Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
1	$z = 1 + \text{i}$ is a root $\Rightarrow z = 1 - \text{i}$ is a root	B1	2.2a	4th Solve quartic equations with two or more complex roots
	$(z - (1 + \text{i}))(z - (1 - \text{i})) = z^2 - 2z + 2$	M1 A1	1.1b 1.1b	
	$\text{f}(z) \div (z^2 - 2z + 2) = z^2 - z - 6$	M1 A1	1.1b 2.2a	
	$(z + 2)(z - 3) = 0 \Rightarrow z = -2, 3$	M1 A1	1.1b 2.2a	
	kite has diagonals of 5 and 2 $\Rightarrow$ area $= \frac{1}{2} \times 5 \times 2$	M1	3.1a	
	$= 5$ (units^2)	A1ft	1.1b	
		(9)		
				(9 marks)

Notes

B1: Conjugate seen as a root

M1: Attempts a quadratic from their pair of roots

A1: Correct quadratic

M1: Uses any viable method (e.g., long division, equating coefficients) to produce further quadratic

A1: Correct quadratic

M1: Solves quadratic (usual rules)

A1: Both correct real roots

M1: Uses correct method for area of a kite (their roots must form a kite)

A1ft: 5 or follow through from their kite

Note that algebra must be used otherwise the first three method marks are not available

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
2	General point on straight line through origin $y = mx$ is (t, mt)	B1	3.1a	5th Find invariant points and lines for a linear transformation
	$\begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} t \\ mt \end{pmatrix} = \begin{pmatrix} -mt \\ 2t+3mt \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$	M1	1.1b	
	$x = -mt \Rightarrow t = -\dfrac{x}{m}$, $y = (2+3m)t \Rightarrow y = -\dfrac{x}{m}(2+3m)$	M1	1.1b	
	$y = \left(-\dfrac{2}{m} - 3\right)x \Rightarrow m = -\dfrac{2}{m} - 3$	A1	2.2a	
	$m^2 + 3m + 2 = 0 \Rightarrow (m+2)(m+1) = 0 \Rightarrow m = -2, -1$	M1	1.1b	
	$y = -2x, y = -x$	A1	2.2a	
		(6)		
				(6 marks)

Notes

B1: Correct form of general point on $y = mx$

M1: Recognisable attempt to multiply their general point vector by matrix

M1: Eliminates t to obtain equation in y, x and m

A1: Correct quadratic in m in any form

M1: Solves their quadratic (usual rules)

A1: Correct invariant lines

Alternative (eigenvectors)

$\begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{pmatrix}$	**B1:** Correct matrix $\mathbf{M} - \lambda\mathbf{I}$
$\begin{vmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix} = -\lambda(3-\lambda) - (-1)(2) = \lambda^2 - 3\lambda + 2$	**M1:** Recognisable attempt at determinant
$(\lambda - 1)(\lambda - 2) = 0 \Rightarrow \lambda = 1, 2$	**M1:** Solves quadratic (usual rules) **A1:** Correct eigenvalues
$\begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ 2x+3y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow y = -x$	**M1:** One use of $\mathbf{Mx} = \lambda\mathbf{x}$ reaching $y = \ldots$
$\begin{pmatrix} -y \\ 2x+3y \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \end{pmatrix} \Rightarrow y = -2x$	**A1:** Correct invariant lines

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
3a	$\sum_{r=1}^{n}(2r+1)=\sum_{r=1}^{n}\left((r+1)^2-r^2\right)$	M1	1.1b	6th Understand and use the method of differences to sum series, including partial fractions
	$= 2^2 - 1^2 + 3^2 - 2^2 + \ldots + (n+1)^2 - n^2$ $= (n+1)^2 - 1^2 = n^2 + 2n$	A1	2.2a	
	$\sum_{r=1}^{n}(2r+1)=2\sum_{r=1}^{n}r+\sum_{r=1}^{n}1=2\sum_{r=1}^{n}r+n=n^2+2n$	B1	1.1b	
	$\sum_{r=1}^{n}r=\frac{1}{2}\left(n^2+n\right)$	M1	1.1b	
	$=\frac{1}{2}n(n+1)$*	A1*	2.1	
		(5)		
3b	$n=1$: LHS $= 1^3 = 1$ RHS $= \frac{1}{4}\times 1^2 \times (1+1)^2 = 1$	B1	1.1b	4th Prove general results for the sum of natural numbers, and squares and cubes of natural numbers
	Assume true for $n=k$: $\sum_{r=1}^{k}r^3=\frac{1}{4}k^2(k+1)^2$			
	When $n=k+1$: $\sum_{r=1}^{k+1}r^3=\sum_{r=1}^{k}r^3+(k+1)^3$	M1	1.1b	
	$=\frac{1}{4}k^2(k+1)^2+(k+1)^3=\frac{1}{4}(k+1)^2(k^2+4k+4)$	M1	1.1b	
	$=\frac{1}{4}(k+1)^2(k+2)^2$ $=\frac{1}{4}(k+1)^2((k+1)+1)^2$	A1	2.2a	
	Since true for $n=1$ and true for $n=k+1$ when assumed true for $n=k$, true for all $n\in\mathbb{Z}^+$	A1*	2.1	
		(5)		

3c	$f(x) = x^2$	**B1**	3.1	4th Prove general results for the sum of natural numbers, and squares and cubes of natural numbers
		(1)		
				(11 marks)

Notes

a **M1:** Attempt to use method of differences with at least $r = 1$ and $r = n$

A1: Correct expression

B1: Replaces $\sum_{r=1}^{n} 1$ with n

M1: Makes $\sum_{r=1}^{n} r$ the subject

A1*: Reaches given answer with no errors

b **B1:** Shows LHS = RHS for $n = 1$

M1: Attempts induction step

M1: Obtains $(k + 1)^2$ as a factor

A1: Correct fully factorised expression

A1*: Gives expression as $f(k + 1)$ and correct full conclusion or narrative

c **B1:** Correct function – allow alternative notation e.g., $f: x \mapsto x^2$, $y = x^2$

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
4a	$\det \mathbf{M} = 1(2(1) - 0(1)) - (-2k)(k(1) - 5(1)) - 1(k(0) - 5(2))$ $= 2 + 2k^2 - 10k + 10$ $2k^2 - 10k + 12 = 0 \Rightarrow k^2 - 5k + 6 = 0 \Rightarrow (k-2)(k-3) = 0$ $k = (p =)\ 2, \quad k = 3$	M1 M1 A1	1.1b 2.2a 2.4	4th Use the determinant of a matrix to find unknown elements
		(3)		
4b	$k = 3$ so $\mathbf{A}$ is singular and $\mathbf{A}^{-1}$ does not exist	B1	2.4	5th Use inverse matrices to solve a system of three simultaneous equations
		(1)		
4c	$x - 6y - z - 5 = 0,\ 3x + 2y + z + 1 = 0 \Rightarrow 4x - 4y - 4 = 0$ $x = y + 1,\ x = \frac{z-1}{-5}\ \left(\Rightarrow x = y + 1 = \frac{z-1}{-5}\right)$ $\mathbf{r} = -\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - 5\mathbf{k})$	M1 M1 A1	1.1b 2.1 1.1b	6th Solve coordinate geometry problems involving planes in three dimensions
		(3)		
4d	The planes form a sheaf	B1	2.2a	6th Interpret geometrically the solution and failure of solution to three simultaneous equations
		(1)		
				(8 marks)

Notes	
a	**M1:** Recognisable attempt at det **M**
	M1: Solves their quadratic (usual rules)
	A1: Correct values for k
b	**B1:** Correct explanation
c	**M1:** Obtains equation in x and y
	M1: Obtains $x = f(y)$ and $x = g(z)$ or obtains $f(x) = g(y) = h(z)$ which could be implied
	A1: Any correct equation (allow column vectors and note that many equivalents are possible)
d	**B1:** Sight of "sheaf"

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
5a	$y=\frac{1}{x^2-9}$ has a discontinuity/vertical asymptote/is not defined at $x=3$	B1	2.4	6th Integrate functions across limits which include values where the function is undefined
		(1)		
5b	$4x^2+8x+13=4\left(x^2+2x+\frac{13}{4}\right)=4\left((x+1)^2-1+\frac{13}{4}\right)$ or $(2x+2)^2-4+13$	M1	3.1a	5th Evaluate integrals that extend to infinity
	$=4\left((x+1)^2+\frac{9}{4}\right)$ or $(2x+2)^2+9$	A1	1.1b	
	$\int\frac{\mathrm{d}x}{4x^2+8x+13}=\frac{1}{4}\int\frac{\mathrm{d}x}{(x+1)^2+\frac{9}{4}}=\frac{1}{4}\times\frac{1}{\frac{3}{2}}\arctan\left(\frac{x+1}{\frac{3}{2}}\right)$	M1	1.1b	
	$=\frac{1}{6}\arctan\frac{2(x+1)}{3}\ (+c)$ or $\frac{1}{6}\arctan\frac{2x+2}{3}\ (+c)$	A1	1.1b	
		(4)		
5c	$\int_{-\frac{5}{2}}^{\infty}\frac{\mathrm{d}x}{4x^2+8x+13}=\lim_{t\to\infty}\left(\frac{1}{6}\arctan\frac{2(t+1)}{3}\right)-\frac{1}{6}\arctan\frac{2\left(-\frac{5}{2}+1\right)}{3}$	M1	3.1a	5th Evaluate integrals that extend to infinity
	$=\frac{1}{6}\left(\frac{\pi}{2}-\left(-\frac{\pi}{4}\right)\right)=\frac{\pi}{8}$ *	A1*	2.1	
		(2)		
		(7 marks)		

Notes

a **B1:** Correct explanation

b **M1:** Valid attempt to complete the square

A1: Correct expression

M1: Integrates to obtain arctan term

A1: Correct answer

c **M1:** Correct use of limits and use of limit as $t\to\infty$

A1*: Obtains given answer with intermediate step seen and no errors

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
6	$\frac{d}{dx}(\text{artanh}\,2x)=\frac{2}{1-(2x)^2}$	B1	1.1b	7th Be able to solve calculus problems with hyperbolic functions in a range of familiar contexts
	$\frac{d}{dx}(8x^2+1)=16x$	B1	1.1b	
	$\frac{dy}{dx}=\frac{\frac{2(8x^2+1)}{1-4x^2}-16x\text{artanh}\,2x}{(8x^2+1)^2}$	M1 A1	3.1a 1.1b	
	$\text{artanh}\left(2\left(\frac{1}{4}\right)\right)=\frac{1}{2}\ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)=\frac{1}{2}\ln 3$	B1	1.1b	
	$m_T=\left(\frac{dy}{dx}\right)_{x=\frac{1}{4}}=\frac{4-4\text{artanh}\frac{1}{2}}{\frac{9}{4}}=\frac{16}{9}-\frac{8}{9}\ln 3=\frac{1}{9}(16-8\ln 3)$	A1	1.1b	
		(6)		
				(6 marks)

Notes

B1: Correct differentiation of artanh $2x$

B1: Correct differentiation of $8x^2 + 1$

M1: Use of correct quotient rule (or product rule on $(8x^2 + 1)^{-1}(\text{artanh}\,2x)$)

A1: Correct expression

B1: Correct exact value for artanh $2x$ at $x = \frac{1}{4}$

A1: Correct answer

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
7a	direction of line = $4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k} - (\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$	M1	3.3	3rd
	$\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j})$ or $\mathbf{r} = 4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j})$	A1	1.1b	Find the vector equation of a line in three dimensions
		(2)		
7b	'$3\mathbf{i} + 4\mathbf{j}$' and $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ are not multiples of each other	B1ft	2.4	5th
	(see Notes)			Solve coordinate geometry problems involving lines in three dimensions
	$1 + 3\lambda = -4 + 2\mu$ (I), $3 + 4\lambda = 2 + 3\mu$ (II), $-2 = -1 - \mu$			
	(III) $\Rightarrow \mu = 1$	M1	1.1b	
	$\mu = 1$ in (I) $\Rightarrow 1 + 3\lambda = -2 \Rightarrow \lambda = -1$			
	Both in (II): LHS = $3 + 4(-1) = -1$, RHS = $2 + 3(1) = 5$	M1	1.1b	
	Lines are not parallel and since (equations are inconsistent $\Rightarrow$) lines do not intersect they are skew*	A1*	2.4	
		(4)		
7c	$\mu = -1 \Rightarrow \mathbf{r} = -4\mathbf{i} + 2\mathbf{j} - \mathbf{k} - 1(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = -6\mathbf{i} - \mathbf{j}$	B1	3.4	7th
	(so P is on l_2)			Find the shortest distance between a point and a line
		(1)		
7d	General point X on l_1 is $(1 + 3\lambda, 3 + 4\lambda, -2)$	M1	3.4	7th
	$\overrightarrow{PX} = (7 + 3\lambda)\mathbf{i} + (4 + 4\lambda)\mathbf{j} - 2\mathbf{k} \Rightarrow 3(7 + 3\lambda) + 4(4 + 4\lambda) = 0$	M1	3.1a	Find the shortest distance between a point and a line
	$25\lambda = -37 \Rightarrow \lambda = -\frac{37}{25}$	M1	1.1b	
	$d = \sqrt{\left(7 + 3\left(-\frac{37}{25}\right)\right)^2 + \left(4 + 4\left(-\frac{37}{25}\right)\right)^2 + (-2)^2}$	M1	3.1a	
	$= \sqrt{\frac{356}{25}} = \frac{2\sqrt{89}}{5} = 3.77$ (m) (3sf)	A1	1.1b	
		(5)		

7e	Answer is unreliable: e.g. No account is taken of the size of the device No account is taken of the thickness of the cables Cables might not be straight Model's positions of cables/device may not be accurate	 **B1** **B1**	 3.5a 3.5b	7th Find the shortest distance between a point and a line
		(2)		
				(14 marks)

Notes

Allow column vectors throughout

a **M1:** Attempts to find direction of line

A1: Correct equation

b **B1ft:** States or demonstrates (e.g., $(3\mathbf{i} + 4\mathbf{j})c = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} \Rightarrow$ inconsistent set: $3c = 2$, $4c = 3$, $0c = -1$) directions are not multiples. Directions must be identified if just a statement. Ft for correct work with their $3\mathbf{i} + 4\mathbf{j}$

Alternatives include (for direction vectors **b** and **d**) showing $\mathbf{b} \times \mathbf{d} \neq 0$, $\mathbf{b}.\mathbf{d} \neq$ bd or by finding $\theta \neq 0$

M1: Obtains the value of one parameter from a system of equations in two parameters

M1: Attempts to show the equations are inconsistent

A1*: Full explanation and conclusion (cao)

c **B1:** Correct verification. If student solves an equation to obtain μ it must be verified in the other two equations

d **M1:** Attempts a general point X (or position vector) on l_1

M1: Uses $\overrightarrow{PX}$.(their $3\mathbf{i} + 4\mathbf{j}$) = 0 (or $\overrightarrow{XP}$)

M1: Obtains a value for λ

M1: Substitutes their value for λ and uses correct Pythagoras

A1: awrt 3.77 (accept exact value)

Alternatives for d:	
Way 2 (λ by differentiation)	
First mark as main scheme	**M1**
$(7+3\lambda)^2+(4+4\lambda)^2+(-2)^2=25\lambda^2+74\lambda+69$	
$\frac{d}{d\lambda}(25\lambda^2+74\lambda+69)=0$	**M1:** Attempts to differentiate their $f(\lambda)$ and sets = 0
$50\lambda+74=0 \Rightarrow \lambda=-\frac{37}{25}$	**M1:** Finds a value for λ
Last two marks as main scheme	**M1 A1**
Way 3 (completing the square)	
First mark as main scheme	**M1**
$25\lambda^2+74\lambda+69=$ $25\left(\left(\lambda+\frac{37}{25}\right)^2-\frac{1369}{625}+\frac{69}{25}\right)$ or $\left(5\lambda+\frac{37}{5}\right)^2-\frac{1369}{25}+69$	**M1:** Valid attempt to complete the square
$=25\left(\left(\lambda+\frac{37}{25}\right)^2+\frac{356}{625}\right)$ or $\left(5\lambda+\frac{37}{5}\right)^2+\frac{356}{25}$	
$d^2=25\times\frac{356}{625}$ or $\frac{356}{25}$	**M1:** Sets squared bracket = 0 and finds numerical expression for d^2
$d=\sqrt{\frac{356}{25}}=\frac{2\sqrt{89}}{5}=3.77$ (m) (3sf)	**M1:** Takes square root **A1:** awrt 3.77 (accept exact value)
Way 4 (Pythagoras)	
Unit vector on l_1 is $\frac{3}{5}\mathbf{i}+\frac{4}{5}\mathbf{j}$	**M1:** Attempts unit vector in direction of l_1
Vector from $A(1, 3, -2)$ on l_1 to P is $\overrightarrow{AP}=-7\mathbf{i}-4\mathbf{j}+2\mathbf{k}$	**M1:** Attempts vector between point on l_1 and P
$AX=\overrightarrow{AP}.\hat{n}=-7\left(\frac{3}{5}\right)-4\left(\frac{4}{5}\right)=-\frac{37}{5}$ and $AP=\sqrt{69}$	**M1:** Obtain values for AX and AP
$d=PX=\sqrt{AP^2-AX^2}=\sqrt{69-\left(-\frac{37}{5}\right)^2}$	**M1:** Uses correct Pythagoras
$d=\sqrt{\frac{356}{25}}=\frac{2\sqrt{89}}{5}=3.77$ (m) (3sf)	**A1:** awrt 3.77 (accept exact value)

Way 5 (cross product)	
For $A(1, 3, -2)$, $\overrightarrow{AP} \times$ direction of l_1	**M1:** Attempts cross product
$= (-7\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) \times (3\mathbf{i} + 4\mathbf{j}) = -8\mathbf{i} + 6\mathbf{j} - 16\mathbf{k}$	**M1:** Finds magnitude of normal vector
magnitude $= \sqrt{(-8)^2 + 6^2 + (-16)^2} = 2\sqrt{89}$	**M1:** Finds distance between points on l_1
For $B(4, 7, -2)$, $AB = 5$	
$\mathrm{d} = \dfrac{2\sqrt{89}}{5} = 3.77$ (m) (3sf)	**M1:** Divides magnitude of normal by AB **A1:** awrt 3.77 (accept exact value)

e	**B1:** "Answer is unreliable" comment and one appropriate reason (or two appropriate reasons with no comment) **B1:** "Answer is unreliable" comment and two appropriate reasons (accept one reason if have also commented that 3.77 is close to 3.75)

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
8a	$y = kx \sin x$ $\frac{dy}{dx} = k \sin x + kx \cos x$ $\frac{d^2y}{dx^2} = k \cos x + k \cos x - kx \sin x$ $2k \cos x - kx \sin x + kx \sin x = \cos x \Rightarrow k = \ldots$ $2k = 1 \Rightarrow k = \frac{1}{2}$ (so PI is $y = \frac{1}{2} x \sin x$)	**B1** **M1** **M1** **A1**	1.1b 1.1b 1.1b 2.1	6th Solve second order non-homogeneous differential equations using a particular integral
		(4)		
8b	$m^2 + 1 = 0 \Rightarrow m^2 = -1, m = \pm i$ CF: $(y =) A \cos t + B \sin t$ PS: $(y =) A \cos t + B \sin t + \frac{1}{2} t \sin t$ $\frac{dy}{dt} = -A \sin t + B \cos t + \frac{1}{2} \sin t + \frac{1}{2} t \cos t$ $t = 0, y = 1, \frac{dy}{dt} = 7 \Rightarrow 1 = A, 7 = B$ $y = \cos t + 7 \sin t + \frac{1}{2} t \sin t$	**M1** **A1** **B1ft** **M1** **M1** **A1**	3.1b 1.1b 1.1b 1.1b 1.1b 2.2a	8th Use second order differential equations to model problems in a range of contexts
		(6)		
8c	$\sin t \approx t, \cos t \approx 1 - \frac{t^2}{2} \Rightarrow y = 1 - \frac{t^2}{2} + 7t + \frac{t^2}{2}$ $y = 7t + 1$	**M1** **A1ft**	3.5a 1.1b	8th Use second order differential equations to model problems in a range of contexts
		(2)		

8d	$t = 0.5 \Rightarrow y_1 = 4.353..., y_2 = 4.5$ $\dfrac{4.5 - 4.353...}{4.353...} \times 100 = 3.367...$ (%) (3.367... < 5) so scientist will use the simplified model	**M1** **A1**	3.2a 3.2b	8th Use second order differential equations to model problems in a range of contexts
		(2)		
				(14 marks)

Notes

a **B1:** Correct derivative

M1: Attempt at second derivative including use of correct product rule

M1: Substitutes into differential equation, equates coefficients and obtains a value for k

A1: $k = \dfrac{1}{2}$

b **M1:** Forms auxiliary equation and solves for m

A1: Correct complementary function

B1ft: Their CF + their PI

M1: Differentiates their PS

M1: Substitutes to obtain values for A and B

A1: Correct particular solution including "$y =$"

c **M1:** Substitutes both correct small angle approximations

A1ft: $y = 7t + 1$ or ft but must be a straight line

d **M1:** Substitutes $t = 0.5$ into both solutions and uses correct method to calculate percentage error

A1: Fully correct and conclusion