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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **1a** | gives | **M1** | 2.1 | 5th  Understand the definition of a probability generating function (p.g.f.) |
|  |  | **A1** | 1.1b |  |
|  |  | **(2)** |  |  |
| **1b** |  | **M1** | 1.1b | 6th  Use the definition of a p.g.f. to find the p.g.f. of a given probability distribution |
|  | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | *x* | 0 | 1 | 2 | 3 | 4 | | P(*X* = *x*) |  |  |  |  |  | | **A1**  **A1** | 1.1b  1.1b |  |
|  |  | **(3)** |  |  |
| (5 marks) | | | | |
| Notes  **1a**: **M1** for use of  **A1** for cso  **1b**: **M1** for expanding and simplifying, at least three terms correct  1st **A1** for top row, 2nd **A1** for bottom row. Accept unsimplified fractions | | | | |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **2** | P(*Y* = *y*) = 0.6 × 0.4*y* – 1 | **B1** | 3.1a | 6th  Use the definition of a p.g.f. to find the p.g.f. of a given probability distribution |
|  | **M1** | 1.1b |
|  | **M1** | 1.1b |
|  | **M1** | 1.2 |
|  | **A1** | 2.1 |
| (5 marks) | | | | |
| Notes  B1 for correct expression for P(*Y* = *y*), can be implied from line 2  1st M1 for using definition of p.g.f  2nd M1 for simplifying summation  3rd M1 for using sum of infinite geometric series  A1 only awarded for complete correct derivation with no errors and at least three distinct steps in working | | | | |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **3a** |  | **B1** | 2.4 | 6th  Be able to check the validity of a p.g.f. |
|  | **(1)** |  |  |
| **3b** |  | **M1**  **A1** | 1.1a  1.1b | 5th  Understand the definition of a probability generating function (p.g.f.) |
|  | **(2)** |  |  |
| **3c** | P(*X* = 2) = 0.1 × 4 | **M1** | 1.1a | 5th  Understand the definition of a probability generating function (p.g.f.) |
| = 0.4 | **A1** | 1.1b |
|  | **(2)** |  |  |
| (5 marks) | | | | |
| Notes  **3a**: **B1** for correct statement , can be in words  **3a: B1** for calculating *GX*(1) = 2  **3b**: **M1** for setting up correct p.g.f. in terms of *k* or other constant and substituting *t* = 1  Award **A1** on finding correct *k*, condone not writing out *G* in full afterwards  **3b:** Allow *GX*(*t*) = 0.1(...) directly from part **a** without working  **3c**: **M1A1ft** *their k* if ‘× 4’ | | | | |

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|  | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **4a** |  | **M1** | 2.1 | 6th  Use the definition of a p.g.f. to find the p.g.f. of a given probability distribution |
| Hence | **A1** | 1.1b |
|  | **(2)** |  |  |
| **4b** |  | **M1** | 1.1a | 6th  Use a given p.g.f. to calculate the mean of a probability distribution |
|  | **M1** | 1.1b |
| Hence | **A1** | 2.1 |
|  | **(3)** |  |  |
| (5 marks) | | | | |
| Notes  **4a**: **M1** for at least three correct coefficients and powers of *t*  **A1** cso  **4b**: **M1** for correct statement for E(*X*), may be implied by subsequent working  **M1** for attempt to differentiate, at least three terms correct  **A1** cso | | | | |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **5** |  | **M1** | 1.1a | 6th  Use a given p.g.f. to calculate the mean of a probability distribution |
|  | **M1** | 1.1b |
|  | **M1** | 1.1a |
|  | **M1** | 1.1b |
|  | **A1** | 1.1b |
|  | **A1** | 1.1b |
| (6 marks) | | | | |
| Notes  1st **M1**: Use of  to form equation in *a* and *b*  2nd **M1**: Use of quotient rule to find  3rd **M1**: Substitutes 1 for *t*  4th **M1**: Attempts to substitute and solve for *a* or *b*  **A1** for *b*, **A1** for *a* | | | | |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **6a** | gives | **M1** | 2.1 | 5th  Understand the definition of a probability generating function (p.g.f.) |
|  | **A1** | 1.1b |
|  | **(2)** |  |  |
| **6b** | P(*Y* = 4) is coefficient of *t*4 so | **M1** | 1.1a | 6th  Use the definition of a p.g.f. to find the p.g.f. of a given probability distribution |
|  | **A1** | 1.1b |
|  | **(2)** |  |  |
| **6c** |  | **M1** | 2.1 | 7th  Use a given p.g.f. to calculate the variance of a probability distribution |
|  | **M1** | 1.1b |
| = 4 | **A1** | 1.1b |
|  | **M1**  **A1** | 2.1  1.1b |
|  | **M1** | 1.1b |
| Var(*Y*) = 13 + 4 – 42 | **M1** | 2.1 |
| = 1 \* | **A1** | 1.1b |
|  | **(8)** |  |  |

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| **6d** |  | **M1** | 3.1a | 8th  Use a p.g.f. to find the p.g.f. of a related random variable |
|  | **A1** | 1.1b |
|  | **(2)** |  |  |
| (14 marks) | | | | |
| Notes  **6a**: **M1** for  **A1** cso  **6b**: **M1** for use of *t*4 term  **A1** cao (accept decimal equivalent or )  **6c**: 1st **M1** for attempt to differentiate using chain or product rule, accept expanded expression for *G* and differentiation using standard rules. Structure must be correct and at least four terms correct  2nd **M1** for substituting 1 into *their* expression for *G*’  **A1** for correct mean  3rd **M1** for second derivative using any method, ft *their* *G*’  **A1ft** for correct second derivative, ft *their* *G*’  4th **M1** for substituting 1 into *their* expression for *G*’’  5th **M1** for correct statement for variance with *their* values  **A1** cso  **6d**: **M1** for either multiplying by *t* or substituting *t*3 for *t*  **A1** cao (accept expanded expression) | | | | |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **7a** |  | **M1** | 1.1b | 7th  Use a given p.g.f. to calculate the variance of a probability distribution |
|  | **M1** | 1.1b |
|  | **A1** | 1.1b |
|  | **M1** | 1.1b |
|  | **A1** | 2.1 |
|  | **(5)** |  |  |
| **7b** |  | **M1** | 3.1a | 8th  Use a p.g.f. to find the p.g.f. of a related random variable |
|  | **A1** | 1.1b |
|  | **(2)** |  |  |
| **7c** |  | **M1** | 3.1a | 8th  Use a p.g.f. to find the p.g.f. of a related random variable |
|  | **M1** | 1.1b |
|  | **A1** | 2.1 |
|  | **(3)** |  |  |
| (10 marks) | | | | |

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| Notes  **7a**: 1st **M1** for attempt to find *G*’  2nd **M1** for attempt to find *G*’’  **A1** for both values (may be implied by correct substitution into variance formula)  3rd **M1** for use of variance formula, ft *their* numerical values  **A1** cso  **7b**: **M1** for either multiplying by *t*3 or substituting *t*4 for *t*  **A1** accept in factorised form as long as indices simplified in bracket  **7c**: **B1** for converting binomial distribution into p.g.f  **M1** for attempt to expand product of two functions, ft *their* binomial p.g.f  **A1** for complete correct derivation with at least two clear steps in working (answer given) |