**Paper 2 Option 2F**

**Further Statistics 1 Mark Scheme (Section A)**

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **1(a)** | H0: There is no association between language and gender | B1 | 1.2 |
|  | **(1)** |  |
| **(b)** | \* | B1\*cso | 1.1b |
|  | **(1)** |  |
| **(c)** | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  | Language | | | | Expected frequencies | | French | Spanish | Mandarin | | Gender | Male | 26.43… | 23.4 | 15.16… | | Female | 34.56… | [30.6] | 19.83… |     Awrt **3.6/3.7** | M1  M1  A1 | 2.1  1.1b  1.1b |
|  | **(3)** |  |
| **(d)** | Degrees of freedom (3 – 1)(2 – 1) → Critical value | M1 | 3.1b |
| As < 9.210, the null hypothesis is not rejected | A1 | 2.2b |
|  | **(2)** |  |
| **(e)** | Still not rejected since | B1 | 2.4 |
|  | **(1)** |  |
| **(8 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **B1:** For correct hypothesis in context | | | |
| **(b)**  **B1\*:** For a correct calculation leading to the given answer and no errors seen | | | |
| **(c)**  **M1:** For attempt at  to find expected frequencies  **M1:** For applying  **A1:** awrt 3.6 or 3.7 | | | |
| **(d)**  **M1:** For using degrees of freedom to set up a model critical value  **A1:** For correct comparison and conclusion | | | |
| **(e)**  **A1ft:** For correct conclusion with supporting reason | | | |
| **Question** | **Scheme** | **Marks** | **AOs** |
| **2(a)** | –4 = 2 – 5E(*X*) | M1 | 3.1a |
| E(*X*) = 1.2 |  |  |
| –1×*c* + 0×*a* + 1×*a* + 2×*b* + 3×*c* = 1.2 | M1 | 1.1b |
| *a* + 2*b* + 2*c* = 1.2 |  |  |
| P(*Y*  – 3) = 0.45 gives P( 2 – 5*X*  – 3) = 0.45  i.e. P( *X*  1 ) = 0.45 | M1 | 2.1 |
| 2*a* + *c* = 0.45 |
| 2*a* + *b* + 2*c* = 1 | M1 | 1.1b |
| or | M1 | 1.1b |
| e.g. |
| *a* = 0.1 *b* = 0.3 *c* = 0.25 | A1  A1 | 1.1b  1.1b |
|  | **(7)** |  |
| **(b)** | Var(*Y*) = 75 – (–4)2 or 59 | M1 | 1.1a |
| [Var(*Y*) = 52Var(*X*) implies] Var(*X*) = 2.36 | A1 | 1.2 |
|  | **(2)** |  |
| **(c)** | P(*Y* > *X*) = P(2 – 5*X* > *X*) → P(*X* < ) | M1 | 3.1a |
| P(*X* < ) = *a* + *c* = 0.35 | A1ft | 1.1b |
|  | **(2)** |  |
| **(11 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **M1:** For using given information to find an expression for E(*X*) i.e. use of E(*Y*) = 2 – 5E(*X*)  **M1:** For use of = ‘1.2’  **M1:** For use of P(*Y*  – 3 ) = 0.45 to set up the argument for solving by forming an equation  in *a* and *c*  **M1:** For use of  **M1:** For solving their 3 linear equations (matrix or elimination)  **A1:** For any 2 of *a*, *b* or *c* correct  **A1:** For all 3 correct values | | | |

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| **Question 2 notes continued:** |
| **Another method for part (a) is:**  **M1:** For using given information to find the probability distribution for *Y* leading to an  expression for E(*Y*)  **M1:** For use of = –4  **M1:** For use of P(*Y*  – 3 ) = 0.45 to set up the argument for solving by forming an equation  in *a* and *c*  **M1:** For use of  **M1:** For solving their 3 linear equations (matrix or elimination)  **A1:** For any 2 of *a*, *b* or *c* correct  **A1:** For all 3 correct values |
| **(b)**  **M1:**  For use of Var(*Y*) = E(*Y*2) – [E(*Y*)]2  (may be implied by a correct answer)  **A1:**  For use of Var(*aX*) = *a*2 Var(*X*) to reach 2.36 or exact equivalent |
| **(c)**  **M1:** For rearranging to the form P(*X* < *k*)  **A1ft:** 0.1’ + ‘025’ (provided their *a* and *c* and their *a* + *c* are all probabilities)  **Another method for part (c) is:**  **M1:** For comparing distribution of *X* with distribution of *Y* to identify *X* = –1 and *X* = 0  **A1ft:** ‘0.1’ + ‘025’ (provided their *a* and *c* and their *a* + *c* are all probabilities) |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **3(a)** | *X* ~ Po(2.6) *Y* ~ Po(1.2) |  |  |
| P(each hire 2 in 1 hour)  = P(*X*=2) × P(*Y*=2) = 0.25104…× 0.21685… | M1 | 3.3 |
| = 0.05444… awrt **0.0544** | A1 | 1.1b |
|  | **(2)** |  |
| **(b)** | *W = X + Y → W ~* Po(3.8) | M1 | 3.4 |
| P(*W* = 3) = 0.20458…. awrt **0.205** | A1 | 1.1b |
|  | **(2)** |  |
| **(c)** | *T* ~ Po((2.6+1.2)×2) | M1 | 3.3 |
| P(*T* < 9) = 0.64819… awrt **0.648** | A1 | 1.1b |
|  | **(2)** |  |
| **(d)** | **(i)** Mean = *np* = **2.4** | B1 | 1.1b |
| **(ii)** Variance = *np*(1 – *p*) = 2.3904 awrt **2.39** | B1 | 1.1b |
|  | **(2)** |  |
| **(e)** | **(i)** [ *D* ~ Po(2.4) P(*D* 4) ]  = 0.9041… awrt **0.904** | B1 | 1.1b |
| **(ii)** Since *n* is large and *p* is small/mean is approximately equal to variance | B1 | 2.4 |
|  | **(2)** |  |
| **(10 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **M1:** For P(*X*=2) × P(*Y*=2) from *X* ~ Po(2.6) and *Y* ~ Po(1.2) i.e. correct models (may be  implied by correct answer)  **A1: awrt 0.0544** | | | |
| **(b)**  **M1:** For combining Poisson distributions and use of Po(‘3.8’) (may be implied by correct  answer)  **A1:** **awrt 0.205** | | | |
| **(c)**  **M1:** For setting up a new model and attempting mean of Poisson distribution (may be implied  by correct answer)  **A1: awrt 0.648** | | | |
| **(d)(i)**  **B1:** For **2.4** | | | |
| **(d)(ii)**  **B1:** For **awrt 2.39** | | | |
| **(e)(i)**  **B1:** For **awrt 0.904** | | | |
| **(e)(ii)**  **B1:** For a correct explanation to support use of Poisson approximation in this case | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **4(a)** | **(i)** P(*X* = 1) = 0.34523… awrt **0.345** | B1 | 1.1b |
| **(ii)** P(*X*  4) = 0.98575… awrt **0.986** | B1 | 1.1b |
|  | **(2)** |  |
| **(b)** | \* | B1\*cso | 1.1b |
|  | **(1)** |  |
| **(c)** | *r* = 40 × ‘0.34523…’ *s* = 40 × ‘1 – 0.986…’ | M1 | 3.4 |
| *r* = **13.81** *s* = **0.57** | A1ft | 1.1b |
|  | **(2)** |  |
| **(d)** | H0: The Poisson distribution is a suitable model  H1: The Poisson distribution is not a suitable model | B1 | 3.4 |
| [Cells are combined when expected frequencies < 5]  So combine the last 3 cells | M1 | 2.1 |
|  | M1 | 1.1b |
| awrt **1.1** | A1 | 1.1b |
| Degrees of freedom = 4 – 1 – 1 = 2 | B1 | 3.1b |
| (Do not reject H0 since 1.10 <). The number of mortgages approved each week follows a Poisson distribution | A1 | 3.5a |
|  | **(6)** |  |
| **(11 marks)** | | | |
| **Notes:** | | | |
| **(a)(i)**  **B1:** awrt 0.345 | | | |
| **(a)(ii)**  **B1:** awrt 0.986 | | | |
| **(b)**  **B1\*:** For a fully correct calculation leading to given answer with no errors seen | | | |
| **(c)**  **M1:** For attempt at *r* or *s* (may be implied by correct answers)  **A1ft:** For both values correct (follow through their answers to part (a)) | | | |
| **(d)**  **B1:** For both hypotheses correct (lambda should not be defined so correct use of the model)  **M1:** For understanding the need to combine cells before calculating the test statistic (may be  implied)  **M1:** For attempt to find the test statistic using  **A1:** awrt 1.1  **B1:** For realising that there are 2 degrees of freedom leading to a critical value  of  **A1:** Concluding that a Poisson model is suitable for the number of mortgages approved each  week | | | |

**Decision Mathematics 1 Mark Scheme (Section B)**

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **5(a)** | T:\SAMS 2016 Material\AS and A level Maths\Pre typesetting graphics\Completed work to QD\D52534A\2nd proof\D52534A_GRAPHICS_SAMs_GCE_Mathematics_9FM0_3G_Sep16 2nd proof (1)-page-004.jpg | M1  A1  A1 | 1.1b  1.1b  1.1b |
| Path: ABECDGF | A1 | 1.1b |
| Length: 55 (metres) | A1ft | 1.1b |
|  | **(5)** |  |
| **(b)** | AB + DG = 13 + 11 = 24 ← | M1 | 1.1b |
| A(BEC)D + B(ECD)G = 34 + 32 = 66 | A1 | 1.1b |
| A(BECD)G + B(EC)D = 45 + 21 = 66 | A1 | 1.1b |
| Repeat arcs: AB, DG | A1ft | 2.2a |
|  | **(4)** |  |
| **(c)** | Length = 189 + 24 = 213 (metres) | B1ft | 1.1b |
|  | **(1)** |  |
| **(d)** |  | M1 | 3.1b |
| so BG is 10 m | A1 | 1.1b |
|  | **(2)** |  |
| **(12 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **M1:** For a larger number replaced by a smaller one in the working values boxes at C, D, F or G  **A1:** For all values correct (and in correct order) at A, B, C and D  **A1:** For all values correct (and in correct order) at E, F & G  **A1:** For the correct path  **A1ft:** For 55 or ft their final value at F | | | |
| **(b)**  **M1:** For 3 correct pairings of the four odd nodes (A,B, D & G)  **A1:** At least two pairings and totals correct  **A2:** All three pairings and totals correct  **A3ft:** Selecting their shortest pairing, and stating that these arcs should be repeated | | | |
| **Question 5 notes continued:** | | | |
| **(c) B1ft:** For 213 or 189 + their shortest repeat  **M1:** For translating the information in the question in to an equation involving *x*, 2*x* and 34  **A1:** For a correct equation leading to BG = 10 (m) | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **6** | Objective line drawn or at least two vertices tested | M1 | 3.1a |
| For solving y = 4*x* and 8*x* + 7*y* = 560 to find the exact co-ordinate of the optimal point, must reach either *x* = or *y* = | M1 | 1.1a |
| and | A1 | 1.1b |
| Finding at least two points with integer co-ordinates from  (15 ± 1, 63 ± 2) | M1 | 1.1b |
| Testing at least two points with integer co-ordinates | M1 | 1.1b |
| *x* = 15 and *y* = 63 | A1 | 2.2a |
| So the teacher should buy 15 pens and 63 pencils | A1ft | 3.2a |
| **(7 marks)** | | | |
| **Notes:** | | | |
| **M1:** Selecting an appropriate mathematical process to solve the problem – either drawing an  objective line with the correct gradient (or reciprocal gradient), or testing at least two  vertices in C  **M1:** Solving simultaneous equations  **A1:** cao  **M1:** Recognition that outcome from this model is non-integer and integer solutions are  required – testing two points with integer co-ordinates in at least one of *y* ≥ 4*x* and  8*x* + 7*y* ≥ 560  **M1:** Testing at least two integer solutions in *y* ≥ 4*x* or 8*x* + 7*y* ≥ 560 and C  **A1:** cao – deducing from tests which integer solution is both valid and optimal  **A1ft:** Interpreting solution in the context of the question – gives their integer values for x and y  in the context of pens and pencils | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **7(a)(b)** | T:\SAMS 2016 Material\AS and A level Maths\Pre typesetting graphics\Completed work to QD\D52534A\2nd proof\D52534A_GRAPHICS_SAMs_GCE_Mathematics_9FM0_3G_Sep16 2nd proof (1)-page-006.jpg  The number(s) at the end of activity E indicate this project can be completed in 21 days  Critical activities: B, G, I | M1  A1  A1 | 1.1b  1.1b  1.1b |
| **(3)** |  |
| M1  A1 | 2.1  1.1b |
| A1ft | 2.2a |
| A1 | 1.1b |
|  | **(4)** |  |
| **(7 marks)** | | | |
| **Notes:** | | | |
| **M1:** At least 5 activities and one dummy, one start  **A1:** A,B,C,D,F,G and first dummy correct  **A1:** E,H,I correct, second dummy correct and one finish | | | |
| **M1:** All boxes completed, number generally increasing L to R (condone one “rogue”)  **A1:** All values cao  **A1:** Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes)  **A1:** Critical activities correct | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **8(a)** | e.g. a graph cannot contain an odd number of odd nodes  e.g. number of arcs | B1 | 2.4 |
|  | **(1)** |  |
| **(b)(i)** |  | M1 | 1.1b |
|  | M1 | 1.1b |
|  | A1 | 1.1b |
|  | **(3)** |  |
| **(b)(ii)** | The order of the nodes are 9, 15, 3, 4, 5 | M1 | 2.1 |
| Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes | A1 | 2.4 |
| A1 | 2.2a |
|  | **(3)** |  |
| **(c)** | T:\SAMS 2016 Material\AS and A level Maths\Pre typesetting graphics\Completed work to QD\D52534A\2nd proof\D52534A_GRAPHICS_SAMs_GCE_Mathematics_9FM0_3G_Sep16 2nd proof (1)-page-003.1.jpgT:\SAMS 2016 Material\AS and A level Maths\Pre typesetting graphics\Completed work to QD\D52534A\2nd proof\D52534A_GRAPHICS_SAMs_GCE_Mathematics_9FM0_3G_Sep16 2nd proof (1)-page-003.jpg | M1  A1 | 2.5  2.2a |
|  | **(2)** |  |
| **(9 marks)** | | | |
| **Notes:** | | | |
| **(a) B1:** Explanation referring to need for an even number of odd nodes oe | | | |
| **(b)** **M1:** Forming an equation involving the orders of the 5 odd nodes and  **M1:** Simplifies to a quadratic in  and attempts to solve  **A1:** 2 cao  **M1:** Construct an argument involving the order of the 5 nodes  **A1:** Explanation considering the number of odd nodes  **A1:** Deduction that therefore it is neither Eulerian nor semi-Eulerian | | | |
| **(c)** **M1:** Interprets mathematical language to construct a disconnected graph  **A1:** Deduce a correct graph | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **9** | Minimise | B1 | 3.3 |
| Subject to: | B1 | 3.3 |
|  | M1  M1 | 3.3  3.3 |
| Which simplifies to  **and** | A1 | 1.1b |
| **(5 marks)** | | | |
| **Notes:** | | | |
| **B1:** A correct objective function + minimise  **B1:** Translate information in to a correct inequality  **M1:** For translating the information given into the LHS inequality  **M1:** For translating the information given in to the RHS inequality  **A1:** Simplifying to the correct inequalities | | | |