# Series

**1** Find $\sum_{r=1}^{n} r^2(r-3)$.

**2** Show that $\sum_{r=1}^{2n} (2r-1)^2 = \frac{2n}{3}(16n^2-1)$.

**3** **a** Show that $\sum_{r=1}^{n} r(r+2) = \frac{n}{6}(n+1)(2n+7)$.

**4** Show that $\sum_{r=n}^{2n} r^2 = \frac{n}{6}(n+1)(an+b)$, where $a$ and $b$ are constants to be found.

**5** **a** Show that $\sum_{r=1}^{n} (r^2-r-1) = \frac{n}{3}(n-2)(n+2)$.

**b** Hence calculate $\sum_{r=10}^{40} (r^2-r-1)$.

**6** **a** Show that $\sum_{r=1}^{n} r(2r^2+1) = \frac{n}{2}(n+1)(n^2+n+1)$.

**b** Hence calculate $\sum_{r=26}^{58} r(2r^2+1)$.

**7** Find **a** $\sum_{r=1}^{n} r(3r-1)$ **b** $\sum_{r=1}^{n} (r+2)(3r+5)$ **c** $\sum_{r=1}^{n} (2r^3-2r+1)$.

**8** **a** Show that $\sum_{r=1}^{n} r(r+1) = \frac{n}{3}(n+1)(n+2)$.

**b** Hence calculate $\sum_{r=31}^{60} r(r+1)$.

**9** **a** Show that $\sum_{r=1}^{n} r(r+1)(r+2) = \frac{n}{4}(n+1)(n+2)(n+3)$.

**b** Hence evaluate $3 \times 4 \times 5 + 4 \times 5 \times 6 + 5 \times 6 \times 7 + \ldots + 40 \times 41 \times 42$.

**10** **a** Show that $\sum_{r=1}^{n} r\{2(n-r)+1\} = \frac{n}{6}(n+1)(2n+1)$.

**b** Hence sum the series $(2n-1) + 2(2n-3) + 3(2n-5) + \ldots + n$

**11** **a** Show that when $n$ is even,

$$1^3 - 2^3 + 3^3 - \ldots - n^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 - 16\left(1^3 + 2^3 + 3^3 + \ldots + \left(\frac{n}{2}\right)^3\right)$$

$$= \sum_{r=1}^{n} r^3 - 16\sum_{r=1}^{\frac{n}{2}} r^3.$$

**b** Hence show that, for $n$ even, $1^3 - 2^3 + 3^3 - \ldots - n^3 = -\frac{n^2}{4}(2n+3)$

**c** Deduce the sum of $1^3 - 2^3 + 3^3 - \ldots - 40^3$.