# Further Mathematics

## Advanced

## Paper 2: Core Pure Mathematics

| Paper 2 Core Pure Mathematics | |
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| **You must have:** Mathematical Formulae and Statistical Tables, calculator | |
| Time | 1 hour 30 minutes |

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|---|---|
| Name | |
| Class | |
| Teacher name | |

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| Total marks | /75 |

**Answer ALL questions.**

1 $$f(z) = z^4 - 3z^3 - 2z^2 + 10z - 12$$

The roots of $f(z) = 0$ are plotted as points on an Argand diagram. The points are connected by straight line segments so that the points form the vertices of a kite.

Given that $z = 1 + i$ is a root of $f(z)$, use algebra to find the vertices and hence area of the kite in square units.

(9)

**(Total for Question 1 is 9 marks)**

**2** Find the equations of any invariant lines through the origin for the transformation represented by the matrix

$$\begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}$$

**(6)**

**(Total for Question 2 is 6 marks)**

**3** **a** Use the identity

$$2r + 1 = (r + 1)^2 - r^2$$

and the method of differences to show that

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n + 1)$$

(5)

**b** Prove by induction that

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+ 1)^2$$

(5)

**c** Hence, given that

$$\sum_{r=1}^{n} r^3 = \mathrm{f}\left(\sum_{r=1}^{n} r\right)$$

write down the the function f($x$).

(1)

**(Total for Question 3 is 11 marks)**

**4**

$$\mathbf{M} = \begin{pmatrix} 1 & -2k & -1 \\ k & 2 & 1 \\ 5 & 0 & 1 \end{pmatrix}$$

**a** By finding and solving an equation, show that the matrix **M** is singular when $k = 3$ or $k = p$ where $p$ is a constant to be found.

(3)

$$\mathbf{A} = \begin{pmatrix} 1 & -6 & -1 \\ 3 & 2 & 1 \\ 5 & 0 & 1 \end{pmatrix}$$

**b** Given that $x$, $y$ and $z$ are variables and **v** is a constant vector with three components, explain clearly why the system of equations given by

$$\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{v}$$

cannot have the solution

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{A}^{-1}\mathbf{v}$$

(1)

Three planes have the equations

$$x - 6y - z - 5 = 0$$

$$3x + 2y + z + 1 = 0$$

$$5x + z - 1 = 0$$

**c** Find the common line of intersection of the three planes, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ where **a** and **b** are vectors to be found.

(3)

**d** State the geometrical configuration of the planes.

(1)

**(Total for Question 4 is 8 marks)**

**5** **a** Explain clearly why

$$\int_0^7 \frac{dx}{x^2 - 9}$$

is an improper integral.

(1)

**b** Giving your answer in its simplest form, find

$$\int \frac{dx}{4x^2 + 8x + 13}$$

(4)

**c** Hence prove that

$$\int_{-\frac{5}{2}}^{\infty} \frac{dx}{4x^2 + 8x + 13} = \frac{\pi}{8}$$

(2)

**(Total for Question 5 is 7 marks)**

**6** The curve $C$ has the equation

$$y = \frac{\operatorname{artanh} 2x}{8x^2 + 1} \qquad |x| < \frac{1}{2}$$

Find the gradient of the tangent to $C$ at the point where $x = \frac{1}{4}$ giving your answer in the form

$\frac{1}{9}(a + b \ln 3)$ where $a$ and $b$ are integers to be found.

**(6)**

**(Total for Question 6 is 6 marks)**

7 A field contains two underground electricity transmission cables.

One cable is modelled as the straight line $l_1$ that passes through the points (1, 3, –2) and (4, 7, –2) relative to a fixed origin $O$, with units in metres.

**a** Find the vector equation of the cable as given by the model.

(2)

The second cable is modelled by the straight line $l_2$ with equation

$$\mathbf{r} = -4\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$$

**b** Prove that $l_1$ and $l_2$ are skew lines.

(4)

A small device that measures the electrical current is attached to the second cable and its position is modelled as the point $P(-6, -1, 0)$.

**c** Verify that the device is modelled as being located on line $l_2$.

(1)

New safety rules mean that if any device on a cable is positioned within 3.75m of any other cable, it must be removed.

**d** Find the shortest distance from point $P$ to the line $l_1$, giving your answer in metres to three significant figures.

(5)

**e** Comment on the reliability of using the answer to part **d** to determine whether the device should be removed, explaining your reasoning clearly.

(2)

**(Total for Question 7 is 14 marks)**

**8** **a** Show that $y = kx\sin x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + y = \cos x$$

where $k$ is a constant to be found.

**(4)**

Military aircraft pilots experience g-forces, which are accelerations given as multiples of the acceleration due to gravity, g.

At a training centre, pilots are revolved at an increasing rate for 30 seconds in a simulator to investigate the g-force that the pilots experience.

A scientist models the situation using the differential equation

$$\frac{d^2y}{dt^2} + y = \cos t \qquad 0 \leqslant t \leqslant 0.5$$

where $y$ is the coefficient of g of the resultant g-force and $t$ is the time in minutes from when the simulator starts to revolve.

**b** Given that when $t = 0$, $y = 1$ and $\frac{dy}{dt} = 7$, find the particular solution of the differential equation.

**(6)**

Statistics reveal that the model is accurate but the scientist wants to simplify this particular solution.

**c** Given that $t$ is sufficiently small, use small angle approximations to show that a simplified solution of the differential equation can be obtained, giving your answer in the form $y = mt + c$ where $m$ and $c$ are constants to be found.

**(2)**

The scientist will not use the simplified solution if the absolute percentage error in using it, compared to the original solution, exceeds 5%.

**d** Given that the solutions differ most at the end of the model's time period, determine whether the scientist will use the simplified solution.

**(2)**

**(Total for Question 8 is 14 marks)**

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**TOTAL FOR PAPER IS 75 MARKS**