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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **1a** |  | **B1** | 1.1b | 4thBe able to differentiate standard hyperbolic functions |
|  | **M1** | 3.1b |
|  | **A1** | 1.1b |
|  | **M1** | 1.1b |
| **Alt** |  | **M1** | 1.1b |
|  | **B1** | 1.1b |
|  | **M1****A1** | 3.1b1.1b |
|  | **M1** | 1.1b |
|  | **A1**\* | 2.1 |
|  | **(6)** |  |  |
| **1b** |  | **B1ft** | 3.1b | 4thBe able to differentiate standard hyperbolic functions |
|  | **B1cao** | 2.4 |
|  | **(2)** |  |  |
| (8 marks) |

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| Notes1a B1: Correct differentiation **M1:** Sets derivative = 0 and obtains tanh 2*t* = *k*,  **A1:** Correct value for *t* **M1:** Uses correct logarithmic form of artanh *x***Alt M1:** Uses correct exponential forms of sinh 2*t* and cosh 2*t* **B1:** Correct differentiation **M1:** Sets derivative = 0 and reaches *t* = ... **A1:** Correct value for *t* **M1:** Uses correct exponential forms of sinh 2*t* and cosh 2*t* **A1\*:** Achieves given answer with no errors**1b B1ft:** Correct differentiation of their derivative *p* sinh 2*t* + *q* cosh 2*t* or  *p*,  **B1:** Obtains correct , shows it is positive and concludes minimum. If no substitution of  is seen there must be justification of  (e.g. an appropriate sketch graph of *y* = 20 cosh 2*t* – 12 sinh 2*t*) |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **2a** |  | **M1** | 1.1b | 3rdUnderstand the definitions of hyperbolic functions |
|  | **M1** | 1.1b |
|  | **A1\*** | 2.1 |
| **Alt** | cosh2 *x* – sinh2 *x* = (cosh *x* + sinh *x*)(cosh *x* – sinh *x*) | **M1** | 1.1b |
|  | **M1** | 1.1b |
|  | **A1\*** | 2.1 |
|  | **(3)** |  |  |
| **2b** |  | **M1** | 1.1b | 3rdUnderstand the definitions of hyperbolic functions |
| 2 sinh *x* = cosh2 *x*2 sinh *x* = 1 + sinh2 *x*sinh2 *x* – 2 sinh *x* + 1 = 0 | **M1** | 1.1b |
| (sinh *x* – 1)2 = 0sinh *x* = 1 | **M1****A1** | 1.1b1.1b |
|  | **A1** | 1.1b |
|  | **(5)** |  |  |
| (8 marks) |

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| Notes**2a M1:** Uses correct exponential forms of sinh *x* and cosh *x* **M1:** Expands at least one bracket correctly **A1\*:** Obtains 1 with no errors. Both LHS and RHS of given answer to have been seen**Alt M1:** Correct factorisation of cosh2 *x* – sinh2 *x* **M1:** Uses correct exponential forms of sinh *x* and cosh *x* **A1\*:** Obtains 1 with no errors. Both LHS and RHS of given answer to have been seen**2b M1:** Uses  **M1:** Uses cosh2 *x* = 1 + sinh2 *x* and obtains 3TQ in sinh *x* **M1:** Solves their 3TQ in sinh *x* (usual rules) **A1:** Correct value for sinh *x* **A1:**  only |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **3a** |  | **B1****B1** | 1.1b1.1b | 3rdSketch the graphs of hyperbolic functions |
| *y* = –1 | **B1** | 2.2a |
| (0, 5) | **B1** | 1.1b |
| (5 ln 6, 0) | **B1** | 1.1b |
|  | **(5)** |  |  |

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| **3b** |  | **M1** | 3.1a | 3rdUnderstand the definitions of hyperbolic functions |
|  | **M1** | 1.1b |
|  | **M1****A1** | 1.1b1.1b |
|  | **A1cao** | 1.1b |
|  | **(5)** |  |  |
| (10 marks) |
| Notes**3a B1:** Uses the correct logarithmic form of artanh *x* with *x* = **B1:** Correct **shape** of *C*2 **B1:** *y* = – 1 only (asymptote must be seen as an equation). Withhold if other asymptotes are given **B1:** (0,5) - allow 5 indicated on *y*-axis **B1:** (5 ln 6, 0) or exact equivalent (e.g. ). Allow if indicated on *x*-axis If only coordinates are seen and answers are given as (5, 0) and (0, 5 ln 6) score B1 B0**3b M1:** Substitutes correct exponential form of sinh  into equation of intersection **M1:** Obtains 3TQ in  **M1:** Solves their 3TQ in  (usual rules) **A1:** Obtains  **A1cao:**  only. Withhold if any incorrect work is seen (e.g. a second answer) |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **4a** | artanh  | **M1** | 1.1b | 4thUnderstand and use the domains and ranges of inverse hyperbolic functions |
| = 1n 25 or 1n 5*k* = 5\* | **A1** | 2.1 |
|  | **(2)** |  |  |
| **4b** | *y* = *x* artanh 3*x* *u* = *x* *v* = artanh 3*x* | **B1** | 1.1b | 5thBe able to differentiate hyperbolic functions using chain, product and quotient rules |
|  | **M1** | 1.1b |
|  | **A1** | 1.1b |
|  | **(3)** |  |  |
| **4c** |  | **M1** | 2.2a | 7thBe able to solve calculus problems with hyperbolic functions in a range of familiar contexts |
|  | **M1****A1** | 1.1b1.1b |
|        | **dM1** | 1.1b |
|  | **M1** | 1.1b |

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|  |  | **A1** | 1.1b |  |
|  | **(6)** |  |  |
| (11 marks) |
| Notes**4a M1:** Uses the correct logarithmic form of artabh *x* with  **A1\*:** Obtains *k* = 5 with an intermediate step and no errors seen**4b B1:**  **M1:** Uses correct product rule **A1:** Correct derivative**4c M1:** Rearranges the result from part **b** or uses integration by parts in the correct direction **M1:** Integrates  to obtain *r* ln(1 + *qx*2), *p*, *q*,  **A1:** Correct derivative **dM1:** Substitutes correct limit. The substitution of the limit 0 must be seen and subtracted if it does not give 0. Dependent on first two method marks **M1:** Uses correct logarithmic form of artanh *x* **A1:**  only |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **5a** |  | **B1** | 1.1b | 6thBe able to integrate functions using hyperbolic substitutions |
|  | **M1** | 1.1b |
|    | **M1** | 1.1b |
|  | **M1** | 1.1b |
|  | **A1** | 1.1b |
|  | **M1** | 2.2a |
|  | **M1** | 1.1b |
|  | **A1\*** | 2.1 |
|  | **(8)** |  |  |

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| **5bi** | For *t* = 2 (*t* < 3),  is undefined since  | **B1** | 3.5b | 8thBe able to solve calculus problems with hyperbolic functions in a range of unfamiliar contexts |
|  | **(1)** |  |
| **5bii** |  | **M1** | **3.1** |
|  | **M1** | **1.1b** |
|  | **M1** | **1.1b** |
|  | **A1** | **3.4** |
|  | **(4)** |  |  |
| (13 marks) |
| Notes**5a B1:** Correct derivative **M1:** Uses their  and *x* = 3 cosh *u* to obtain a complete substitution from *x* to *u* **M1:** Uses  **M1:** Uses  **A1:** Correct integration **M1:** Uses the correct hyperbolic identities sinh 2*u* = sinh *u* cosh *u* and sinh  to obtain an expression in *u* and cosh *u* only **M1:** Uses cosh  and  to obtain an expression in *x* only **A1\*:** Obtains given answer with no errors**5bi B1:** Accept equivalents but answer must explain why the model is unsuitable, i.e., not just, e.g., “*v* is undefined”**5bii M1:** A correct method for the distance to owner. This statement is sufficient (may be in *x*). Subtraction from 15 may be seen later **M1:** Substitutes their limits into the result from part **a**. Subtraction of lower limit to be seen if limit is not 3 **M1:** Uses correct logarithmic form of arcosh *x* **A1:** Exact answer only. Accept simplified equivalents, e.g.  |