

Further Mathematics

Advanced

Paper 3B: Further Statistics 1

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You must have: Mathematical Formulae and Statistical Tables, calculator	
Time	1 hour 30 minutes

Name	
Class	
Teacher name	

Total marks	/75
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Answer ALL questions.

- 1 Flaws in a certain type of cloth occur at random and at an average of 0.5 per 100 metres. Assuming that the flaws in the cloth follow a Poisson distribution, find the probability that in a piece of cloth of length 400 metres,
- a there will be at least two flaws, (2)
 - b there will be at most three flaws. (1)
- In a batch there are 5 pieces of cloth, each of length 400 metres.
- c Find the probability that each of the pieces of cloth in a batch will contain fewer than 4 flaws. (2)
- A new type of cloth is developed and in 1 kilometre of this new cloth, 2 flaws were found.
- d Stating your hypotheses clearly, test at the 5% significance level whether this new type of cloth is an improvement on the previous one. (5)
 - e If the true average rate of flaws occurring in one kilometre of the new type of cloth was 3, calculate the P(Type II error) for this test. (4)

(Total for Question 1 is 14 marks)

2 A single observation x is taken from a Poisson distribution with parameter λ . The observation is to be used to test $H_0 : \lambda = 8$ against $H_1 : \lambda < 8$ using a 5% level of significance.

a Find the critical region for this test.

(2)

b Find the size of this test.

(1)

c Show that the power function of this test is given by

$$e^{-\lambda} \left(1 + \lambda + \frac{1}{2} \lambda^2 + \frac{1}{6} \lambda^3 \right)$$

(4)

(Total for Question 2 is 7 marks)

3 Bob is a footballer. On average he scores a goal with 70% of his penalty kicks, so his probability of scoring with a penalty kick is 0.70.

During the season, find the probability that

a i Bob scores his third penalty goal with his fifth penalty kick.

(2)

ii Bob scores his sixth penalty goal with his tenth penalty kick, given that he scored with his first penalty kick.

(2)

b State two assumptions that have to be made for the model used in part a to be valid.

(2)

(Total for Question 3 is 6 marks)

4 A random variable X has a geometric distribution with variance 6.

a Find $E(X)$

(6)

b Find

i $P(X = 4)$

(2)

ii $P(X > 4)$

(2)

A random variable Y has the distribution $\text{Geo}(p)$.

c Show that

$$P(Y \text{ is odd}) = \frac{1}{2-p}$$

(5)

(Total for Question 4 is 15 marks)

5 The probability generating function of the discrete random variable X is given by

$$G_X(t) = k(1 + t^2 + t^5)$$

a Find the value of k .

(2)

b Write down the probability distribution of X .

(2)

c Work out $E(X)$.

(2)

d Work out $\text{Var}(X)$.

(3)

(Total for Question 5 is 9 marks)

- 6 An office building has a new fire alarm system installed but seems to be suffering from a large number of false fire alarms during office hours. Office hours are from 8:30 am to 5:30 pm. The times at which these false alarms have occurred are summarised in the table below.

Time interval	8:30 am to 11:30 am	11:30 am to 1:30 pm	1:30 pm to 3:30 pm	3:30 pm to 5:30 pm
Number of false alarms	20	3	6	7

A technician believes that the false alarms are caused by electrical faults and are equally likely to occur at any time throughout the day.

- a i Using a χ^2 distribution, at the 10% level of significance, determine whether a uniform distribution provides an adequate model for the data. (8)
- ii Comment on the technician's belief. (2)

When the fire alarm goes off after 4 pm, the office closes for the day.

The manager believes that some of the false alarms are deliberately caused by office staff wishing to go home early.

- b Comment on the manager's belief. (3)

The manager argues that, in order to investigate his belief thoroughly, it would be necessary to break the table down into 9 one-hour slots before applying a χ^2 test.

- c Explain why this would not be appropriate. (2)

(Total for Question 6 is 15 marks)

- 7 The discrete random variable X has probability distribution given by

x	0	1	2	3
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$

Find an approximation to the probability that a random sample of 500 observations on X will have a total less than 520.

(9)

(Total for Question 7 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS