Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
1a	$400\text{ m} \sim \text{Po}(2)$ $\text{P}(X \geqslant 2) = 1 - \text{P}(X \leqslant 1) = 0.5940$	M1 A1	1.1b 1.1b	3rd Use the Poisson distribution to model real-world situations
		(2)		
1b	$\text{P}(X \leqslant 3) = 0.8571$	B1	1.1a	3rd Use the Poisson distribution to model real-world situations
		(1)		
1c	$(0.8571)^5 = 0.4625$	M1 A1	2.1 1.1b	3rd Use the Poisson distribution to model real-world situations
		(2)		
1d	$\text{H}_0 : \lambda = 5 \qquad \text{H}_1 : \lambda < 5$ $\text{P}(X \leqslant 2) = 0.1247$ Do not reject H_0. There is no evidence to suggest that the new cloth is an improvement on the previous one	B1 M1 A1 M1 A1	2.5 1.1b 1.1b 2.4 2.2b	4th Carry out one-tailed tests for the mean of a Poisson distribution
		(5)		
1e	From tables Po(5) CR $= X \leqslant 1$ $\text{P(Type II error)} = \text{P}(X \geqslant 2 \mid \lambda = 3)$ $1 - \text{P}(X \leqslant 1)$ $1 - 0.1991 = 0.8009$	B1 M1 M1 A1	2.1 3.1a 1.1b 1.1b	6th Calculate the value of a Type II error using conditional probability
		(4)		
				(14 marks)

Notes
1a **M1:** $\mathrm{P}(X \geqslant 2) = 1 - \mathrm{P}(X \leqslant 1)$
A1: 0.5940
1b **B1:** 0.8471
1c **M1:** $(\text{part(b)})^5$
A1: 0.4625
1d **B1:** Both hypotheses
M1: Writing or using $\mathrm{P}(X \leqslant 2)$
A1: 0.1247
M1: A non-contextual statement follow through $\mathrm{P}(X \leqslant 2)$
A1: There is no evidence to suggest that the new cloth is an improvement on the previous one
1e **B1:** CR $X \leqslant 1$
M1: $\mathrm{P}(X \geqslant 2)$
M1: $1 - \mathrm{P}(X \leqslant 1)$
A1: 0.8009

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
2a	$P(X \leqslant 3) = 0.0424$ $\quad P(X \leqslant 4) = 0.0996$	M1	3.3	3rd
	$X \leqslant 3$	A1	1.1b	Understand the language of hypothesis testing
		(2)		
2b	$P(X \leqslant 3) = 0.0424$	B1 cao	1.2	5th Know the definition of the size of a test
		(1)		
2c	Power function = $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$	M1	2.1	7th
	$e^{-\lambda} + e^{-\lambda}\lambda + \frac{e^{-\lambda}\lambda^2}{2!} + \frac{e^{-\lambda}\lambda^3}{3!}$	M1 A1	3.4 1.1b	Use standard probability distributions to find the power of a test
	$e^{-\lambda}\left(1 + \lambda + \frac{1}{2}\lambda^2 + \frac{1}{6}\lambda^3\right)$ *	A1 cso	1.1b	
		(4)		
				(7 marks)

Notes

2a **M1:** $P(X \leqslant 3) = 0.0424$ or $P(X \leqslant 4) = 0.0996$

A1: $X \leqslant 3$ oe

2b **B1:** 0.0424 cao

2c **M1:** $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

M1: Use of $\frac{e^{-\lambda}\lambda^r}{r!}$ (may be implied by 2 correct terms)

A1: All 4 correct terms

A1: cso A correct solution with no incorrect working seen

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
3ai	$X \sim$ Negative B(3, 0.7) $P(X=5)=\binom{4}{2}0.7^3 0.3^2=0.18522$	M1 A1	3.3 1.1b	5th Use the negative binomial distribution to model real-world situations
		(2)		
3aii	$Y \sim$ Negative B(5, 0.7) $P(Y=9)=\binom{8}{4}0.7^5 0.3^4=0.095295...$	M1 A1	3.3 1.1b	5th Use the negative binomial distribution to model real-world situations
		(2)		
3b	Penalty kicks are independent Probability of success is the same for each penalty kick	B1 B1	3.5b 3.5b	5th Understand the assumptions necessary for the negative binomial distribution
		(2)		
				(6 marks)

Notes

3ai M1: Use of Negative B(3, 0.7) with $P(X = 5)$

A1: awrt 0.185

3aii M1: Use of Negative B(5, 0.7) with $P(Y = 9)$

A1: awrt 0.0953

3b B1: Penalty kicks are independent

B1: Probability of success is the same for each penalty kick

NB: Context only needs to be seen in one answer to allow B2

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
4a	$\frac{1-p}{p^2}=6$	M1	3.1b	6th Calculate the mean of a geometric distribution
	$6p^2+p-1=0$	M1	1.1b	
	$(3p-1)(2p+1)=0$	M1	1.1b	
	$p=\frac{1}{3}$	A1	1.1b	
	$E(X)=\frac{1}{\frac{1}{3}}=3$	M1 A1	3.4 1.1b	
		(6)		
4bi	$P(X=4)=\frac{1}{3}\left(\frac{2}{3}\right)^3=\frac{8}{81}$	M1 A1	2.1 1.1b	5th Use the geometric distribution to model real-world situations
		(2)		
4bii	$P(X>4)=\left(\frac{2}{3}\right)^4=\frac{16}{81}$	M1 A1	2.1 1.1b	5th Use the geometric distribution to model real-world situations
		(2)		
4c	$P(Y=1)=p$ $\quad P(Y=3)=(1-p)^2p$ $\quad P(Y=5)=(1-p)^4p$	B1	3.3	7th Solve problems using the geometric distribution in a range of contexts
	Common ratio $=(1-p)^2$	M1	3.1a	
	$S_\infty=\frac{p}{1-(1-p)^2}$	M1	2.1	
	$=\frac{p}{2p-p^2}=\frac{1}{2-p}$ *	M1 A1 cso	1.1b 1.1b	
		(5)		
				(15 marks)

Notes

4a **M1:** $\frac{1-p}{p^2}=6$

M1: Rearranging to a 3 term quadratic

M1: Solving a 3 term quadratic

A1: $p=\frac{1}{3}$ must reject the other root

M1: Attempting $\mathrm{E}(X)=\frac{1}{p}$

A1: 3

4bi **M1:** $p(1-p)^3$

A1: $\frac{8}{81}$

4bii **M1:** for $(1-p)^4$

A1: $\frac{16}{81}$

4c **B1:** At least 2 terms from $\mathrm{P}(Y=1)=p$ $\mathrm{P}(Y=3)=(1-p)^2p$ $\mathrm{P}(Y=5)=(1-p)^4p$

M1: Recognising that the common ratio is $(1-p)^2$

M1: Use of the sum to infinity

M1: Simplifying to $\frac{p}{2p-p^2}$ oe

A1: cso A correct solution with no incorrrect working seen

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
5a	$k(1 + 1 + 1) = 3$	M1	2.1	5th Understand the definition of a probability generating function (p.g.f.)
	$k = \frac{1}{3}$	A1	1.1b	
		(2)		
5b	x: 0, 2, 5; $\mathrm{P}(X = x)$: $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$	B1	1.1b	6th Use the definition of a p.g.f. to find the p.g.f. of a given probability distribution
		B1	1.1b	
		(2)		
5c	$\mathrm{G}'_X(t) = \frac{1}{3}\left(2t + 5t^4\right)$	M1	2.1	6th Use a given p.g.f. to calculate the mean of a probability distribution
	$\mathrm{E}(X) = \mathrm{G}'_X(1) = \frac{7}{3}$	A1	1.1b	
		(2)		
5d	$\mathrm{G}''_X(t) = \frac{1}{3}\left(2 + 20t^3\right)$	M1	2.1	7th Use a given p.g.f. to calculate the variance of a probability distribution
	$\mathrm{Var}(X) = \frac{22}{3} + \frac{7}{3} - \left(\frac{7}{3}\right)^2 = \frac{38}{9}$ oe, e.g. $\frac{114}{27}$	M1 A1	1.1b 1.1b	
		(3)		
				(9 marks)

Notes

5a **M1:** Use of $G_X(1) = 1$

A1: $k = \frac{1}{3}$

5b **B1:** Top row

B1: Bottom row

5c **M1:** An attempt to find $G'_X(t)$

A1: $G'_X(1) = \frac{7}{3}$

5d **M1:** An attempt to find $G''_X(t)$

M1: $\text{Var}(x) = G''_X(1) + G'_X(1) - \left(G'_X(1)\right)^2$

A1: $\frac{38}{9}$

<table>
<tr><th>Q</th><th>Scheme</th><th>Marks</th><th>AOs</th><th>Pearson Progression Step and Progress descriptor</th></tr>
<tr><td rowspan="2">6ai</td><td>
<table>
<tr><th>Time Interval</th><th>O</th><th>E</th></tr>
<tr><td>8:30 – 11:30</td><td>20</td><td>12</td></tr>
<tr><td>11:30 – 1:30</td><td>3</td><td>8</td></tr>
<tr><td>1:30 – 3 :30</td><td>6</td><td>8</td></tr>
<tr><td>3:30 – 5:30</td><td>7</td><td>8</td></tr>
</table>

H_0 : Uniform distribution is a suitable model
H_1 : Uniform distribution is not a suitable model

$\sum\frac{(O-E)^2}{E}=\frac{8^2}{12}+\frac{5^2}{8}+\frac{2^2}{8}+\frac{1^2}{8}=9.08...$

CV $\chi^2=6.251$

Reject H_0. Significant evidence that the uniform distribution does not model the distribution of daily times of false alarms</td><td>M1
A1

B1

M1
A1

B1

M1
A1</td><td>3.4
1.1b

2.5

1.1b
1.1b

1.1a

2.4
2.2b</td><td>6th

Know how to apply the goodness of fit test to a discrete uniform distribution</td></tr>
<tr><td></td><td>(8)</td><td></td><td></td></tr>
<tr><td rowspan="2">6aii</td><td>If the technician's belief was correct a uniform distribution would provide an adequate model

As we have rejected H_0 we reject the technician's belief</td><td>M1

A1</td><td>2.2b

3.5a</td><td>6th

Know how to apply the goodness of fit test to a discrete uniform distribution</td></tr>
<tr><td></td><td>(2)</td><td></td><td></td></tr>
<tr><td rowspan="2">6b</td><td>Number of false alarms between 3:30 to 5:30 is slightly less than expected

No evidence to support the manager's belief as the number between 4 pm and 5:30 pm is unknown</td><td>B1

B1

B1</td><td>3.3

3.5a

3.4</td><td>7th

Apply chi-squared tests in context and in unfamiliar situations</td></tr>
<tr><td></td><td>(3)</td><td></td><td></td></tr>
</table>

6c	Inappropriate as expected value for a single hour would be 4, which is < 5 and so test would be invalid	**M1** **A1**	3.1a 3.2b	4th Understand the principle of a goodness of fit test
		(2)		
				(15 marks)

Notes

6ai M1: Calculating expected values (may be implied by one correct expected value)

A1: All expected values

B1: Both hypotheses

M1: Attempting to find $\sum\frac{(O-E)^2}{E}$

A1: awrt 9.08

B1: CV = 6.251

M1: A non-contextual statement follow through their CV and test statistic

A1: Significant evidence that the uniform distribution does not model the distribution of daily times of false alarms

6aii M1: If the technician's belief was correct a uniform distribution would provide an adequate model

A1: Reject the technician's belief

6b B1: The number of false alarms between 3:30 to 5:30 is slightly less than expected

B1: No evidence to support the manager's belief

B1: The number between 4 pm and 5:30 pm is unknown

6c M1: Expected value for a single hour would be 4

A1: Which is < 5 and so the test would be invalid

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
7	$E(X)=\left(0\times\frac{1}{4}\right)+\left(1\times\frac{1}{2}\right)+\left(2\times\frac{1}{8}\right)+\left(3\times\frac{1}{8}\right)=\frac{9}{8}$	M1	1.1b	8th Recognise and apply the central limit theorem in contextualised situations
	$E(X^2)=\left(0^2\times\frac{1}{4}\right)+\left(1^2\times\frac{1}{2}\right)+\left(2^2\times\frac{1}{8}\right)+\left(3^2\times\frac{1}{8}\right)=\frac{17}{8}$	M1	1.1b	
	$\text{Var}(X)=\frac{17}{8}-\left(\frac{9}{8}\right)^2=\frac{55}{64}$	M1 A1	1.1b 1.1b	
	Let T be the RV corresponding to the total of the 500 observations			
	$E(T)=500\times\frac{9}{8}=562.5$	M1	3.1a	
	$\text{Var}(T)=500\times\frac{55}{64}=\frac{6875}{16}$	M1	1.1b	
	By CLT $T\sim N\left(562.5,\frac{6875}{16}\right)$			
	$P(T<520)=P\left(Z<\frac{520-562.5}{\sqrt{\frac{6875}{16}}}\right)=P(Z<-2.05)$	M1 A1	3.4 1.1b	
	$1-P(Z<2.05)=0.0202$	A1	1.1b	
		(9)		
				(9 marks)

Notes

7 **M1:** $\left(0\times\frac{1}{4}\right)+\left(1\times\frac{1}{2}\right)+\left(2\times\frac{1}{8}\right)+\left(3\times\frac{1}{8}\right)=\frac{9}{8}$

M1: $\left(0^2\times\frac{1}{4}\right)+\left(1^2\times\frac{1}{2}\right)+\left(2^2\times\frac{1}{8}\right)+\left(3^2\times\frac{1}{8}\right)=\frac{17}{8}$

M1: Use of $\text{Var}(X)=E\left(X^2\right)-(E(X))^2$

A1: $\frac{55}{64}$

M1: Finding E(T)

M1: Finding Var(T)

M1: Standardising using 520 and E(T) and Var(T)

A1: $\frac{520-562.5}{\sqrt{\frac{6875}{16}}}$ or awrt –2.05

A1: awrt 0.0202