Further Mathematics

Advanced

Paper 3C: Further Mechanics 1

Paper 3C Further Mechanics 1	
You must have: Mathematical Formulae and Statistical Tables, calculator	
Time	1 hour 30 minutes

Name	
Class	
Teacher name	

Total marks	/75

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Whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Answer ALL questions.

1 A body of mass m is moving with an initial velocity u and is then acted upon by a constant force F for time t. This results in a constant acceleration a producing a final velocity v such that $v = u + at$.

Using Newton's second law, and the usual units for impulse,

a Derive the formula for impulse, $I = m(v - u)$.

(2)

Two spheres, A and B, of masses m_1 and m_2 kg respectively, are travelling towards each other in a straight line along a smooth horizontal surface, so that they collide directly.

Before the collision, A has a speed of $3\,\text{m}\,\text{s}^{-1}$ and B has a speed of $1\,\text{m}\,\text{s}^{-1}$.

After the collision A continues in the same direction but at a speed of $1\,\text{m}\,\text{s}^{-1}$.

After the collision B reverses its direction of travel and has a speed of $3\,\text{m}\,\text{s}^{-1}$.

The magnitude of the impulse received by A due to its impact with B is $1.2\,\text{N}\,\text{s}$.

b Find m_1.

(2)

c Find m_2.

(3)

(Total for Question 1 is 7 marks)

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2 A light elastic string, of natural length 12 cm, with modulus of elasticity λ N, is connected at one end to a fixed point X.

A particle of mass 200 g is attached to the other end of the string such that the particle hangs freely in equilibrium, vertically below X.

In this equilibrium position the string is 16 cm long.

a Show that $\lambda = 5.88$

(4)

The string and particle are now detached from X and attached to a new fixed point, Y, which lies at one edge of a rough horizontal surface with coefficient of friction $\mu = 0.7$

The string is extended x cm beyond its natural length so that the string lies in a straight line parallel to the surface. The particle lies on the surface, at rest, in equilibrium.

b Find the maximum possible value of x.

(3)

c Explain why this value of x is a maximum.

(1)

(Total for Question 2 is 8 marks)

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3 A high speed train, of total mass 4.2×10^5 kg, is travelling along a straight horizontal track.

The train's engine is working at a constant rate of 8.1×10^6 W.

At the instant the train has a speed of $30\,\mathrm{m\,s^{-1}}$ it has an acceleration of $0.3\,\mathrm{m\,s^{-2}}$.

At this same instant, the total resistance to motion experienced by the train is R Newtons.

Calculate the value of R.

(4)

(Total for Question 3 is 4 marks)

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4 A straight, rigid slide d metres long is inclined at $30°$ to the horizontal. This slide is modelled as the line of greatest slope of an inclined plane.

A child, Gemma, of mass m kg, pushes herself from the top of the slide so she begins her descent along the line of greatest slope with an initial velocity of $0.1\,\text{m s}^{-1}$.

The coefficient of friction between Gemma and the slide is 0.5.

Gemma's speed is $v\,\text{m s}^{-1}$ when she reaches the bottom of the slide.

a Show, by considering the work–energy principle, that

$$v^2 = \left(1 - \frac{\sqrt{3}}{2}\right)gd + \frac{1}{100}$$

(5)

b Given that $v = 2\,\text{m s}^{-1}$, find the length of the slide. Give your answer to the nearest centimetre.

(1)

Gemma now chooses to sit on a mat to try and **eliminate** friction as she slides for a second time down the same slide.

She again begins her descent with an initial velocity of $0.1\,\text{m s}^{-1}$.

Gemma's speed is now $v = 5\,\text{m s}^{-1}$ when she reaches the bottom of the slide.

c Prove, by considering conservation of mechanical energy, that friction was not eliminated.

(4)

(Total for Question 4 is 10 marks)

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5 Two particles of equal mass, m kg, are moving with constant velocity in a two dimensional plane.

The first particle has initial velocity $u_1 = (2\mathbf{i} + 4\mathbf{j})\,\text{m}\,\text{s}^{-1}$.

The second particle has initial velocity $u_2 = (3\mathbf{i} - 7\mathbf{j})\,\text{m}\,\text{s}^{-1}$.

An impulse of $(4m\mathbf{i} - 18m\mathbf{j})\,\text{kg}\,\text{m}\,\text{s}^{-1}$ is then applied to the first particle.

a Prove that the first particle travels in a direction parallel to the second particle as a result of this impulse.

(4)

The second particle now collides with a third particle, of mass $3m$ kg, which is travelling with initial constant velocity $u_3 = (5\mathbf{i} + 8\mathbf{j})\,\text{m}\,\text{s}^{-1}$.

At the point of collision, the two particles coalesce and the new larger particle thus formed begins to move with velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$.

b Determine the speed of this new larger particle.

(4)

(Total for Question 5 is 8 marks)

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6 Two particles A and B lie at rest in a straight line on a smooth horizontal plane. The particles A and B have masses, in kilograms, of $4m$ and $7m$ respectively.

Particle A is projected towards particle B with constant speed $u\,\text{m s}^{-1}$. The coefficient of restitution between these two particles is e.

After the collision with A, B then collides directly with a smooth vertical wall.

The coefficient of restitution between B and the wall is $\frac{1}{6}e$.

a Show that the speed, $w\,\text{m s}^{-1}$, of particle B after colliding with the wall is given by:

$$w = \frac{2}{33}eu(1+e)$$

(4)

b Given that particle B collides with particle A for a second time, determine the range of possible values for e.

(5)

Particle A comes to instantaneous rest as a result of first colliding with particle B.

c Find an exact expression in terms of m and u for the kinetic energy lost by B when B first **collides** with the wall.

(3)

(Total for Question 6 is 12 marks)

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7 A spring, of natural length 0.25 m and modulus of elasticity $20\,\text{N m}^{-2}$, is attached at one end to a fixed point P so that it hangs vertically.

A particle of mass 1.2 kg is attached to the spring's free end and this system is held in position so that the particle hangs 0.3 m vertically below P.

From this position, the particle is projected vertically downwards with an initial velocity of $u\,\text{m s}^{-1}$.

For modelling purposes it may be assumed there are no resistance forces to the motion.

The particle first comes to rest when it has travelled a distance of 20 cm.

a Using a suitable model for the energy in this system, show that $u = 0.28\,\text{m s}^{-1}$.

(5)

The above starting conditions for the same particle are now repeated but the initial downward velocity given to the particle is increased to $1\,\text{m s}^{-1}$.

The particle now first comes to rest a distance d cm below P.

b Find d to the nearest centimetre.

(5)

The model's prediction for d in part **b** is now checked experimentally.

The distance d is measured to be 54 cm.

c **i** Calculate the extra loss in energy suggested by this result.

(3)

ii Explain the likely nature of this extra energy loss.

(1)

(Total for Question 7 is 14 marks)

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8 Two smooth spheres, P, of mass 0.5 kg, and Q, of mass 1.2 kg, have equal radii.
P and Q are moving on a smooth horizontal surface so that they collide obliquely.

Immediately before the collision:

- P has speed $2u$ inclined at $\theta°$ to the line of centres of the spheres at the instant of impact.
- Q has speed u also inclined at $\theta°$ to the line of centres of the spheres.

This is shown as a plan view in Figure 1.

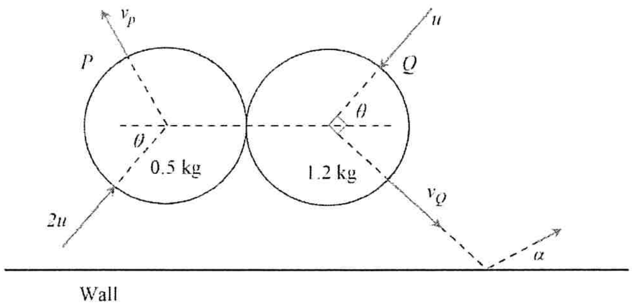

Figure 1

Immediately after the collision:
P has speed v_P.
Q travels at 90° to its path before the collision and has speed v_Q.

At the instant of impact, a smooth vertical wall lies parallel to the line of centres of the spheres.
After colliding with P, Q collides with this wall.
After its collision with the wall, Q moves away from the wall at an angle $\alpha°$ as shown.

The coefficient of restitution between P and Q is e.

The coefficient of restitution between Q and the wall is $\frac{1}{5}e$.

Given that $\tan\theta = \frac{1}{6}$

a Show that $e = \frac{89}{540}$

(8)

b Find the exact value of $\tan\alpha$.

(4)

(Total for Question 8 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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