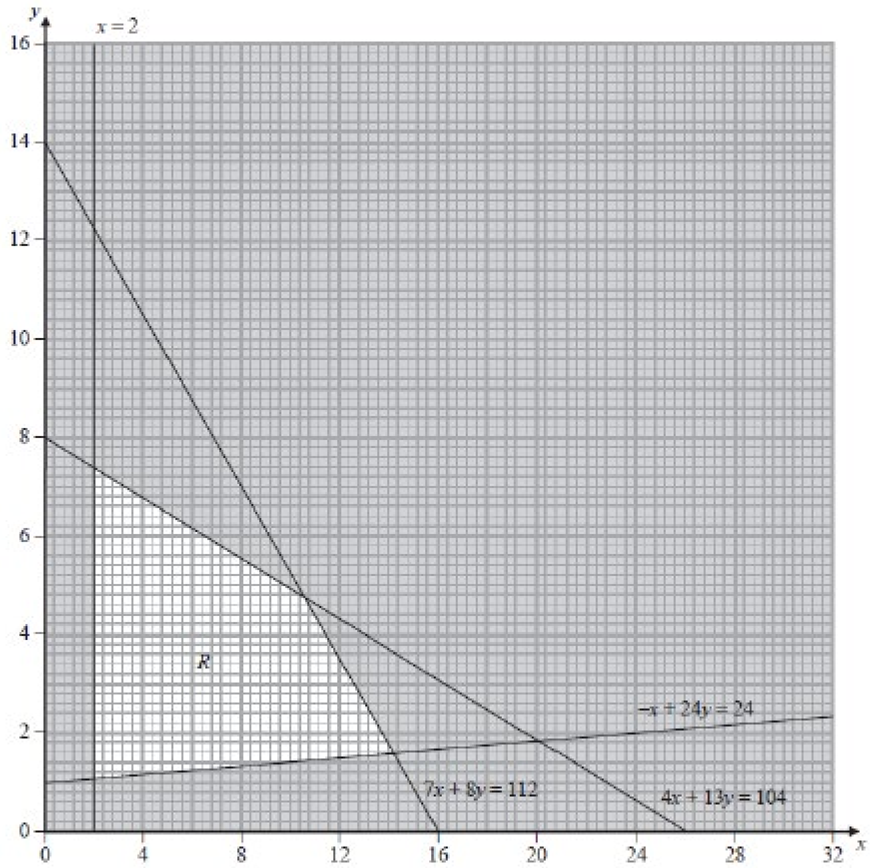
Question	Scheme	Marks
5(a)	Bin 1: 24 14 8 Bin 2: x 19 6 Bin 3: 25 17 Bin 4: 9	M1 A1 A1
		(3)
(b)	e.g. using middle right 24 14 8 x 19 25 6 17 9 pivot 19 24 x 25 19 14 8 6 17 9 pivots x 6 24 25 x 19 14 8 17 9 6 pivots 25 17 25 24 x 19 17 14 8 9 6 pivots (24) 8 25 24 x 19 17 14 9 8 6 pivot 9 25 24 x 19 17 14 9 8 6 (sort complete)	M1 A1(1st pass/pivots for 2nd) A1ft(2nd /3rd pass/pivots for 2nd) A1cso
		(4)
(c)	(i) Bin 1: 25 24 Bin 2: x 19 9 Bin 3: 17 14 8 6 (ii) Bin 1: 25 24 Bin 2: x 19 8 Bin 3: 17 14 9 6	M1 A1 A1 A1
		(4)
(d)	$x + 19 + 9 = 50 \Rightarrow x = 22$ $x + 19 + 8 = 50 \Rightarrow x = 23$	B2,1,0
		(2)
		(13 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(a)		M1 A1 M1 A1
		(4)
(b)	Critical:- e.g. zero float, no delay, immediate, if delayed project will finish late, etc.	B1
	Path:- e.g. from start event to end event continuously, the event forming end of one activity forms the start of the next, etc.	B1
		(2)
(c)	Critical path: AGIK	A1ft
		(1)
d(i)	4 day delay on J – no effect on the project completion date (float on J is 4)	B1
(ii)	4 day delay on M – project finishes 1 day late (float on M is 3)	B1
		(2)
(e)	$\frac{84}{27} = 3.111...$, so a minimum of 4 workers needed	M1A1
		(2)

Question	Scheme	Marks
6(f)		M1 A1 A1
		(3)
		(14 marks)

Mark scheme for Question 7 (Examiner comment) (Return to Question 7)

Question	Scheme	Marks
7(a)	$20x + 65y \leqslant 520$ or $4x + 13y \leqslant 104$ oe	B1
		(1)
(b)		B1 B1 B1 dB1(R)
		(4)
(c)	e.g . (P =) $2x + 3y$	B1
		(1)
(d)	Drawing an objective line accept reciprocal gradient	M1
	Correct objective line minimum length equivalent to (0, 1) to (1.5, 0)	A1
	V correctly labelled	A1
		(3)
(e)	$V\left(\frac{624}{59}, \frac{280}{59}\right)$	M1A1
		(2)

Question	Scheme	Marks
7(f)	Testing integer solutions around V, $x = 11$ and $y = 4$ is optimal integer solution, so they should buy 11 standard containers and 4 deluxe containers	**M1A1**
	Cost is (£) 480	**B1**
		(3)
		(14 marks)

A level Further Mathematics – Decision 1 – Practice Paper 02 – Examiner report –

Examiner comment for Question 1 **(Mark scheme)** **(Return to Question 1)**

1. Candidates generally showed a good understanding of the process of constructing an activity network from a precedence table, using arcs drawn with arrows and labelled for activities. Some responses lacked a sink node at the end and a small number did not have a single source node. Some of the diagrams and labels were challenging to read, especially when they were very small and/or drawn with lines that crossed over. Some candidates were unsure about the placement of their dummies, putting them in 'anywhere' so that they had three dummies included. Some also had four or more dummies even though the question clearly stated that they were to use exactly three. A very small number of candidates put activity on node, and some failed to check that they had all activities present, with activity K being the activity that was missing most often.

Examiner comment for Question 2 **(Mark scheme)** **(Return to Question 2)**

2. This question proved to be a good source of marks for many candidates with 49.8% of candidates scoring 6 marks or more and only 23.1% scoring 3 marks or fewer.

   Part (a) was generally answered well by most candidates with the vast majority stating the correct three distinct pairings of the correct four odd nodes. There were a few candidates who only gave two pairings of the four odd nodes or who gave several pairings but not three distinct pairings. There were however many instances where the totals were incorrect. The majority of such mistakes occurred for the pairing of D with F. There were also some instances where no totals were given which lost candidates a significant number of marks. Candidates should be advised to be thorough when checking the shortest route between each odd pairing. Many candidates did not explicitly state the arcs that should be repeated instead stating that DE and FK should be repeated instead of the correct arcs DA, AE, FJ, JK. Furthermore, a number of candidates did not state the length of a shortest inspection route. Only a minority of candidates were able to answer part (b) correctly with the majority stating that J would appear 6 times (6 was the order of vertex J once the two additional arcs were added) rather than the correct answer of 3 times. In part (c) many candidates identified DE, DF and EF as the paths that needed to be considered, although they often missed stating the fact that DF was the shortest path that did not including vertex K. Many candidates, even with the correct selection of arcs, either did not state a route or gave an incorrect route. Some candidates misunderstood the reasoning altogether and focused on the fact that DK had the greatest weight of the previous pairings and therefore should be avoided and so EF should be repeated. Others still said that 'FK is the least therefore start and end at F and K'. Those that did state DF usually went on to score the mark for stating the length of a shortest inspection route.

Examiner comment for Question 3 (Mark scheme) (Return to Question 3)

3. Part (a) was usually well done with most candidates applying Dijkstra's algorithm correctly. The boxes at each node in part (a) were usually completed correctly. When errors were made it was either an order of labelling error (some candidates repeated the same labelling at two different nodes) or working values were either missing, not in the correct order or simply incorrect (usually these errors occurred at nodes C, D, F, and/or H). The most common errors were seen at vertex H (due to two of the arcs leading into H having weights in terms of x) with many candidates not having the three correct working values of 37 $13+2x$ and $21+x$. This question also highlighted, yet again, the need for candidates to read the question carefully as almost 10% of candidates started the application of the algorithm from vertex C rather the correct vertex A. Furthermore, the question explicitly asked for candidates to: *Use Dijkstra's algorithm to find the possible routes that minimise the driving time from A to H. State the length of each route, leaving your answer in terms of x where necessary* – very few stated all three routes and their corresponding lengths.

The most common incorrect response in part (b) was to have $21+x=13+2x+2$ which gives $x=6$ which scored no marks as the question explicitly stated that $x>7$.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. Most candidates applied Kruskal's algorithm correctly in part (a), but some did not demonstrate the correct handling of rejected arcs, which is essential in this algorithm. Candidates would be advised to list all the arcs (from the network) in ascending order and then state 'add' or 'reject' next to each arc (or some other clear indication of which arcs are being included/not included in the MST). Some candidates lost the final mark by omitting one or more rejected arcs (usually BD) while a small minority scored no marks in this part as they failed to record any rejections.

Part (b) was generally well answered with the majority of candidates applying Prim's algorithm correctly starting from vertex G. A few candidates attempted to construct a table to perform Prim, clearly believing that this algorithm can only be performed when expressed in matrix form. Finally, there is still a small minority of candidates who appear to be rejecting arcs when applying Prim's algorithm so scoring only one of the three possible marks in this part.

Most candidates could correctly work out the weight of the minimum spanning tree in part (c), with mistakes at this point usually following on from earlier mistakes when applying either Prim's and/or Kruskal's algorithm. Some candidates did not seem to be aware that the weight of the minimum spanning tree should be the same irrespective of the algorithm used. This should have provided evidence to the candidate that either part (a) and/or part (b) was incorrect if two different weights/trees were found in the first two parts.

Part (d) was often not attempted, and those that did usually failed to gain much credit. Very few candidates in part (i) showed an understanding that the sum of the order of nodes is equal to twice the number of arcs (giving an answer of $m/2$). Part (ii) was handled better as more candidates appreciated that the number of arcs in a minimum spanning tree is one less than the number of nodes (therefore giving an answer of $n-1$). When parts (i) and (ii) were answered correctly it was common to see a strict inequality incorrectly used to link these two answers in part (iii).

Examiner comment for Question 5 (Mark scheme) (Return to Question 5)

5. This question discriminated well leading to a good spread of marks. The modal mark was 11, 19.2% scored full marks and 25% scored 7 marks or fewer.

The inclusion of x as an unknown value threw a significant number of candidates but the larger percentage took it in their stride. A few chose a value for x at the beginning (usually 23) and then forgot it was a variable, using the same value throughout. One or two left out x completely or felt it needed to be isolated in a bin of its own.

Part (a) was answered well with many correct responses seen. A common error was to put the 6 in the wrong bin. Only a few candidates just worked with x as a chosen value within the range at this stage.

In part (b) most candidates' selected middle-right pivots and many were able to carry out the sort correctly. Errors cropped up in the ordering of the sublists after the first (and subsequent) iterations. The most common occurrence of this tended to be that x and 25 were interchanged after the first pass. Other errors included failing to select the 9 as a pivot for the fifth pass probably because the sublist of length two after the fourth pass appeared to already be in order. Very occasionally, candidates selected only one pivot for each iteration or failed to sort the list into (values greater than the pivot), (the pivot), (values less than the pivot) after the first iteration. There were only a few cases where candidates selected the first or last items as the pivot. Pivots were usually chosen consistently although the spacing and notation on some solutions made these difficult for examiners to follow. Some candidates over complicated the process by insisting on using a different 'symbol' to indicate the pivots for each pass. Those candidates who sorted into ascending order usually remembered to reverse their list at the end to gain full credit although a number of candidates left their list in ascending order and then went on to apply first-fit increasing in part (c).

Part (c) challenged many candidates with most only stating one correct allocation. Often candidates did not even seem to consider a second possible allocation despite the question clearly saying there were two. Some candidates did not know how to handle the value of x in this part and used a value instead. A common error was omitting the values of 8 or 9 from bin 2 and putting in the value of 6 instead.

Many candidates could only score one mark in part (d) due to the lack of two correct allocations stated in part (c). Of those that did have two correct allocations and both answers for x, most also supported their answers with relevant calculations, and so were able to gain full credit.

Examiner comment for Question 6 (Mark scheme) (Return to Question 6)

6. Parts (a), (c) and (e) were generally well answered with many candidates scoring full marks in all three parts. The most common error in part (a) was to have an incorrect value at the end of either activity E or J, and in part (e) the most common error was a failure to correctly sum the length of all fifteen activities. Part (b) was extremely challenging with very few candidates scoring any marks in this part. It was expected that candidates would realise that one of the marks in this part was for explaining what was meant by ‘critical’ and the other mark was for explaining the word ‘path’. The delay on activities J and M were well understood and a lot of correct answers were given in part (d). Some candidates’ responses were more succinct than others and a significant number of candidates described the effect these delays would have on the activity which followed, or on the float time and did not, as requested, comment on the effect on the project completion date. For part (f) quite a few candidates drew a Gantt chart instead of a scheduling diagram, and so scored no marks. There were also quite a few instances of this part being left blank. For candidates who knew what a scheduling diagram was this part was generally well answered, although candidates should be reminded to check that they include all the activities in the network and that the activities have the correct lengths. In this part many candidates did not include all fifteen activities, or they scheduled using five (or three) workers rather than the correct four. Some candidates made a good attempt but failed to fully check the precedencies for each activity. There were some good solutions seen to this part, with a number of different but valid solutions seen.

Examiner comment for Question 7 (Mark scheme) (Return to Question 7)

7. The majority of candidates answered part (a) correctly. A very small number used the wrong inequality sign or wrote $x + y \leqslant 520$.

Most candidates were able to draw the required lines correctly in part (b) although some were unable to draw lines sufficiently accurately (some drew lines without a ruler) or sufficiently long enough. The following general principle should always be adopted by candidates:

- lines should always be drawn which cover the entire graph paper supplied in the answer book and therefore,
- lines with negative gradient should always be drawn from axis to axis.

The rationale behind this is that until all the lines are drawn (and shaded accordingly) it is unclear which lines (or parts of lines) will define the boundary of the feasible region. If candidates only draw the line segments that they believe define the boundary of the feasible region then examiners are unaware of the order in which the lines were drawn and therefore it is unclear to examiners why some parts of the lines have been omitted. In general, the lines $x = 2$, $7x + 8y = 112$ and $20x + 65y = 520$ were correctly drawn. The most common error when drawing the line $-x + 24y = 24$ was drawing this line with a negative rather than a positive gradient.

In part (c), only a minority of candidates were able to state a correct objective function. Some transposed the coefficients, and many had the ratio of coefficients as 1: 2. A common error was $P = x + 2y$ but other functions stated were $3y = 2x$, $P = 2x + y$ and $y = 2x$.

In part (d), the majority of candidates drew the correct objective line, however, a line with reciprocal gradient was often seen or, in a number of cases, no objective line was drawn (and therefore no marks could be awarded in this part). Some used obscure constant values to plot the objective line.

In part (e), some candidates gave an estimate of the optimal vertex using a reading from their graph, rather than solving the relevant equations simultaneously.

Only a few candidates attempted part (f). Few candidates tested at least two valid integer points, either in a correct pair of constraint inequalities or in a correct objective function. Simply stating "in region" or "not in region" therefore gaining no marks. Some candidates just stated one pair of integer coordinates despite being asked to "make your method clear". The final mark was independent of all others, so a handful of candidates scored the mark for correctly stating the cost as £480.