**1** The random variable *X* has a Poisson distribution *X* ~ Po(5). A sample is taken, and it is desired to test H0: *λ* = 5 against H1: *λ* > 5, using a 5% level of significance.

**a** Find the critical region for this test. **(2 marks)**

**b i** Define a Type I error. **(1 mark)**

**ii** State the probability of a Type I error. **(1 mark)**

**2** The random variable *Y* is binomially distributed. A sample of size 10 is taken and it is desired to test H0: *p* = 0.35 against H1: *p* ≠ 0.35, using a 5% level of significance.

**a** Calculate the probability of a Type I error **(4 marks)**

Given that true probability is later found to be 0.4

**b i** define a Type II error. **(1 mark)**

**ii** calculate the probability of a Type II error. **(4 marks)**

**3** A manager of a canning factory claims that the amount of soup in a can, measured
in ml, is normally distributed with a mean of 550 ml and a variance of 12 ml2

A random sample of 20 cans is taken to check if there has been a decrease in the mean amount of soup per can. You can assume that the variance has remained unchanged.

**a** Find, at the 5% level of significance, the critical region for the test statistic  the mean amount of soup in a sample of 20 cans. **(4 marks)**

Given that the actual mean amount has decreased to 549 ml,

**b** find the probability of a Type II error. **(2 marks)**

The manager would like to see the probability of a Type II error reduced.

She considers changing the significance level.

**c** State, giving a reason, what she should do to the significance level to achieve her aim. **(1 mark)**

**4** Explain briefly what you understand by

**a** the size of a hypothesis test **(1 mark)**

**b** the power of a hypothesis test. **(1 mark)**

The random variable *X* has a Poisson distribution. A sample is taken, and it is desired to test H0: *λ* = 7 against H1: *λ* < 7, using a 10% level of significance.

**c** Find

**i** the critical region for the test and state the size of the test. **(3 marks)**

**ii** the power of the test, given that *λ* is in fact 7.5. **(2 marks)**

**5** It is thought that a biased coin lands on heads with probability *p*. The coin is tossed
15 times and the random variable *Y* represents the number of heads thrown.

In a test of H0: *p* = 0.4 against H1: *p* < 0.4, the null hypothesis is rejected if the number of successes is less than 3

**a** Show that the power function for this test is given by

 **(4 marks)**

**b** Find the power of this test when *p* = 0.3 **(2 marks)**

**6** Jack and George are playing a computer game where they play the role of a cat who has to catch a virtual mouse. They are testing to see if the probability of catching the mouse is less than 0.15

They both use a 5% significance level.

Jack plays the game until he catches the mouse and he records the number of attempts.

**a** Find the critical region for Jack’s test. **(5 marks)**

**b** Find the size of Jack’s test. **(1 mark)**

**c** Find the power function for Jack’s test. **(1 mark)**

George plays the game 31 times and records the number of times he catches the mouse.

**d** Find the size of George’s test. **(3 marks)**

**e** Show that the power function for George’s test is

 **(3 marks)**

**f** With reference to your answers to parts **b**, **c**, **d** and **e**, state, giving reasons, whether you would recommend Jack’s test or George’s test when *p* = 0.12 **(4 marks)**