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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **1a** |  gives | **M1** | 2.1 | 5thUnderstand the definition of a probability generating function (p.g.f.) |
|  |  | **A1** | 1.1b |  |
|  |  | **(2)** |  |  |
| **1b** |   | **M1** | 1.1b | 6thUse the definition of a p.g.f. to find the p.g.f. of a given probability distribution |
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| *x* | 0 | 1 | 2 | 3 | 4 |
| P(*X* = *x*) |  |  |  |  |  |

 | **A1****A1** | 1.1b1.1b |  |
|  |  | **(3)** |  |  |
| (5 marks) |
| Notes**1a**: **M1** for use of  **A1** for cso**1b**: **M1** for expanding and simplifying, at least three terms correct 1st **A1** for top row, 2nd **A1** for bottom row. Accept unsimplified fractions |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **2** | P(*Y* = *y*) = 0.6 × 0.4*y* – 1  | **B1** | 3.1a | 6thUse the definition of a p.g.f. to find the p.g.f. of a given probability distribution |
|   | **M1** | 1.1b |
|   | **M1** | 1.1b |
|   | **M1** | 1.2 |
|   | **A1** | 2.1 |
| (5 marks) |
| NotesB1 for correct expression for P(*Y* = *y*), can be implied from line 21st M1 for using definition of p.g.f2nd M1 for simplifying summation3rd M1 for using sum of infinite geometric seriesA1 only awarded for complete correct derivation with no errors and at least three distinct steps in working |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **3a** |   | **B1** | 2.4 | 6thBe able to check the validity of a p.g.f. |
|  | **(1)** |  |  |
| **3b** |   | **M1****A1** | 1.1a1.1b | 5thUnderstand the definition of a probability generating function (p.g.f.) |
|  | **(2)** |  |  |
| **3c** | P(*X* = 2) = 0.1 × 4 | **M1** | 1.1a | 5thUnderstand the definition of a probability generating function (p.g.f.) |
| = 0.4 | **A1** | 1.1b |
|  | **(2)** |  |  |
| (5 marks) |
| Notes**3a**: **B1** for correct statement , can be in words**3a: B1** for calculating *GX*(1) = 2**3b**: **M1** for setting up correct p.g.f. in terms of *k* or other constant and substituting *t* = 1 Award **A1** on finding correct *k*, condone not writing out *G* in full afterwards**3b:** Allow *GX*(*t*) = 0.1(...) directly from part **a** without working**3c**: **M1A1ft** *their k* if ‘× 4’ |

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|  | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **4a** |   | **M1** | 2.1 | 6thUse the definition of a p.g.f. to find the p.g.f. of a given probability distribution |
| Hence  | **A1** | 1.1b |
|  | **(2)** |  |  |
| **4b** |   | **M1** | 1.1a | 6thUse a given p.g.f. to calculate the mean of a probability distribution |
|   | **M1** | 1.1b |
| Hence   | **A1** | 2.1 |
|  | **(3)** |  |  |
| (5 marks) |
| Notes**4a**: **M1** for at least three correct coefficients and powers of *t* **A1** cso**4b**: **M1** for correct statement for E(*X*), may be implied by subsequent working **M1** for attempt to differentiate, at least three terms correct **A1** cso |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **5** |   | **M1** | 1.1a | 6thUse a given p.g.f. to calculate the mean of a probability distribution |
|   | **M1** | 1.1b |
|   | **M1** | 1.1a |
|   | **M1** | 1.1b |
|   | **A1** | 1.1b |
|   | **A1** | 1.1b |
| (6 marks) |
| Notes1st **M1**: Use of  to form equation in *a* and *b*2nd **M1**: Use of quotient rule to find  3rd **M1**: Substitutes 1 for *t*4th **M1**: Attempts to substitute and solve for *a* or *b***A1** for *b*, **A1** for *a* |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **6a** |  gives | **M1** | 2.1 | 5thUnderstand the definition of a probability generating function (p.g.f.) |
|  | **A1** | 1.1b |
|  | **(2)** |  |  |
| **6b** | P(*Y* = 4) is coefficient of *t*4 so   | **M1** | 1.1a | 6thUse the definition of a p.g.f. to find the p.g.f. of a given probability distribution |
|   | **A1** | 1.1b |
|  | **(2)** |  |  |
| **6c** |   | **M1** | 2.1 | 7thUse a given p.g.f. to calculate the variance of a probability distribution |
|   | **M1** | 1.1b |
| = 4 | **A1** | 1.1b |
|   | **M1****A1** | 2.11.1b |
|   | **M1** | 1.1b |
| Var(*Y*) = 13 + 4 – 42 | **M1** | 2.1 |
| = 1 \* | **A1** | 1.1b |
|  | **(8)** |  |  |

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| **6d** |   | **M1** | 3.1a | 8thUse a p.g.f. to find the p.g.f. of a related random variable |
|   | **A1** | 1.1b |
|  | **(2)** |  |  |
| (14 marks) |
| Notes**6a**: **M1** for  **A1** cso**6b**: **M1** for use of *t*4 term **A1** cao (accept decimal equivalent or )**6c**: 1st **M1** for attempt to differentiate using chain or product rule, accept expanded expression for *G* and differentiation using standard rules. Structure must be correct and at least four terms correct 2nd **M1** for substituting 1 into *their* expression for *G*’ **A1** for correct mean 3rd **M1** for second derivative using any method, ft *their* *G*’ **A1ft** for correct second derivative, ft *their* *G*’ 4th **M1** for substituting 1 into *their* expression for *G*’’ 5th **M1** for correct statement for variance with *their* values **A1** cso**6d**: **M1** for either multiplying by *t* or substituting *t*3 for *t* **A1** cao (accept expanded expression) |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **7a** |   | **M1** | 1.1b | 7thUse a given p.g.f. to calculate the variance of a probability distribution |
|   | **M1** | 1.1b |
|   | **A1** | 1.1b |
|   | **M1** | 1.1b |
|   | **A1** | 2.1 |
|  | **(5)** |  |  |
| **7b** |   | **M1** | 3.1a | 8thUse a p.g.f. to find the p.g.f. of a related random variable |
|   | **A1** | 1.1b |
|  | **(2)** |  |  |
| **7c** |   | **M1** | 3.1a | 8thUse a p.g.f. to find the p.g.f. of a related random variable |
|  | **M1** | 1.1b |
|  | **A1** | 2.1 |
|  | **(3)** |  |  |
| (10 marks) |

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| Notes**7a**: 1st **M1** for attempt to find *G*’ 2nd **M1** for attempt to find *G*’’ **A1** for both values (may be implied by correct substitution into variance formula) 3rd **M1** for use of variance formula, ft *their* numerical values **A1** cso**7b**: **M1** for either multiplying by *t*3 or substituting *t*4 for *t* **A1** accept in factorised form as long as indices simplified in bracket**7c**: **B1** for converting binomial distribution into p.g.f **M1** for attempt to expand product of two functions, ft *their* binomial p.g.f **A1** for complete correct derivation with at least two clear steps in working (answer given) |