Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

A level Further Mathematics

Decision Mathematics 1
Practice Paper 4

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **6** questions in this question paper. The total mark for this paper is **75**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

30	11	21	53	50	39	16	4	60	43

The numbers in the list above represent the lengths, in cm, of some pieces of electrical wire. The wire is sold in one metre lengths.

(a) Use the first-fit bin packing algorithm to determine how these pieces could be cut from one metre lengths. You should ignore wastage due to cutting.

(3)

The list of numbers above is to be sorted into **ascending** order.

Starting at the left-hand end of the list, after three passes of the bubble sort, the list is

11	21	30	16	4	39	43	50	53	60

(b) (i) Write down the list that results at the end of the fourth pass.

(ii) Write down the number of comparisons and swaps performed during the fourth pass.

(3)

The **original** list of numbers is now to be sorted into **descending** order.

(c) Perform a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

(d) Use the first-fit decreasing bin packing algorithm to determine how these pieces could be cut from one metre lengths. You should ignore wastage due to cutting.

(3)

(Total 13 marks)

Mark scheme for Question 1

Examiner comment

2. (a) Explain the difference between the classical travelling salesperson problem and the practical travelling salesperson problem.

(2)

	A	B	C	D	E	F	G
A	–	31	15	12	24	17	22
B	31	–	20	25	14	25	50
C	15	20	–	16	24	19	21
D	12	25	16	–	21	32	17
E	24	14	24	21	–	28	41
F	17	25	19	32	28	–	25
G	22	50	21	17	41	25	–

The table above shows the least direct distances, in miles, between seven towns, A, B, C, D, E, F and G. Yiyi needs to visit each town, starting and finishing at A, and wishes to minimise the total distance she will travel.

(b) Show that there are two nearest neighbour routes that start from A. State these routes and their lengths.

(3)

(c) Starting by deleting A, and all of its arcs, find a lower bound for the length of Yiyi's route.

(3)

(d) Use your results to write down the smallest interval which you can be confident contains the optimal length of Yiyi's route.

(2)

(Total 10 marks)

[Mark scheme for Question 2](#)

[Examiner comment](#)

3. (a) In the context of the critical path analysis, define the term '*total float*'.

(2)

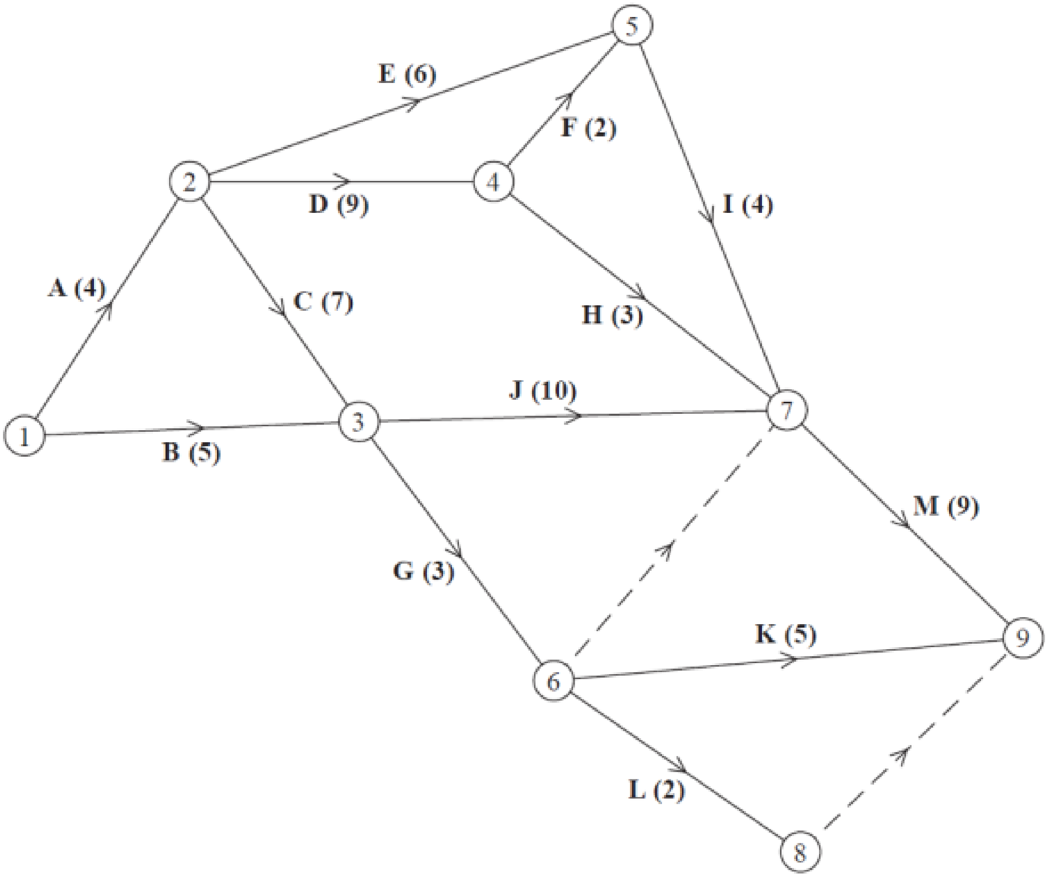


Figure 3

Figure 3 is the activity network for a building project. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(3)

(c) State the critical activities.

(1)

(d) Calculate the maximum number of days by which activity G could be delayed without affecting the shortest possible completion time of the project. You must make the numbers used in your calculation clear.

(2)

(e) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(2)

Question 3 continued

The project is to be completed in the minimum time using as few workers as possible.

(f) Schedule the activities using Grid 1 in the answer book.

(4)

(Total 14 marks)

Mark scheme for Question 3

Examiner comment

Leave blank

3. (a)

(b)

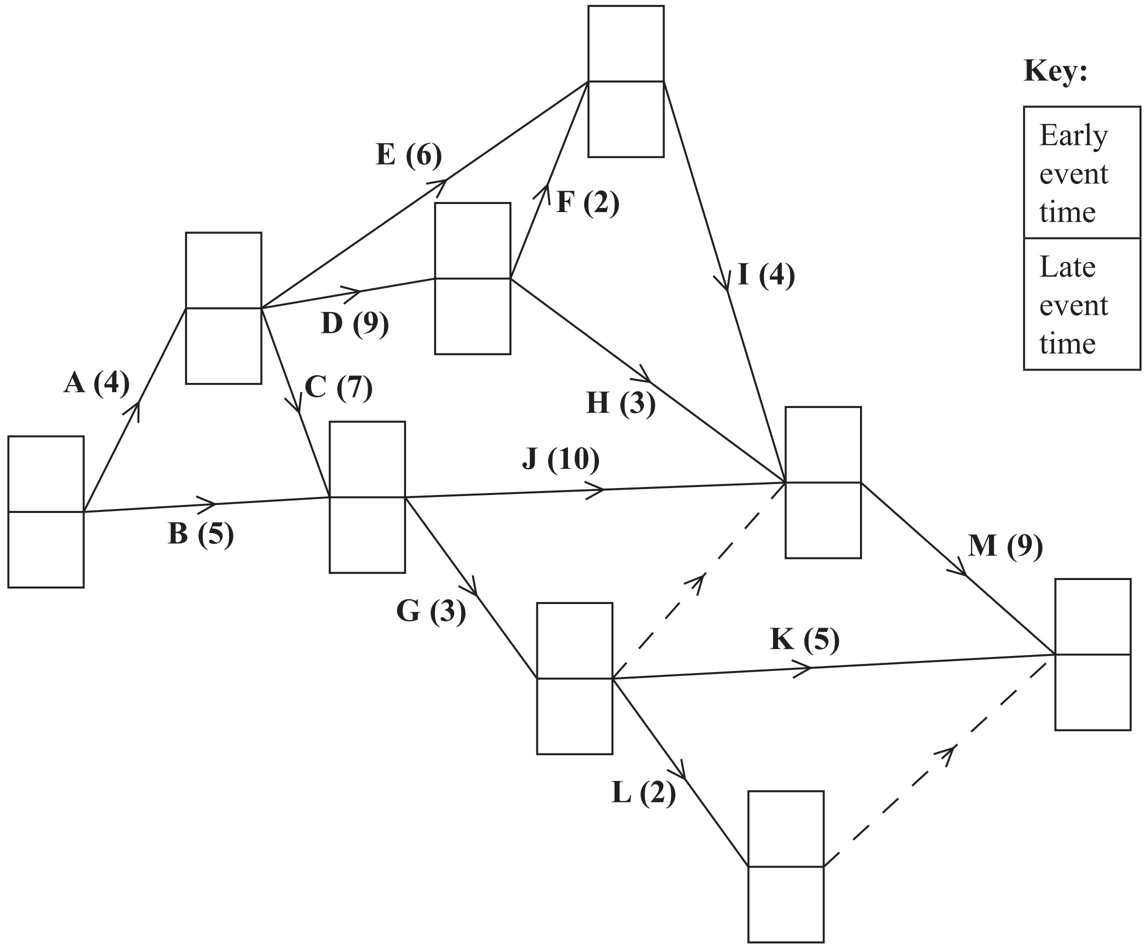


Diagram 1

Leave blank

Question 3 continued

(c) Critical activities ____________________

(d) ____________________

____

____

(e) ____________________

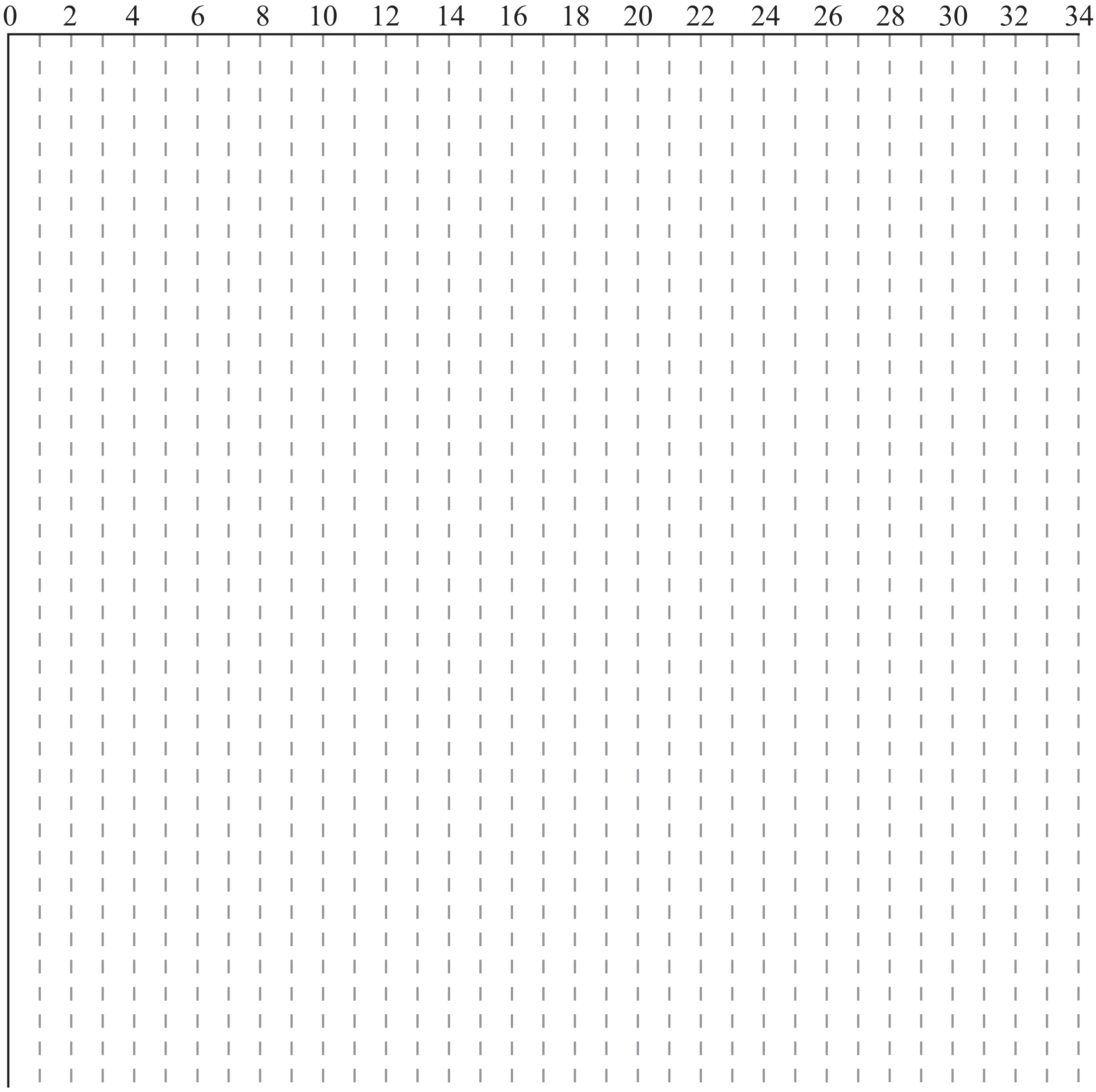
____

____

____

____

(f)



Grid 1

Q7

(Total 14 marks)

4. The tableau below is the initial tableau for a three-variable linear programming problem in x, y and z. The objective is to maximise the profit, P.

Basic variable	x	y	z	r	s	t	Value
r	15	–2	3	1	0	0	180
s	10	1	1	0	1	0	80
t	1	6	–2	0	0	1	100
P	–1	–2	–5	0	0	0	0

(a) Using the information in the tableau, write down

(i) the objective function,

(ii) the three constraints as inequalities.

(3)

(b) Taking the most negative number in the profit row to indicate the pivot column at each stage, solve this linear programming problem. Make your method clear by stating the row operations you use.

(8)

(c) State the final values of the objective function and each variable.

(2)

(Total 13 marks)

Mark scheme for Question 4

Examiner comment

5. A linear programming problem in x and y is described as follows.

Minimise $C = 2x + 3y$

subject to

$$x + y \geq 8$$
$$x < 8$$
$$4y \geq x$$
$$3y \leq 9 + 2x$$

(a) Add lines and shading to Diagram 1 in the answer book to represent these constraints.
(4)

(b) Hence determine the feasible region and label it R.
(1)

(c) Use the objective line (ruler) method to find the exact coordinates of the optimal vertex, V, of the feasible region. You must draw and label your objective line clearly.
(3)

(d) Calculate the corresponding value of C at V.
(1)

The objective is now to maximise $2x + 3y$, where x and y are integers.

(e) Write down the optimal values of x and y and the corresponding maximum value of $2x + 3y$.
(2)

A further constraint, $y \leq kx$, where k is a positive constant, is added to the linear programming problem.

(f) Determine the least value of k for which this additional constraint does not affect the feasible region.
(2)

(Total 13 marks)

[Mark scheme for Question 5]

[Examiner comment]

Leave blank

5.

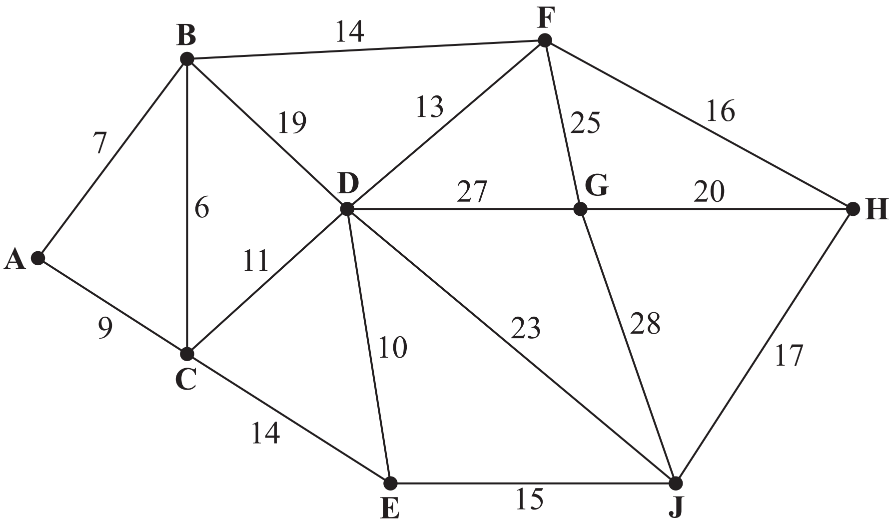


Figure 5

Question 5 continued

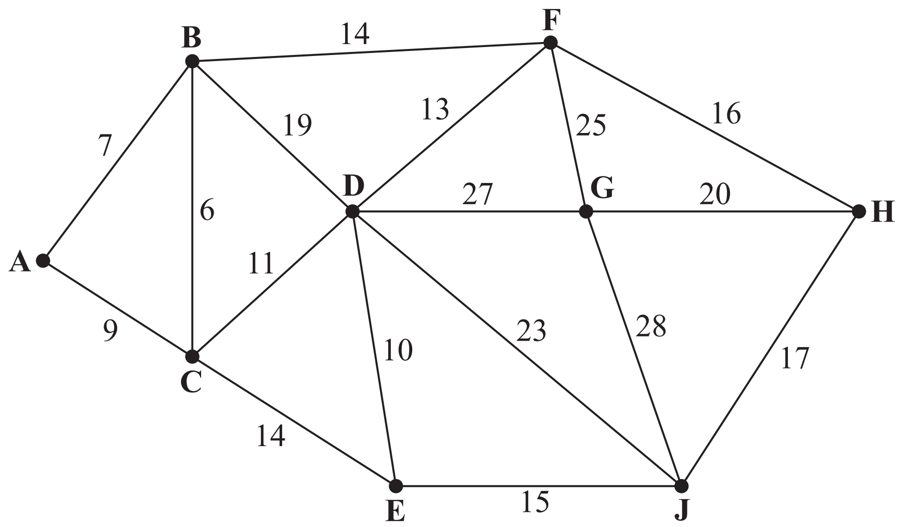


Figure 5

(Total 10 marks)

6.

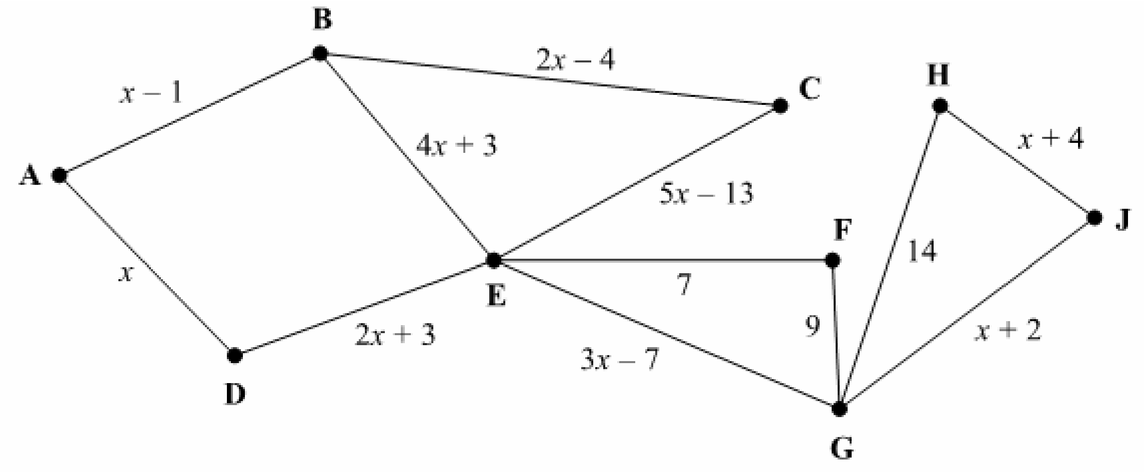


Figure 3

[*The weight of the network is* $20x + 17$]

(a) Explain why it is not possible to draw a network with an odd number of vertices of odd valency.

(2)

Figure 3 represents a network of 12 roads in a city. The expression on each arc gives the time, in minutes, to travel along the corresponding road.

(b) During rush hour one day $x = 9$

(i) Starting at A, use Prim's algorithm to find a minimum spanning tree. You must state the order in which you select the arcs of your tree.

(ii) Calculate the weight of the minimum spanning tree.

(4)

You are now given that $x > 3$

A route that minimises the total time taken to traverse each road at least once needs to be found. The route must start and finish at the same vertex.

The route inspection algorithm is applied to the network in Figure 3 and the time taken for the route is 162 minutes.

(c) Determine the value of x, showing your working clearly.

(6)

(Total 12 marks)

Mark scheme for Question 6

Examiner comment

TOTAL FOR PAPER: 75 MARKS

A level Further Mathematics – Decision Mathematics 1 –
Practice Paper 04 – Mark scheme –

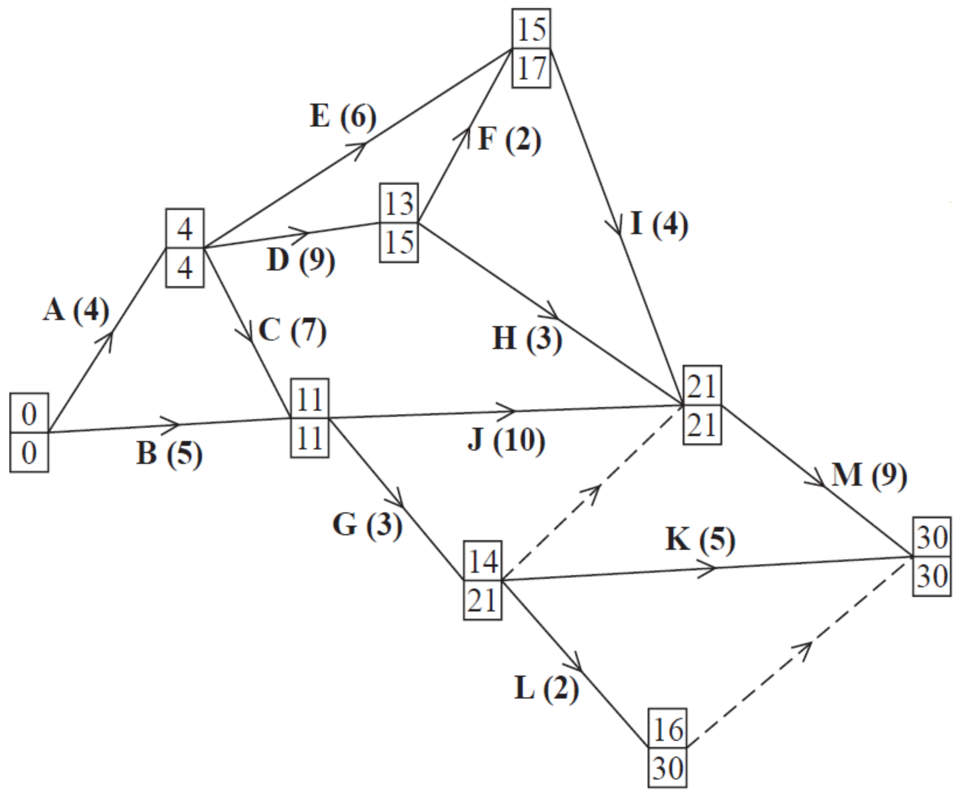
Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

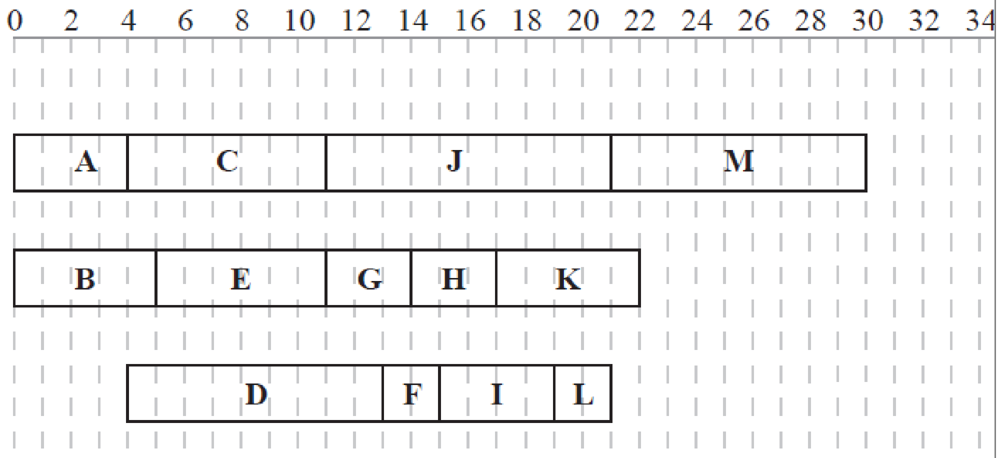
Question	Scheme	Marks
1(a)	Bin 1: 30 11 21 16 4 Bin 2: 53 39 Bin 3: 50 43 Bin 4: 60	M1 A1 M1
		(3)
(b)(i)	11 21 16 4 30 39 43 50 53 60	B1
(ii)	Comparisons: 6 Swaps: 2	B1B1
		(3)
(c)	e.g. middle right 30 11 21 53 50 39 16 4 60 43 53 50 60 43 39 30 11 21 16 4 60 53 50 43 39 30 21 11 16 4 60 53 50 43 39 30 21 16 11 4 60 53 50 43 39 30 21 16 11 4	M1 A1 A1ft A1
		(4)
(d)	Bin 1: 60 39 Bin 2: 53 43 4 Bin 3: 50 30 16 Bin 4: 21 11	M1 A1 M1
		(3)
		(13 marks)

Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2(a)	e.g. in the practical problem each vertex must be visited at least once. In the classical problem each vertex must be visited just once	**B2, 1, 0**
		(2)
(b)	A – D – C – F – B – E – G – A 12+16+19+25+14+41+22 = 149 A – D – C – F – G – E – B – A 12+16+19+25+41+14+31 = 158	**M1** **A1** **A1**
		(3)
(c)	RMST weight = 86 (miles)	**B1**
	86 + 12 + 15 = 113 (miles)	**M1A1**
		(3)
(d)	113 $\leq$ optimal distance $\leq$ 149	**B2, 1, 0**
		(2)
		(10 marks)

Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

Question	Scheme	Marks
3(a)	The total float $F(i,j)$ of activity (i,j) is defined to be $F(i,j) = l_j - e_i -$ duration (i,j), where e_i is the earliest time for event i and l_j is the latest time for event j **(see note below)**	**M1A1**
		(2)
(b)		**M1** **A1** **A1**
		(3)
(c)	Critical activities: A C J M	**B1**
		(1)
(d)	G can be delayed by 21 – 11 – 3 = 7 (days)	**M1A1**
		(2)
(e)	$\frac{69}{30} = 2.3$ so lower bound is 3 workers	**M1A1**
		(2)

Question	Scheme	Marks
3(f)		M1 A1 A1 A1
		(4)
		(14 marks)

Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

Question	Scheme	Marks
4(a)(i)	$P = x + 2y + 5z$	B1
(ii)	$15x - 2y + 3z \leqslant 180$ $10x + y + z \leqslant 80$ $x + 6y - 2z \leqslant 100$	M1A1
		(3)
(b)	(tables below)	M1A1 M1A1 M1 A1ft M1A1

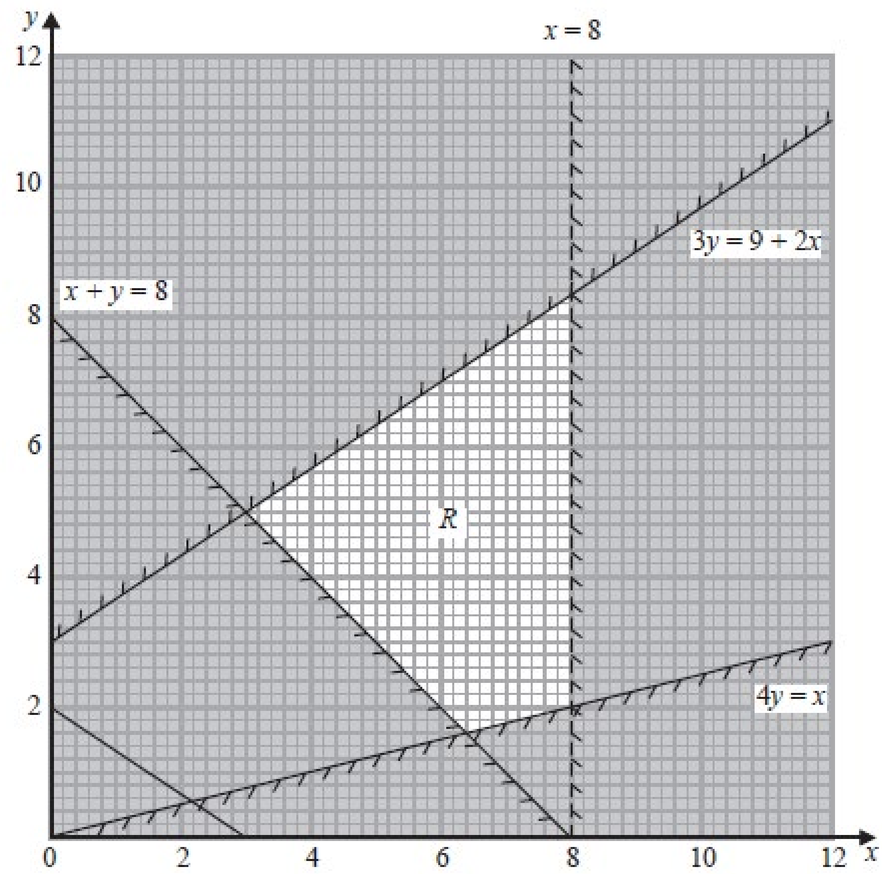
b.v	x	y	z	r	s	t	value
r	15	-2	3	1	0	0	180
s	10	1	1	0	1	0	80
t	1	6	-2	0	0	1	100
P	-1	-2	-5	0	0	0	0

b.v	x	y	z	r	s	t	value	row ops
z	5	$-\frac{2}{3}$	1	$\frac{1}{3}$	0	0	60	$R_1 \div 3$
s	5	$\frac{5}{3}$	0	$-\frac{1}{3}$	1	0	20	$R_2 - R_1$
t	11	$\frac{14}{3}$	0	$\frac{2}{3}$	0	1	220	$R_3 + 2R_1$
P	24	$-\frac{16}{3}$	0	$\frac{5}{3}$	0	0	300	$R_4 + 5R_1$

b.v	x	y	z	r	s	t	value	row ops
z	7	0	1	$\frac{1}{5}$	$\frac{2}{5}$	0	68	$R_1 + \frac{2}{3}R_2$
y	3	1	0	$-\frac{1}{5}$	$\frac{3}{5}$	0	12	$R_2 \div \frac{5}{3}$
t	-3	0	0	$\frac{8}{5}$	$-\frac{14}{5}$	1	164	$R_3 - \frac{14}{3}R_2$
P	40	0	0	$\frac{3}{5}$	$\frac{16}{5}$	0	364	$R_4 + \frac{16}{3}R_2$

Question	Scheme	Marks
		(8)
(c)	$P = 364$; $x = 0$; $y = 12$; $z = 68$; $r = s = 0$; $t = 164$	M1A1
		(2)
		(13 marks)

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme	Marks
5(a)		
	$(x+y=8)$	B1
	$(3y=9+2x)$	B1
	$(4y=x)$	B1
	$(x=8)$	B1
		(4)
(b)	Correct R labelled	B1
		(1)
(c)	Objective line drawn	B1
	$V\left(\frac{32}{5},\frac{8}{5}\right)$ (oe)	M1 dA1
		(3)
(d)	$(C=)\frac{88}{5}$ (oe)	B1
		(1)

Question	Scheme	Marks
5(e)	(7,7)	**B1**
	35	**B1**
		(2)
(f)	$y \leqslant \frac{5}{3}x \therefore k = \frac{5}{3}$	**M1A1**
		(2)
		(13 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(a)	e.g. (each arc contributes 1 to the orders of two nodes, and so) the sum of the orders of all the nodes is equal to twice the number of arcs Which implies that the sum of the orders of all the nodes is even and therefore there must be an even (or zero) number of vertices of odd order hence there cannot be an odd number of vertices of odd order	**B2, 1, 10**
		(2)
(b)(i)	Prim: AB, AD, BC, DE; EF, FG; GJ, HJ	**M1 A1 A1**
(ii)	Weight = 92 (mins)	**B1**
		(4)
(c)	$20x + 17$ + repeart arcs = 162 B and E are the only two odd nodes so must be apired	**B1**
	$4x + 3$ (BE) is clearly greater in value than $x + (2x + 3) + x - 1 = 4x + 2$ (AB, AD, DE) so repeated arcs are either AB, AD, DE ($4x + 2$) or BC, CE ($7x - 17$)	**B1**
	$4x + 2 = 7x - 17 \Rightarrow 19/3$	**B1**
	$x < 19/3 \Rightarrow 20x + 7 - 17 = 145 \therefore x = 6$	**M1A1**
	$x < 19/3 \Rightarrow 20x + 4x + 2 = 145 \therefore x = 143/24 < 19/3$ so $x \neq 143/24$	**A1**
		(6)
		(12 marks)

A level Further Mathematics – Decision Mathematics 1 –
Practice Paper 04 – Examiner report –

Examiner comment for Question 1 (Mark scheme) (Return to Question 1)

1. Examiners reported that a small number of students struggled in applying the first-fit bin packing algorithm in part (a). This was mainly down to not applying the algorithm correctly. First fit is just that; students must decide if the current item under consideration will fit in the first bin rather than the most recent bin used. In this part a number of students placed the 39 in the third bin (and not the second bin) and others did not place the 4 in the first bin.

   Full marks in part (b) was rare. Some students completed the full sort rather than stopping after the fourth pass. Others completed four passes on the provided list so effectively obtaining the seventh pass. For those who did carry out the correct number of passes or who made their fourth pass clear, errors were rare. In part (ii) of (b), only the more able students were able to correctly establish that 6 comparisons were needed with most stating that 9 comparisons were required. This demonstrated something of a lack of understanding of how the Bubble Sort works. Deducing that 2 swaps took place seemed more straightforward for most students.

   Many correct solutions were seen in part (c), but a number of students did not choose their pivots consistently, switching between middle-left and middle-right pivots during the course of the quick sort algorithm. A number of students either lost an item or changed an item during the sort, and in a small number of cases only one pivot was chosen per iteration. As stated in previous examiners' reports, in cases such as this in which the list appears to be in the correct order after three passes a fourth pass (pivoting on the 4) is required to successfully complete the algorithm. Students should be reminded that items should remain in the order from the previous pass as they move into sub-lists. Pivots were usually chosen consistently although the spacing and notation on some solutions made these difficult for examiners to follow. Some students over complicated the process by insisting on using a different 'symbol' to indicate the pivots for each pass. Those students who sorted into ascending order usually remembered to reverse their list at the end to gain full credit although a number of students left their list in ascending order.

   The first-fit decreasing in part (d) was well carried out with only a small minority failing to attempt this part. There were a large number of wholly correct answers. A small number performed first-fit increasing therefore scoring no marks. A small minority of students lost all three marks by placing the 43 in the 3rd rather than 2nd bin (so failing to apply the algorithm at its first real test). Some students wrote totals in the bin rather than the next value. A variety of different layouts were used but in nearly all cases were easy to read and decipher.

Examiner comment for Question 2 (Mark scheme) (Return to Question 2)

2. For part (a), many candidates could accurately describe the difference between the practical and classical problem, with accurate and correct terminology seen in most responses. However, this initial question differentiated between those that had a secure grasp of the topic and those that were much less secure in their knowledge, with a significant number referring to 'arcs needing to be traversed', rather than the need to visit every vertex. Those that only scored partial marks on this part had the correct general idea without being able to pinpoint the need to visit every vertex.

Whilst part (b) was well answered by the majority of candidates, the most common error was to fail to return to the starting vertex. There were also a significant number of candidates who found an upper bound by doubling the length of a minimum spanning tree, so gave answers of 298 and 316, instead of finding nearest neighbour routes.

Part (c) was also well answered by most candidates, with many securing full marks for an answer of 113. A significant number of candidates did not find the correct RMST (found from deleting vertex A and all arcs incident to this vertex), but still managed to secure a mark for adding the correct two least weighted arcs.

Most candidates scored at least one mark in part (d) on the follow through from an earlier stated upper bound found in part (b). It was common, however, to lose the final mark in this part for either the use of a strict inequality for the upper limit, or because of inaccurate earlier working. It was rare but nonetheless worrying to see statements along the lines of $149 \leq$ optimal distance ≤ 113.

Examiner comment for Question 3 (Mark scheme) (Return to Question 3)

3. This question also discriminated well leading to a good spread of marks. The modal mark was 12, only 4.8% scored full marks and 29.5% scored 6 marks or fewer.

Part (a) was poorly answered on the whole. Many definitions of total float muddled the terms 'events' and 'activities'. A large majority of candidates gave incomplete answers, for example, not clearly stating earliest start time or latest finish time (either saying start and finish time or early and late times). Some candidates tried to state the mathematical formula but did not fully explain all the symbols they used. The majority of candidates understood the idea that it was the time an activity could be delayed for but in many cases, candidates thought it was the total of the floats for all activities in a network.

In part (b) the dummy between events 6 and 7 caused the most difficulty, with a number of candidates going via K and getting a late event time of 25 rather than the correct value of 21. Most could gain the first mark by having only one rogue value and the early event times were largely correct. In general more mistakes were seen in the bottom half of the boxes. Candidates should be advised to take time checking their values as a significant number of subsequent marks can be lost if errors are made in this part.

The majority of students successfully identified the correct critical activities in part (c).

In part (d) the majority of candidates knew the method for the float calculation and showed it clearly. Others just gave the calculation of 21 – 14 which could have been the latest finish time – the earliest start time for the event between G and L rather than the correct calculation 21 – 11 – 3. Finding the lower bound in part (e) had more variable success; some did not do a calculation and tried to argue for a lower bound based on scheduling despite the question asking for a calculation. Others made either basic arithmetical errors or conceptual errors (the most common being calculating the ratio of the earliest possible finish time (30) to the number of activities (13)) in their calculation.

In part (f) quite a few candidates drew a Gantt chart instead of a scheduling diagram, and so scored no marks. There were also quite a few instances where this part was left blank. Those that did schedule tended to make errors on activity H, which needed to take place after activity D. There were also minor errors in duration lengths seen meaning few scored full marks in this part. It would be advisable for candidates to check their working carefully to ensure that preceding activities are completed and that activities do not start before their earliest start time or continue beyond their latest finish time. Also it was common for at least one activity to be missing from the scheduling diagram.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. In Part (a) was answered well by the majority of students with many correctly stating the objective function and the constraints as inequalities. The most common errors were:

- Stating the objective as either $P = -x - 2y - 5z$ or $P = x + 2y + 5z = 0$,
- stating the constraints as either

  $15x - 2y + 3z < 180$, $15x - 2y + 3z \geqslant 180$, $15x - 2y + 3z + r = 180$ or even

  $15x - 2y + 3z + r \leqslant 180$.

In part (b), many students were able to correctly identify the pivot row and divide and replace r with z as a basic variable. Most students defined row operations in terms of the new row 1. There were of course a number of students who made errors in the subsequent row operation calculations. A significant number stopped after one iteration, in some cases stating that the solution was not optimal due to negative values in the profit row despite the fact that the question had asked for the problem to be solved. Of those who did proceed into the second iteration very few picked negative pivots and many were able to proceed correctly albeit with some errors in the pivot row calculations.

In part (c), it was clear that students either did not read the question carefully or assumed it was asking for students to write down the profit equation from the optimal tableau. Therefore many students did not state $P = 364$ but instead stated that $P + 40x + 0.6r + 3.2s = 364$. Furthermore, many did not state the final values of the slack variables and examiners noted that it was quite common to see $x = 40$ rather than $x = 0$ stated as one of the final values.

Examiner comment for Question 5 (Mark scheme) (Return to Question 5)

5. This was the most challenging question on the paper for many candidates, with very few scoring full marks. Most candidates were able to draw the required lines correctly in part (a) although some were unable to draw lines sufficiently accurately (some drew lines without a ruler) or sufficiently long enough. The following general principle should always be adopted by candidates:

- lines should always be drawn which cover the entire graph paper supplied in the answer book and therefore,
- lines with negative gradient should always be drawn from axis to axis.

The rationale behind this is that until all the lines are drawn (and shaded accordingly) it is unclear which lines (or parts of lines) will define the boundary of the feasible region. If candidates only draw the line segments that they believe define the boundary of the feasible region then examiners are unaware of the order in which the lines were drawn and therefore it is unclear to examiners why some parts of the lines have been omitted.

In general the lines $x + y = 8$ and $3y = 9 + 2x$ were correctly drawn. One of the most common errors was with the line $4y = x$ (many candidates drew $y = 4x$), the second most common error was not showing the line $x = 8$ as distinct from the other three in anyway (many candidates either ignored or did not notice the strict inequality constraint $x < 8$). In part (b) a significant number of candidates were unable to select the correct feasible region.

In part (c), the majority of candidates drew the correct objective line, however, a line with reciprocal gradient was often seen or, in a number of cases, no objective line was drawn (and therefore no marks could be awarded in this part). Some used obscure constant values to plot the objective line. Some candidates gave an estimate of the optimal vertex using a reading from their graph, rather than solving the relevant equations simultaneously.

The majority of candidates who correctly answered part (c) usually went on to score the mark in part (d).

Parts (e) and (f) were often not attempted and many who did attempt part (e) identified the maximum point as (8, 8) (even though this was not in the feasible region) or gave a point with non-integer values despite the question explicitly asking for integer values. A minority of candidates identified the point (7,7) but failed to substitute the values into the objective function.

Part (f) differentiated well between candidates. Once again the management of the inequality provided a source of errors for those who had an understanding of what was required in this part but gave the incorrect answer of $k = 1/4$.

Examiner comment for Question 6 (Mark scheme) (Return to Question 6)

6. No examiner report available.