Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
1	$\frac{1}{\alpha}+\frac{1}{\beta}=-\frac{b}{a}=\frac{4}{5}$ $\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)=\frac{c}{a}=\frac{1}{5}$	M1	1.1b	3rd Use the relationship between the roots and coefficients of quadratics to solve problems
	$\frac{1}{\alpha\beta}=\frac{1}{5}\Rightarrow\alpha\beta=5$	A1	2.2a	
	$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{4}{5}\Rightarrow\alpha+\beta=4$	A1ft	2.2a	
	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=4^2-2(5)=6$	M1	1.2	
	$\alpha^2\beta^2=(\alpha\beta)^2=5^2=25$			
	$-\frac{q}{p}=6,\ \frac{r}{p}=25\Rightarrow q=-6p,\ r=25p\Rightarrow px^2-6px+25p=0$	M1	1.1b	
	$x^2-6x+25=0$	A1	2.2a	
	Alternative:			
	Since root$^{-1}\to$ root^2 then let $w=x^{-2}$	M1	1.1b	
	$5x^2-4x+1=0,\ x=w^{-0.5}\Rightarrow 5w^{-1}-4w^{-0.5}+1=0$	A1	2.2a	
	$5w^{-1}+1=4w^{-0.5}\Rightarrow 25w^{-2}+10w^{-1}+1=16w^{-1}$	A1ft	2.2a	
	$25w^{-2}-6w^{-1}+1=0\Rightarrow w^2-6w+25=0$	M1	1.2	
	$x^2-6x+25=0$	M1	1.1b	
		A1	2.2a	
		(6)		
				(6 marks)

Notes

M1: Attempts at least one equation in α and β

A1: Correct value for $\alpha\beta$

A1ft: Correct value for $\alpha+\beta$ or ft from their $\alpha\beta$

M1: Uses correct identity for $\alpha^2+\beta^2$

M1: Recognisable attempt to form new quadratic

A1: Correct quadratic or integer multiple including "= 0"

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
2a	$\cosh \mathrm{i}z = \dfrac{\mathrm{e}^{\mathrm{i}z} + \mathrm{e}^{-\mathrm{i}z}}{2}$	M1	1.2	3rd Understand the definitions of hyperbolic functions
	$\mathrm{e}^{\mathrm{i}z} + \mathrm{e}^{-\mathrm{i}z} = \cos z + \mathrm{i} \sin z + \cos(-z) + \mathrm{i} \sin(-z)$	M1	1.1b	
	$\cosh \mathrm{i}z = \dfrac{1}{2}(\cos z + \mathrm{i} \sin z + \cos z - \mathrm{i} \sin z) = \dfrac{1}{2}(2 \cos z) = \cos z$*	A1*	2.1	
		(3)		
2b	$\sinh \mathrm{i}z = \mathrm{i} \sin z$	B1	1.2	4th Know how to use sine and cosine in terms of the exponential form of a complex number
	$\cosh^2 w - \sinh^2 w = \cos^2 z - \mathrm{i}^2 \sin^2 z$	M1	1.1b	
	$= \cos^2 z + \sin^2 z = 1$*	A1*	2.1	
		(3)		
2c	$3 \cosh^2 x - 5 \sinh x - 1 = 0$			5th Be able to derive or use the logarithmic form of inverse hyperbolic functions
	$3(1 + \sinh^2 x) - 5 \sinh x - 1 = 0 \Rightarrow 3 \sinh^2 x - 5 \sinh x + 2 = 0$	M1	1.1b	
	$(3 \sinh x - 2)(\sinh x - 1) = 0$	M1	1.1b	
	$\sinh x = 1, \dfrac{2}{3}$	A1	1.1b	
	$x = \text{arsinh } 1, \text{arsinh } \dfrac{2}{3} = \ln\left(1+\sqrt{1^2+1}\right), \ln\left(\dfrac{2}{3}+\sqrt{\left(\dfrac{2}{3}\right)^2+1}\right)$	M1	3.1	
	$= \ln(1 + \sqrt{2}), \ln\left(\dfrac{2}{3}+\dfrac{\sqrt{13}}{3}\right)$	A1	2.2a	
		(5)		
				(11 marks)

Notes

a **M1:** Uses correct exponential form for cosh

M1: One correct use of Euler's formula

A1*: Fully correct proof with intermediate step

b **B1:** $\sinh iz = i \sin z$ seen or used

M1: Replaces w with iz and obtains an expression in z for $\cosh^2 w - \sinh^2 w$

A1*: Obtains "1" with $\cos^2 z + \sin^2 z$ seen explicitly and no errors

c **M1:** Replaces $\cosh^2 x$ with $1 + \sinh^2 x$ to reach 3TQ in $\sinh x$

M1: Solves their 3TQ by usual rules

A1: Correct values for $\sinh x$

M1: Correct use of logarithmic formula for arsinh (or substitutes correct exponential formula for sinh and reaches $x = \ldots$)

A1: Both correct exact answers and no extra

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
3a	$f(x) = \ln(1+x) \Rightarrow f'(x) = (1+x)^{-1}$	B1	1.1b	5th
	$f''(x) = -(1+x)^{-2}$ $f'''(x) = 2(1+x)^{-3}$ $f''''(x) = -6(1+x)^{-4}$	M1	1.1b	Find and recognise the series expansions of standard functions
	$f(0) = 0, f'(0) = 1, f''(0) = -1, f'''(0) = 2, f''''(0) = -6$			
	$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f''''(0) \Rightarrow$	M1	3.1a	
	$f(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$	A1*	2.1	
		(4)		
3b	$g(x) = \ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$	M1	3.1a	5th
	$= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 - \left(-x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4\right)$	B1	1.1b	Express functions as an infinite series using Maclaurin's expansion
	$= 2x + \frac{2}{3}x^3$	A1	1.1b	
		(3)		
				(7 marks)

Notes

a **B1:** Correct $f'(x)$

M1: Obtains three further derivatives of the correct form

M1: Finds values for f and their four derivatives at $x = 0$ and uses a correct Maclaurin expansion

A1*: Obtains given answer with no errors seen

b **M1:** Uses subtraction law of logarithms

B1: Correct expansion of $\ln(1-x)$ seen

A1: Correct answer

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
4a	$p=-1, q=1$ $p=q=-2$ $p=3, q=1$	B1 B1 B1	1.1b 1.1b 1.1b	4th Represent transformations in two dimensions using matrices
		(3)		
4b	$n=1 \Rightarrow \text{LHS}=\begin{pmatrix}1 & r\\0 & 1\end{pmatrix}^1=\begin{pmatrix}1 & r\\0 & 1\end{pmatrix}=\begin{pmatrix}1 & (1)(r)\\0 & 1\end{pmatrix}=\text{RHS}$	B1	1.1b	6th Prove results involving powers of matrices using induction
	$n=k \Rightarrow \begin{pmatrix}1 & r\\0 & 1\end{pmatrix}^k=\begin{pmatrix}1 & kr\\0 & 1\end{pmatrix}$			
	$n=k+1 \Rightarrow \begin{pmatrix}1 & r\\0 & 1\end{pmatrix}^{k+1}=\begin{pmatrix}1 & r\\0 & 1\end{pmatrix}\begin{pmatrix}1 & r\\0 & 1\end{pmatrix}^k=\begin{pmatrix}1 & r\\0 & 1\end{pmatrix}\begin{pmatrix}1 & kr\\0 & 1\end{pmatrix}$	M1	3.1	
	$=\begin{pmatrix}1 & kr+r\\0 & 1\end{pmatrix}$	M1 A1	1.1b 1.1b	
	$=\begin{pmatrix}1 & (k+1)r\\0 & 1\end{pmatrix}$			
	Statement shown to be true for $n=1$ and true for $n=k+1$ when assumed true for $n=k$. Hence true for all $n\in\mathbb{Z}^+$	A1*	2.1	
		(5)		
4c	$\mathbf{B}=\begin{pmatrix}1 & -10\\0 & 1\end{pmatrix}$	B1	1.1b	5th Represent successive transformations in two dimensions using matrices
	$\mathbf{C}=\mathbf{BA}=\begin{pmatrix}1 & -10\\0 & 1\end{pmatrix}\begin{pmatrix}4 & 0\\0 & 3\end{pmatrix}$ $=\begin{pmatrix}4 & -30\\0 & 3\end{pmatrix}$	M1 A1	1.1b 1.1b	
		(3)		

4d	det **BA** = 12 so area $S = 39 \div 12$ $= 3.25$ oe (square units)	**M1** **A1**	3.1a 2.2a	5th Represent successive transformations in two dimensions using matrices
		(2)		
				(13 marks)

Notes

a Correct values for both p and q for each mark

b **B1:** Correctly shows LHS = RHS for $n = 1$

M1: Attempts induction step

M1: Recognisable attempt at matrix multiplication

A1: Correct matrix

A1: Final matrix achieved and full conclusion or narrative

c **B1:** Correct matrix **B**

M1: Recognisable attempt at multiplication of matrices in the correct order

A1: Correct matrix

d **M1:** Divides 39 by correct (non-zero) determinant for their matrix (could just use det **A** since det **B** = 1)

A1: 3.25 or equivalent

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
5a	$d = \frac{1}{2}\text{ arsinh}\left(\frac{9^2}{16}\right)$	M1	3.4	5th Be able to derive or use the logarithmic form of inverse hyperbolic functions
	$= 1.162$ (m)	A1	1.1b	
		(2)		
5b	$y = \frac{1}{2}\text{ arsinh}\left(\frac{x^2}{16}\right) \Rightarrow x^2 = 16\sinh 2y$	B1	3.4	6th Solve problems involving volumes of revolution
	$V = (\pi)\int x^2\ \mathrm{d}y$	M1	3.3	
	$\int \sinh 2y\ \mathrm{d}y \Rightarrow \frac{1}{2}\cosh 2y$	M1	1.1b	
	$V = 8\pi\left[\cosh 2y\right]_0^{"1.162"} = 8\pi(\cosh(2(“1.162”)) - 1)$	M1	3.4	
	= awrt 104 or 105 (m^3)	A1	1.1b	
		(5)		
5c	$y = kx^2$	M1	3.5	5th Model problems using volumes of revolution
	$k = \frac{"1.162"}{81} = 0.0143$	A1ft	2.2a	
		(2)		
				(9 marks)

Notes

a **M1:** Uses model to obtain a numerical expression for d (could be implied)

A1: awrt 1.162

b **B1:** Correctly makes x^2 the subject of the model equation

M1: Correct method for volume of revolution about y-axis allowing for incorrect or missing π

M1: $\int \sinh 2y\ \mathrm{d}y = k\cosh 2y$

M1: Correct use of “1.162” and 0 as limits

A1: awrt 104 or 105

c **M1:** Suggests correct form of quadratic model

A1ft: Correct value for k or ft for “1.162” $\div$ 81 correct to 3 sf

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
6a	P is (12, 0)	B1	2.2a	4th Know how to sketch standard polar curves
		(1)		
6b	$\frac{d}{d\theta}(r\sin\theta) = 0$	M1	1.2	6th Know how to find tangents parallel and perpendicular to the initial line
	$\frac{d}{d\theta}(6\sin\theta + 6\sin\theta\cos\theta) = 6(\cos\theta + \cos^2\theta - \sin^2\theta)$	M1 A1	3.4 1.1b	
	$\cos\theta + \cos^2\theta - (1 - \cos^2\theta) = 0$			
	$2\cos^2\theta + \cos\theta - 1 = 0$	M1	1.1b	
	$(2\cos\theta - 1)(\cos\theta + 1) = 0$			
	$\cos\theta = \frac{1}{2} \quad \cos\theta = -1$	M1	1.1b	
	$\theta = \frac{\pi}{3}, -\frac{\pi}{3}, \pi$	A1	2.2a	
	$\Rightarrow (9, \frac{\pi}{3}), (9, -\frac{\pi}{3}), (0, \pi)$	A1	2.2a	
		(7)		

6c	Area bounded by curve $= 2\int_0^{\pi}\frac{1}{2}r^2\,d\theta$ or $\int_0^{2\pi}\frac{1}{2}r^2\,d\theta$	**M1**	3.3	7th Know how to find compound areas enclosed by polar curves
	$= 36\int_0^{\pi}(1 + 2\cos\theta + \cos^2\theta)\,d\theta$			
	$= 36\int_0^{\pi}(1 + 2\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta)\,d\theta$	**M1**	3.4	
	$= 36\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^{\pi}$	**M1** **A1**	1.1b 1.1b	
	$OQ = -\frac{3}{2}\sec\pi\ \left(=\frac{3}{2}\right)$	**B1**	2.2a	
	$AQ = \text{'}9\text{'} \times \sin\text{'}\frac{\pi}{3}\text{'}\ \left(=\frac{9\sqrt{3}}{2}\right)$	**M1**	2.2a	
	Area wasted $= \left(\frac{3}{2}+2\right)\left(2\times\frac{9\sqrt{3}}{2}\right) - 36\left(\frac{3\pi}{2} + 0 + 0 - (0)\right)$	**M1**	3.3	
	$\frac{243\sqrt{3}}{2} - 54\pi\ (\text{cm}^2)$	**A1**	1.1b	
		(8)		
				(16 marks)

Notes

a **B1:** (12, 0) only (but allow mark for $r = 12$, $\theta = 0$)

b **M1:** Uses $\frac{d}{d\theta}(r\sin\theta) = 0$

M1: Uses model and differentiates to obtain correct form of derivative

A1: Correct derivative

M1: Uses $\sin^2\theta = 1 - \cos^2\theta$ to obtain 3TQ

M1: Solves quadratic by usual rules

A1: At least two correct values for θ

A1: Fully correct coordinates and no extra

c **M1:** Applies area formula to model (allow this mark for half these integrals i.e. half the bounded area)

M1: Uses model and replaces $\cos^2\theta$ with $\frac{1}{2} + \frac{1}{2}\cos 2\theta$ condoning sign errors

M1: Integrates to obtain expression of the correct form

A1: Correct integraton

B1: Correct OQ in any form

M1: Correct attempt at AQ (could be doubled at this stage)

M1: Fully correct method for shaded area; i.e. their rectangle area – their area bounded by curve

A1: $\dfrac{243\sqrt{3}}{2} - 54\pi$ only

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
7a	$t\frac{dP}{dt} - P - t^2 \ln t + P_0 = 0 \Rightarrow \frac{dP}{dt} - \frac{1}{t}P = t\ln t - \frac{P_0}{t}$	B1	3.1a	8th Use first order differential equations to model problems in a wide range of contexts
	$\text{IF} = e^{\int -\frac{1}{t} dt}$	M1	1.2	
	$= e^{-\ln t} = \frac{1}{t}$	A1	2.2a	
	$\frac{1}{t}P = \int(\ln t - \frac{P_0}{t^2})\ dt$	M1	1.1b	
	$\int \ln t\ dt \Rightarrow t\ln t - t$	M1	1.1b	
	$\frac{1}{t}P = t\ln t - t + \frac{P_0}{t}\ (+c)$	A1	1.1b	
	$P = t^2 \ln t - t^2 + P_0 + ct$			
	100 days: $t = 1, P = 1.5, P_0 = 2.5 \Rightarrow 1.5 = -1 + 2.5 + c \Rightarrow c = 0$	M1	3.3	
	$P = t^2 \ln t - t^2 + 2.5 = t^2(\ln t - 1) + 2.5$*	A1*	2.1	
		(8)		
7b	$\frac{dP}{dt} = 2t(\ln t - 1) + t^2\left(\frac{1}{t}\right)$ oe	M1 A1	3.4 1.1b	8th Use first order differential equations to model problems in a wide range of contexts
	$2t\ln t - t = 0$			
	$t(2\ln t - 1) = 0$			
	$(t \neq 0)\ \ln t = \frac{1}{2} \Rightarrow t = e^{0.5}\ (= 1.6487...)$	M1	2.2a	
	$P = (e^{0.5})^2(\ln(e^{0.5}) - 1) + 2.5 = 2.5 - 0.5e = 2.5 - 1.359...$			
	$= 1.14.. \Rightarrow 114$ (birds)	A1	3.4	
		(4)		
7c	[($t > e^{0.5}$) As t increases, P increases (at an increasing rate)] The model is unrealistic as it predicts a limitless increase in the bird population oe	B1	3.5b	8th Use first order differential equations to model problems in a wide range of contexts
		(1)		
				(13 marks)

Notes

a **B1:** Uses model and correctly divides through by t

M1: Correct method for integrating factor

A1: Correct integrating factor

M1: Applies IF $\times$ $B = \int(\text{IF} \times Q)\,\mathrm{d}t$

M1: Attempts integration of $\ln t$ by parts in the correct direction

A1: Fully correct integration

M1: Uses model to substitute and obtain c

A1*: Correct proof with no errors seen

b **M1:** Attempts differentiation of model equation using product rule

A1: Correct differentiation

M1: Sets derivative = 0 and reaches $t = \ldots$

A1: 114 only

c **B1:** Statement that makes clear the model is unrealistic with a suitable supporting reason

TOTAL FOR PAPER IS 75 MARKS