Further Mathematics
Advanced
Paper 1: Core Pure Mathematics

Paper 1 Core Pure Mathematics	
You must have:	
Mathematical Formulae and Statistical Tables, calculator	
Time	1 hour 30 minutes

Name	
Class	
Teacher name	

Total marks	/75

Answer ALL questions.

1   The equation

$$5x^2 - 4x + 1 = 0$$

has roots $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$

Without solving this equation, find a quadratic equation with roots α^2 and β^2, giving your answer in the form $px^2 + qx + r = 0$ where p, q, $r \in \mathbb{Z}$

(6)

(Total for Question 1 is 6 marks)

2 **a** Starting with the definition of cosh in terms of exponentials, show clearly that

$$\cosh iz = \cos z$$

(3)

b By setting $w = iz$ and using the result from part **a** together with an equation relating sinh iz and sin z, prove that

$$\cosh^2 w - \sinh^2 w = 1$$

(3)

c Hence find the exact values of x for which

$$3\cosh^2 x - 5\sinh x - 1 = 0 \quad x \in \mathbb{R}$$

(5)

(Total for Question 2 is 11 marks)

3

$$f(x) = \ln(1+x) \quad |x| < 1$$

$$g(x) = \ln\left(\frac{1+x}{1-x}\right) \quad |x| < 1$$

a Use differentiation to show that the Maclaurin expansion of $f(x)$ in ascending powers of x up to and including the term in x^4 is

$$f(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$$

(4)

b Using the expansion from (a), find the first two non-zero terms in ascending powers of x in the Maclaurin expansion of $g(x)$.

(3)

(Total for Question 3 is 7 marks)

4
$$A = \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}$$

a State the values of p and q if the transformation represented by the matrix A

   i produces a reflection in the y-axis

   ii produces an enlargement of scale factor -2

   iii produces a stretch of scale factor 3 parallel to the x-axis only.

(3)

b Use the method of mathematical induction to prove that for $n \in \mathbb{Z}^+$, where r is a constant,

$$\begin{pmatrix} 1 & r \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & nr \\ 0 & 1 \end{pmatrix}$$

(5)

The shape S is transformed by the matrix $A = \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix}$ to produce the image S'. This image is then

transformed by the matrix $B = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^5$ to produce the image S''.

S is also transformed into S'' by the matrix C.

c Find the matrix C.

(3)

The area of shape S'' is 39 square units.

d Find the area of shape S.

(2)

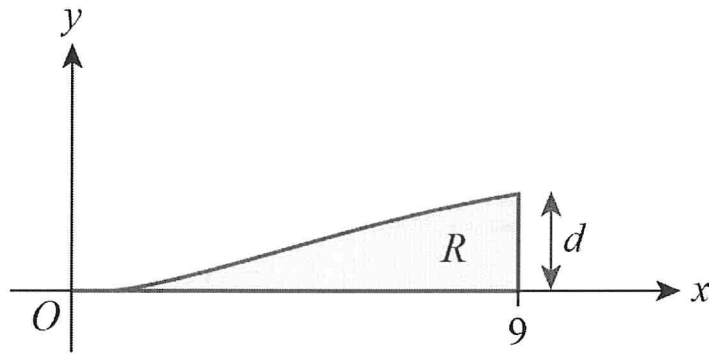**(Total for Question 4 is 13 marks)**

5

Figure 1

A circular concrete pond is modelled by the shape obtained by rotating the region R in Figure 1 by 2π radians about the y-axis.

Region R is bounded by the line $x = 9$, the x-axis and the curve with equation

$$y = \frac{1}{2} \operatorname{arsinh}\left(\frac{x^2}{16}\right)$$

Both x and y are measured in metres.

a Find, to four significant figures, the maximum depth of the pond d as predicted by the model.

(2)

b Use algebraic integration to find the volume of concrete that the model predicts would have been required to construct the pond, giving your answer in m^3 to 3 significant figures.

(5)

It is suggested that this volume prediction could be improved by modelling the curve of the cross-section of the pond as a quadratic function.

c Use your answer to part **a** to suggest an appropriate function.

(2)

(Total for Question 5 is 9 marks)

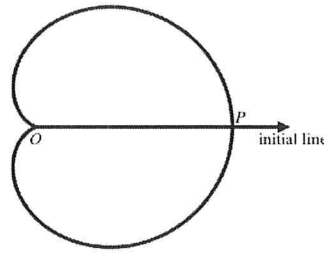6   A graphic designer is making a birthday card.

Figure 2

The shape of the flat card is modelled by the area enclosed by the curve with polar equation

$$r = 6(1 + \cos\theta) \quad 0 \leqslant \theta < 2\pi$$

as shown in Figure 2. The curve crosses the initial line at the point P.

a   Find the polar coordinates of P.

(1)

b   Find the exact polar coordinates of those points on the curve where tangents to the curve are parallel to the initial line.
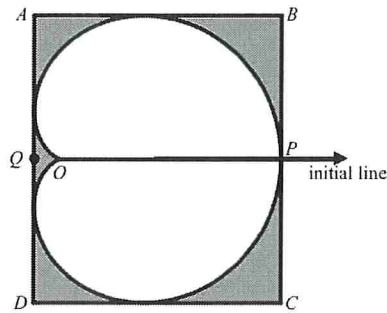
(7)

Figure 3

A birthday card is made by cutting out the shape of the curve from a rectangle of card, as shown in Figure 3. The shaded regions of card will be wasted.

Straight lines AB and DC are tangents to the curve that are parallel to the initial line.

Straight lines AD and BC are tangents to the curve that are perpendicular to the initial line. Point Q is the midpoint of AD. Measurements are in centimetres.

Given that AD is a segment of the line with polar equation

$$r = -\frac{3}{2}\sec\theta \qquad \frac{\pi}{2} < \theta < \frac{3\pi}{2}$$

c find, according to the model, the exact total area of card that will be wasted, giving your answer in square centimetres in the form $a\sqrt{3} + b\pi$ where a and b are constants to be determined.

(8)

(Total for Question 6 is 16 marks)

7 A biologist is conducting a study of the population of a species of bird on an island. The biologist models the situation using the differential equation

$$t\,\frac{dP}{dt} - P - t^2 \ln t + P_0 = 0 \quad t > 0$$

where P is the population of birds in hundreds,

P_0 is the population of birds in hundreds at the start of the study, and

t is the time from the start of the study in hundreds of days.

At the start of the study, there are 250 birds on the island.

a Given that after 100 days there are 150 birds on the island, solve the differential equation to show that

$$P = t^2 (\ln t - 1) + 2.5$$

(8)

b Use calculus to find, to the nearest whole number, the minimum number of birds on the island as predicted by the model.

(4)

c Comment on the long-term performance of the model, justifying your answer.

(1)

(Total for Question 7 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

Pearson Edexcel AS and A level Further Mathematics