2.3 On the road

Learning objectives:

- Why do we seem to be thrown outwards if a car rounds a bend too quickly?
- What happens to the force between a passenger and his seat on a curved bridge?
- What forces provide the centripetal force on a banked track?

Specification reference: 3.4.1

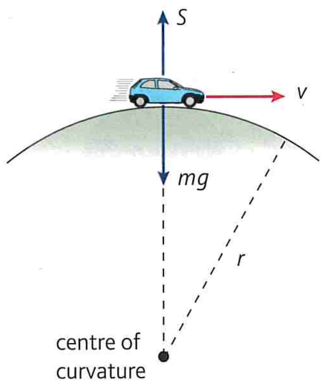

Figure 1 *Over the top*

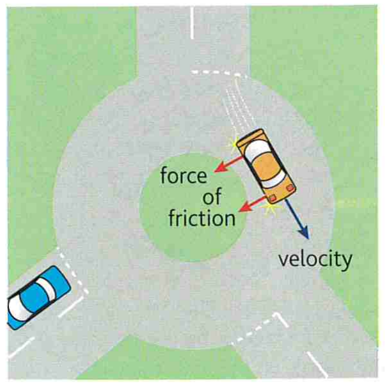

Figure 2 *On a roundabout*

Even on a very short journey, the effects of circular motion can be important. For example, a vehicle that turns a corner too fast could skid or topple over. A vehicle that goes over a curved bridge too fast might even lose contact briefly with the road surface. To make any object move on a circular path, the object must be acted on by a resultant force which is always towards the centre of curvature of its path.

Over the top of a hill

Consider a vehicle of mass m moving at speed v along a road that passes over the top of a hill or over the top of a curved bridge.

At the top of the hill, the support force S from the road on the vehicle is directly upwards in the opposite direction to its weight, mg. The resultant force on the vehicle is the difference between the weight and the support force. This difference acts towards the centre of curvature of the hill as the centripetal force. In other words,

$$mg - S = \frac{mv^2}{r},$$

where r is the radius of curvature of the hill

The vehicle would lose contact with the road if its speed is equal to or greater than a certain speed, v_0. If this happens, then the support force is zero (i.e. $S = 0$) so $mg = \frac{mv_0^2}{r}$.

Therefore, the vehicle speed should not exceed v_0, where $v_0^2 = gr$, otherwise the vehicle will lose contact with the road surface at the top of the hill. Prove for yourself that a vehicle that travels over a curved bridge of radius of curvature 5 m would lose contact with the road surface if its speed exceeded $7\,\mathrm{m\,s^{-1}}$.

On a roundabout

Consider a vehicle of mass m moving at speed v in a circle of radius r as it moves round a roundabout on a level road. The centripetal force is provided by the force of friction between the vehicle's tyres and the road surface. In other words,

$$\text{force of friction } F = \frac{mv^2}{r}$$

For no skidding or slipping, the force of friction between the tyres and the road surface must be less than a limiting value F_0 which is proportional to the vehicle's weight.

Therefore, for no slipping, the speed of the vehicle must be less than a certain value v_0 which is given by the equation,

$$\text{limiting force of friction } F_0 = \frac{mv_0^2}{r}$$

Note: As F_0 is proportional to the vehicle's weight, then $F_0 = \mu mg$, where μ is the coefficient of friction.

Therefore $\mu mg = \frac{mv_0^2}{r}$.

The maximum speed for no slipping, $v_0 = (\mu gr)^{1/2}$

(μ is not on this A2 specification, but you might meet it if you are studying A level maths.)

On a banked track

A race track is often banked where it curves. Motorway slip roads often bend in a tight curve. Such a road is usually banked to enable vehicles to drive round without any sideways friction on the tyres. Rail tracks on curves are usually banked to enable trains to move round the curve without slowing down too much. Imagine you are an engineer and you have to design a banked track for a horizontal motorway curve.

- Without any banking, the centripetal force on a road vehicle is provided only by sideways friction between the vehicle wheels and the road surface. As explained in the previous example, the vehicle on a bend slips outwards if its speed is too high.
- On a banked track, the speed can be higher. To understand why, consider Figure 3 which represents the front-view of a racing car of mass m on a banked track, where θ = the angle of the track to the horizontal. For there to be no sideways friction on the tyres due to the road, the horizontal component of the support forces N_1 and N_2 must act as the centripetal force.

Resolving these forces into horizontal components ($= (N_1 + N_2)\sin\theta$) and a vertical component ($= (N_1 + N_2)\cos\theta$), then

- because $(N_1 + N_2)\sin\theta$ acts as the centripetal force,

  then $(N_1 + N_2)\sin\theta = \frac{mv^2}{r}$,
- because $(N_1 + N_2)\cos\theta$ balances the weight (mg), then

$$(N_1 + N_2)\cos\theta = mg$$

$$\text{Therefore } \tan\theta = \frac{(N_1 + N_2)\sin\theta}{(N_1 + N_2)\cos\theta} = \frac{mv^2}{mgr}$$

Simplifying this equation gives the condition for no sideways friction:

$$\tan\theta = \frac{v^2}{gr}$$

In other words, there is no sideways friction if the speed v is such that $\boldsymbol{v^2 = gr\tan\theta}$

Prove for yourself that if the banking angle θ is not to exceed 5° and the radius of curvature is 360 m, the speed for zero sideways friction is $18\,\mathrm{m\,s^{-1}}$.

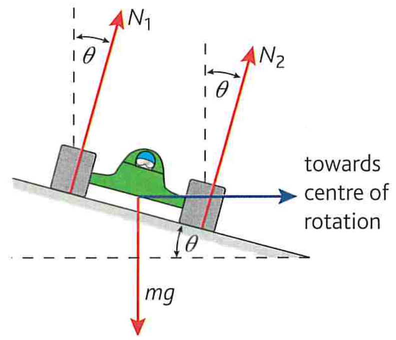

Figure 3 *A racing car taking a bend*

Link

The horizontal and vertical components of a force F which is at angle θ to the vertical must be F sin θ and F cos θ respectively. See *AS Physics* Topic 7.1.

Summary questions

$g = 9.8\,\mathrm{m\,s^{-2}}$.

1. A vehicle of mass 1200 kg passes over a bridge of radius of curvature 15 m at a speed of $10\,\mathrm{m\,s^{-1}}$. Calculate:
   a the centripetal acceleration of the vehicle on the bridge,
   b the support force on the vehicle when it was at the top.
2. The maximum speed for no skidding of a vehicle of mass 750 kg on a roundabout of radius 20 m is $9.0\,\mathrm{m\,s^{-1}}$. Calculate:
   a the centripetal acceleration,
   b the centripetal force on the vehicle when moving at this speed.
3. Explain why a circular athletics track is banked for sprinters but not for marathon runners.
4. At a racing car circuit, the track is banked at an angle of 25° to the horizontal on a bend which has a radius of curvature of 350 m.
   a Use the formula $v^2 = gr\tan\theta$ to calculate the speed of a vehicle on the bend if there is to be no sideways friction on its tyres.
   b Discuss and explain what could happen to a vehicle that took the bend too fast.