Chapter 1 The discovery of the electron

1.1 Thermionic emission of electrons

Learning objectives:

- What are cathode rays and how were they discovered?
- Why does the gas in a discharge tube emit light of a certain colour?
- How is a beam of electrons produced in a vacuum tube?

Cathode rays

Discharge tubes used in colourful advertising displays were first developed by William Crookes in the 1870s. He was able to reduce the pressure of the gas in a gas-filled glass tube by removing most of the gas from the tube using a vacuum pump. He discovered that gases at sufficiently low pressure in such tubes conduct electricity and emit light of a characteristic colour. For example, neon gas in such a tube emits red light.

Changing the gas pressure causes the glow of light to come from different parts along the tube, as shown in Figure 1. Investigations into the cause of the glow showed that:

- the glowing gas near the anode, the positive column, was easily distorted when a magnet was brought near the tube. This observation showed charged particles move through the gas when it conducts electricity.
- radiation from the direction of the cathode forced 'a paddle wheel' placed in the tube to rotate. The radiation was referred to as **cathode rays**. Placing a magnet near the tube stopped the paddle wheel rotating by deflecting the cathode rays away from it.

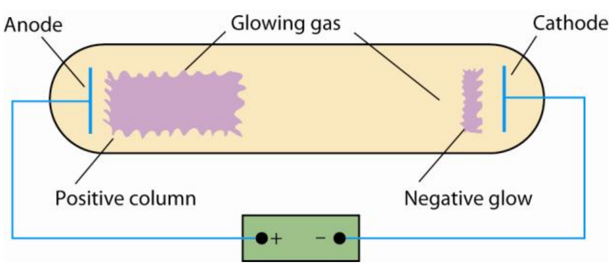


Figure 1 *A discharge tube*

Further tests showed that the cathode rays to be negatively charged particles. Exactly what cathode rays are puzzled scientists for over 20 years until J J Thomson carried out a series of experiments and proved that the negative particles are the same, regardless of which gas is used in the tube. The cathode ray particles were referred to as **electrons**.

The emission of light from a discharge tube

The emission of light from a discharge tube happens because the voltage applied to the tube is so high that it pulls electrons out of some of the gas atoms in the tube. In other words, some of the gas atoms in the tube are ionised. Positive ions created near the cathode are attracted onto the cathode surface, causing free electrons from the cathode surface to be emitted.

- The electrons from the cathode are accelerated towards the anode and collide with gas atoms causing them to be ionised. The glowing gas near the cathode, the 'negative glow', is due to photons emitted when some of the positive ions and electrons produced by ionisation recombine.
- Some of the electrons pulled out of the gas atoms do not recombine and are attracted to the anode and therefore further move away from the cathode – hence the term 'cathode rays'. These electrons move towards the anode and cause **excitation by collision** of gas atoms in the tube. The positive column of glowing gas is due to de-excitation of these excited gas atoms.

The processes of recombination and de-excitation result in the emission of photons of visible and ultraviolet light.

Link

Look back at *AS Physics A* Topic 3.3 Ionisation and de-excitation; photons and energy levels.

The principle of thermionic emission

Thermionic emission is a much simpler way of producing an electron beam than using a discharge tube. When a metal is heated, some of the electrons that move about freely inside the metal (referred to as 'free' or 'conduction' electrons) gain sufficient kinetic energy to leave the metal at its surface. In practice, the metal is a wire filament which is heated by passing an electric current through it. The filament or 'cathode' is at one end of an evacuated glass tube which has a metal plate or 'anode' at the other end. This is shown in Figure 2.

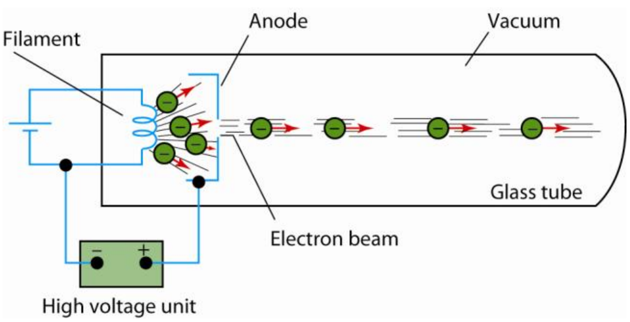


Figure 2 *Thermionic emission*

The electrons emitted from the filament are attracted to the anode by connecting a high voltage power supply between the anode and the cathode, with the anode positive relative to the filament. There are no gas molecules in the tube to stop the electrons, so the electrons are accelerated to the anode where they pass through a small hole to form a narrow beam.

The **speed** of each electron passing through the hole can be worked out by equating:

- the work done by the pd V between the anode and the cathode on each electron = eV, where e is the charge of the electron, and
- the kinetic energy of each electron passing through the hole = $\frac{1}{2}mv^2$, where v is the speed of each electron at this position.

Since the work done on each electron increases its kinetic energy from a negligible value at the cathode, the speed, v, of each electron on leaving the anode is given by

$$\frac{1}{2}mv^2 = eV$$

Notes

1 The speed of each electron in the beam from the anode is given by the above equation. This is because the electric field between the anode and the cathode does not act on the electrons once they have passed through the hole in the anode. Their kinetic energy does not change after leaving the anode.

2 The above equation assumes that:

- each electron starts from the filament with negligible kinetic energy in comparison with the work done on it by the accelerating pd V
- the speed of the electrons in the beam, v, is much less than the speed of light in free space, c, so the non-relativistic formula for kinetic energy ($\frac{1}{2}mv^2$) applies.

Links

The equation **work done = charge × pd** was looked at in Topic 4.2 of *AS Physics A*.

The equation **kinetic energy = $\frac{1}{2}$ mv^2** was looked at in Topic 10.2 of *AS Physics A*.

Worked example

$e = 1.60 \times 10^{-19}$ C

Calculate the speed of an electron after it has been accelerated from rest through a pd of 5000 V.

The rest mass of an electron = 9.11×10^{-31} kg.

Solution

Rearranging $\frac{1}{2}mv^2 = eV$ gives $v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.60\times10^{-19}\times5000}{9.11\times10^{-31}}} = 4.19\times10^{7}\ \text{m s}^{-1}$

AQA Examiner's tip

Be careful not to mix up the symbols v and V.

Summary questions

1 When a suitable pd is applied between the anode and the cathode of a discharge tube, explain why the gas in the tube conducts electricity and emits light.

2 Explain why:

a the glowing gas in a discharge tube is distorted when a magnet outside the tube is placed near the glowing gas

b the negative particles in a discharge tube are identical, regardless of the gas in the tube.

3 **a** Look at Figure 2, and state the function of:

i the cell connected to the filament

ii the high voltage unit.

b Explain why the tube in Figure 2 must contain a vacuum.

4 Calculate the speed of an electron after it has been accelerated in a vacuum tube from rest through a pd of 4000 V. Give your answer to 3 significant figures.

1.2 Deflection of an electron beam

Learning objectives:

- How can electron beams be controlled and deflected?
- What happens to the deflection of an electron beam if the speed of the electrons is increased?
- How can we determine the speed of the electrons in a beam?

An electron beam can be deflected by an electric field or a magnetic field. By adjusting the strength of the field in each case, the extent of the deflection can be controlled. In this topic, we will consider the principles behind the use of electric and magnetic fields to control an electron beam before moving on in the next topic to consider how these principles may be used to determine the specific charge e/m of the electron.

Using a uniform electric field

Figure 1 shows a glass vacuum tube in which an electron beam is produced so that it can be deflected by an electric field or a magnetic field or both.

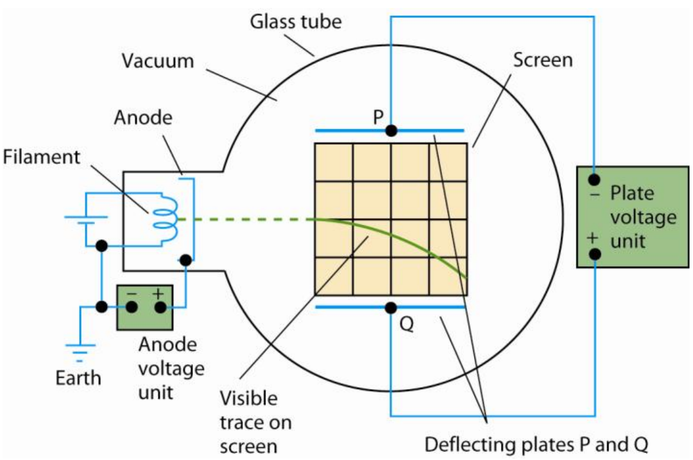


Figure 1 *Deflecting electrons*

The electric field used to deflect the beam is produced in Figure 1 by applying a constant pd between the metal deflecting plates P and Q. The beam enters the field at right angles to the field lines and is deflected towards the positive plate.

The force on each electron in the field is constant in magnitude and direction because the field is uniform so the beam curves in a parabolic path (just as a projectile projected horizontally does). The fluorescent screen in the tube enables the path of the beam to be observed and measured.

If the pd between plates P and Q is V_P, the force F on each electron is given by the equation

$$F = eE = \frac{eV_P}{d}$$

where d is the perpendicular distance between plates P and Q, and E $(= V_P/d)$ is the electric field strength between the plates.

Notes

1 The acceleration of each electron towards the positive plate, $a = \frac{F}{m} = \frac{eV_P}{md}$, where m is the mass of the electron.

2 The time taken, t, by each electron to cross the field = $\frac{L}{v}$, where L is the length of each plate and v is the initial speed of the electron on entry to the field. This is the horizontal component of the electron's velocity in the field as it has no horizontal acceleration.

3 The deflection, y, of the electron on leaving the field is given by the equation $y = \frac{1}{2}at^2$. The deflection may be measured directly from the trace on the screen in the tube. If the speed of the electrons is increased (by increasing the anode voltage), each electron spends less time in the electric field so the deflection is less.

4 The above equations can be combined to show that the deflection y is directly proportional to the plate pd.

Link

Horizontal projection was looked at in Topic 8.7 of *AS Physics A*.

Using a uniform magnetic field

A uniform magnetic field can be applied to the electron beam by placing a pair of large circular coils coaxially either side of the tube such that the plane of each coil is parallel to the screen, as shown in Figure 2.

When the two coils are connected in series and a current is passed through them in the same direction, a uniform magnetic field perpendicular to the screen is produced. The magnetic flux density can be varied by varying the current in the coils.

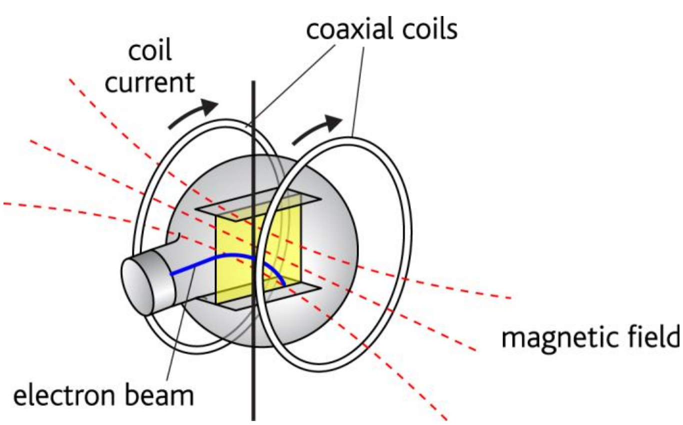


Figure 2 *Using coaxial coils*

Each electron in the beam experiences a force due to the magnetic field at right angles to the direction of motion of the electrons and to the direction of the magnetic field. In accordance with Fleming's left-hand rule, the result is that each electron is forced on a circular path so the beam traces a circular arc on the screen.

The force F on each electron in the field is given by

$$F = Bev$$

where B = the magnetic flux density of the magnetic field and v = the speed of the electron.

Notes

The direction of the magnetic force on each electron is perpendicular to the direction of velocity of the electron. Therefore,

- the magnetic force does no work on the electron so its speed remains constant,
- the electron moves on a circular path of radius r with a centripetal acceleration $a = \frac{v^2}{r}$

Using $F = ma$ therefore gives $Bev = \frac{mv^2}{r}$

Hence the radius of curvature of the beam path, $\boldsymbol{r = \frac{mv}{Be}}$

The equation shows that the radius of the beam can be increased by:

- increasing the anode pd so the electron speed v is increased, or
- decreasing the coil current so the magnetic flux density is decreased.

Balanced fields

With a magnetic field applied as outlined above to deflect the beam downwards, the deflection can be cancelled out by applying an electric field of suitable strength with the top plate P **positive** relative to the lower plate Q.

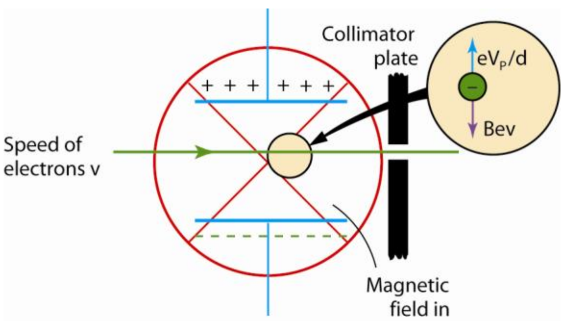


Figure 3 *Balanced fields*

- If the electric field is too strong, the beam will be deflected upwards because the magnetic force acting downwards on each electron is weaker than the electric force acting upwards.
- If the electric field is too weak, the beam will be deflected downwards because the magnetic force acting downwards on each electron is stronger than the electric force acting upwards.

The beam passes through the fields without being deflected when the electric force $\frac{eV_P}{d}$ is equal and opposite to the magnetic force Bev on each electron. The two fields may be said to be 'balanced' because they produce zero resultant force on each electron. In this situation, the speed v of each electron is given by

$$Bev = \frac{eV_P}{d}$$

Rearranging this equation gives $v = \frac{V_P}{Bd}$

Worked example

e = 1.60 × 10^{-19} C

A beam of electrons is directed horizontally into a uniform electric field which acts vertically downwards. The electric field is due to a pd of 4500 V applied between two parallel deflecting plates 60 mm apart. With the electric field on, a uniform magnetic field is applied to the beam perpendicular to the electric field and to the initial direction of the beam. The magnetic flux density is adjusted to a value of 2.4 mT so the beam is undeflected.

a State the direction of:

i the electric force

ii the magnetic force.

b Calculate the speed of the electrons as they pass through the electric and magnetic fields.

Solution

a **i** The electric force is vertically upwards (in the opposite direction to the electric field lines because the electron has a negative charge).

ii The magnetic force is vertically downwards (in the opposite direction to the electric force).

b Using $Bev = \frac{eV_P}{d}$ gives speed $v = \frac{V_P}{Bd} = \frac{4500}{2.4\times10^{-3}\times60\times10^{-3}}$ = 3.2 × 10^{7} m s^{-1}

Summary questions

1 In Figure 1, if the top plate is at a negative potential relative to the lower plate, state the direction of:

a the electric field between the plates

b the force on an electron in the field

c the acceleration of an electron in the field.

2 An electron beam is directed horizontally into a uniform electric field which acts vertically downwards. Explain why the electron beam curves upwards with increasing speed.

3 An electron beam is directed horizontally into a uniform magnetic field that is perpendicular to the initial direction of the beam. Explain why the beam curves round in a vertical circle at constant speed.

4 A beam of electrons moving at a speed of 2.8 × 10^{7} m s^{-1} passes through perpendicular electric and magnetic fields without being deflected. The magnetic field has a flux density 3.2 × 10^{-3} T. Calculate the strength of the electric field.

1.3 Use of electric and magnetic fields to determine *e*/*m*

Learning objectives:

- How can *e*/*m* be measured?
- What measurements are needed to determine *e*/*m*?
- What was the significance of the first accurate determination of *e*/*m*?

The determination of the specific charge of the electron, *e*/*m*

After the discovery of cathode rays, J J Thomson in 1895 measured the specific charge, $\frac{e}{m}$, of the electron (i.e. its charge *e* divided by its mass *m*, sometime referred to as its charge/mass ratio). Using a deflection tube, he measured the speed of the electrons by passing them through balanced electric and magnetic fields, adjusted so the electron beam was undeflected. The speed of the electrons could then be calculated, as explained in Topic 1.2, by measuring the deflecting pd V_P, the plate spacing *d* and the magnetic flux density *B* and using the equation $v = \frac{V_P}{Bd}$.

With one of the fields switched off, the deflection of the beam due to the field still on was then measured and used with the known value of the initial speed to determine $\frac{e}{m}$.

Using the electric field only, as shown in Figure 1, with electrons of known speed *v*

- the time taken, *t*, for the electrons to travel the length *L* of the deflecting plates = $\frac{L}{v}$
- the acceleration, *a* towards the positive plate = $\frac{2y}{t^2}$ (from $y = \frac{1}{2}at^2$)

Since the acceleration $a = \frac{eV_P}{md}$, rearranging this equation gives $\frac{e}{m} = \frac{ad}{V_P}$

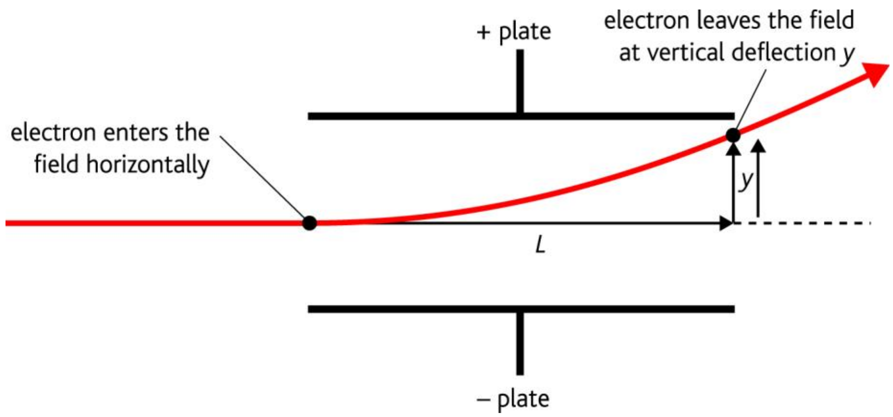


Figure 1 *Deflection by a uniform electric field*

Link

Projectile motion was looked at in Topic 8.7 of *AS Physics A*.

Worked example

Electrons moving at a constant speed are directed horizontally into a uniform electric field due to two parallel plates of length 60.0 mm. The plates are spaced 50.0 mm apart and have a pd of 3000 V between them. The electrons are deflected by 22.0 mm as a result. When a uniform magnetic field of flux density 2.05 mT is applied at right angles to the beam and the electric field, the beam is undeflected. Calculate the specific charge *e*/*m* of the electron.

Solution

The speed of the beam, $v = \frac{V_P}{Bd} = \frac{3000}{2.05\times10^{-3}\times50\times10^{-3}} = 2.93\times10^{7}\,\mathrm{m\,s^{-1}}$

time taken, $t = \frac{L}{v} = \frac{60.0\times10^{-3}}{2.93\times10^{7}} = 2.05\times10^{-9}\,\mathrm{s}$

acceleration, $a = \frac{2y}{t^2} = \frac{2\times22.0\times10^{-3}}{(2.05\times10^{-9})^2} = 1.05\times10^{16}\,\mathrm{m\,s^{-2}}$

$e/m = \frac{ad}{V_P} = \frac{1.05\times10^{16}\times50\times10^{-3}}{3000} = 1.75\times10^{11}\,\mathrm{C\,kg^{-1}}$

Using the magnetic field only, with electrons of known speed *v*, the radius of curvature *r* of the beam path must be measured in a magnetic field of known flux density *B*.

Rearranging the equation $r = \frac{mv}{Be}$ from Topic 1.2 gives $\frac{e}{m} = \frac{v}{Br}$. Using the measured values of $\frac{v}{Br}$ then enables $\frac{e}{m}$ to be calculated.

Note

The radius of curvature of the beam may be calculated using $r = \dfrac{(L^2 + y^2)}{2y}$. Knowledge or use of this formula is not required in the specification for this option.

Worked example

Electrons moving at a speed of $1.60 \times 10^7\,\mathrm{m\,s^{-1}}$ are directed into a uniform magnetic field of flux density $1.40 \times 10^{-3}\,\mathrm{T}$ in such a direction as to form a circle of diameter 65 mm. Calculate the specific charge, e/m, of the electron.

Solution

$$e/m = \frac{v}{Br} = \frac{1.60\times10^7}{1.40\times10^{-3}\times0.065} = 1.76 \times 10^{11}\,\mathrm{C\,kg^{-1}}$$

Measurement of *e*/*m* using the fine beam tube

The ‘fine beam’ tube shown in Figure 2 is designed to make the beam visible as a result of collisions between some of the electrons in the beam and the small amount of gas in the tube. The amount of gas is small enough so that most electrons are unaffected by its presence in the tube. With a sufficiently strong magnetic field, provided the initial direction of the beam is at right angles to the magnetic field lines, the beam path can be seen as a complete circle, enabling its diameter and its radius to be measured. Details of the measurement of the magnetic flux density are not required in the specification for this option.

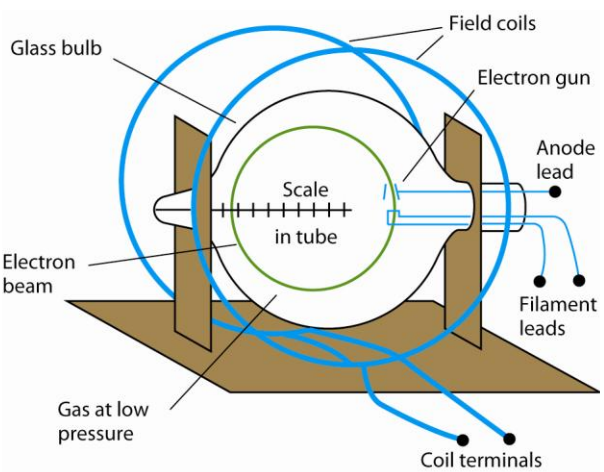


Figure 2 *The fine beam tube*

The stronger the field, the smaller the circle. With the anode voltage V_A constant, the coil current is adjusted in steps so the beam diameter can be measured directly from a scale inside the tube. In this way, the magnetic flux density B is measured for different values of the beam radius, r.

As explained in Topic 1.1, the speed of the electrons depends on the anode voltage, V_A, in accordance with the equation $\frac{1}{2} mv^2 = eV_A$.

Combining this with the equation $r = \dfrac{mv}{Be}$ rearranged as $v = \dfrac{Ber}{m}$ gives

$$\tfrac{1}{2}m\left(\frac{Ber}{m}\right)^2 = eV_A$$

Making $\frac{e}{m}$ the subject of this equation therefore gives

$$\frac{e}{m} = \frac{2V_A}{B^2r^2}$$

Notes

1 Rearranging the above equation gives $\boldsymbol{r = k/B}$ where the constant $k = \sqrt{\frac{2mV_A}{e}}$

Using the measurements of r and B to plot a graph of r against $\frac{1}{B}$ gives a straight line through the origin with a gradient equal to k. Therefore, $\frac{e}{m}$ can be determined if the value of k is measured from the graph and used in the equation $\frac{e}{m} = \frac{2V_A}{k^2}$

2 B can be measured using a Hall probe without the tube present. V_A is measured directly using a suitable voltmeter connected across the high voltage supply unit.

Worked example

Electrons emitted from the heated filament are accelerated through a pd of 490 V to form a beam which is then directed into a uniform magnetic field of magnetic flux density 3.4 mT. The magnetic field deflects the beam into a circle of diameter 44 mm. Use these data to calculate the specific charge of the electron.

Solution

beam radius r = 22 mm

$$e/m = \frac{2V_A}{B^2r^2} = \frac{2 \times 490}{(3.4\times10^{-3} \times 22 \times 10^{-3})^2} = 1.75 \times 10^{11}\ \mathrm{C\,kg^{-1}}$$

The significance of Thomson's determination of *e*/*m*

The specific charge of the electron is $1.76 \times 10^{11}\ \mathrm{C\,kg^{-1}}$. The value was first determined by J J Thomson in 1895. Before Thomson made this measurement, the hydrogen ion was known to have the largest specific charge of any charged particle at $9.6 \times 10^{7}\ \mathrm{C\,kg^{-1}}$. Thomson therefore showed that the electron's specific charge was 1860 times larger than that of the hydrogen ion. However, Thomson could not conclude that the electron has a much smaller mass than the hydrogen ion as neither the mass nor the charge of the electron was known at that time. The charge of the electron was measured by R A Millikan in 1915.

AQA Examiner's tip

Concentrate on the principles of the above methods and make sure you know when to use the relevant equations.

Summary questions

1 Electrons moving at a constant speed are directed horizontally into a uniform electric field due to two parallel plates of length 85.0 mm and spaced 40.0 mm apart which have a pd of 4000 V between them.

a When a uniform magnetic field of flux density 2.95 mT is applied at right angles to the beam and to the electric field, the beam is undeflected. Calculate the speed of the electrons.

b When the magnetic field is switched off, the beam is deflected vertically by the electric field by 55 mm where it leaves the field. Show that the each electron takes 2.51 ns to pass through the field and calculate the specific charge *e*/*m* of the electron.

2 Electrons moving at a speed of 1.35×10^{7} m s^{-1} are directed into a uniform magnetic field of flux density 1.54×10^{-3} T in such a direction as to form a circle of diameter 50 mm. Calculate the specific charge, *e*/*m*, of the electron.

3 Electrons emitted from the heated filament are accelerated through a pd of 550 V to form a beam which is then directed into a uniform magnetic field of magnetic flux density 2.8 mT. The magnetic field deflects the beam into a circle of radius 28 mm. Use these data to calculate the specific charge of the electron.

4 Outline the significance of Thomson's determination of the specific charge of the electron, *e*/*m*.

1.4 The determination of the charge of the electron, *e*, by Millikan's method

Learning objectives:

- How can *e* be measured?
- What measurements are needed to determine *e*?
- Why was Millikan's determination of *e* important?

Measurement of the charge of a charged oil droplet of known mass

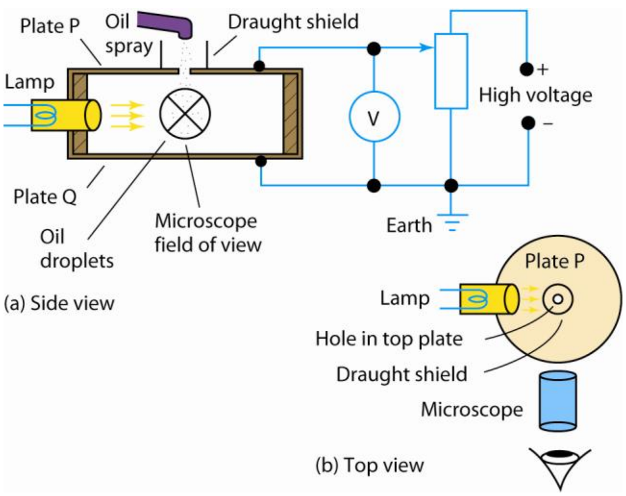


Figure 1 *Millikan's method for finding e*

The accurate measurement of the charge, *e*, of the electron was first successfully made by R A Millikan in 1915. He discovered he could control the motion of charged oil droplets from an oil spray using the electric field between oppositely charged parallel metal plates. He found that if the plates were horizontal so the field was vertical, he could slow down or stop or reverse the descent of an oil droplet falling between the plates. Using a microscope as shown in Figure 1, he was able to observe individual charged oil droplets.

He was able to make any charged droplet stay in a stationary position by adjusting the pd between the plates until the electric force on the droplet acting vertically upwards was equal and opposite to the weight of the droplet. Figure 2 shows the forces on such a charged droplet.

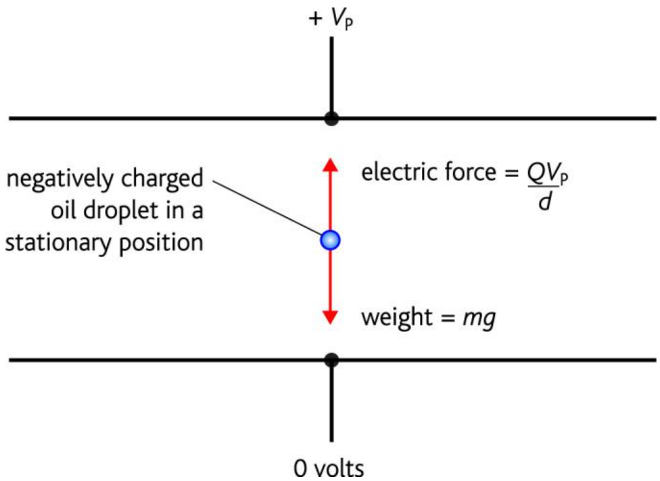


Figure 2 *A stationary oil droplet*

For a droplet of charge Q and mass m

- the electric force on the droplet $=\frac{QV_P}{d}$ where V_P is the plate pd and d is the perpendicular distance between the plates
- the weight of the droplet = mg

Therefore, when the droplet is stationary,

$$\frac{QV_P}{d} = mg$$

Hence the charge on the droplet $\frac{QV_P}{d} = \frac{mgd}{V_P}$

If the mass of the droplet is known, the charge Q on the droplet can therefore be calculated.

Notes

1 The polarity of the top plate must be opposite to the sign of the charge on the droplet in order that the force due to the electric field acts upwards. If the droplet is positive, the top plate must be negative with respect to the bottom plate. If the droplet is negative, the top plate must be positive with respect to the bottom plate.

2 The upthrust on the droplet due to buoyancy is equal to about 0.1% of its weight and therefore may be considered to be negligible. Knowledge of the upthrust is not required in this option specification.

3 The electric field strength between the plates $E = V_P/d$

The significance of Millikan's results

Millikan was able to measure the mass of each droplet accurately by measuring its terminal speed with the electric field switched off. He was then able to calculate its charge as explained above. After making many measurements on many charged oil droplets, he found that the charge Q was always a whole number × 1.6×10^{-19} C. In other words, he showed that electric charge is **quantised** in whole number multiples of 1.6×10^{-19} C. He concluded that the charge of the electron is 1.6×10^{-19} C and that the whole number n corresponds to how many electrons on the droplet are responsible for its charge.

Worked example

$g = 9.81\,\text{m s}^{-2}$, $e = 1.6 \times 10^{-19}\,\text{C}$

A charged oil droplet of mass 3.8×10^{-15} kg is held stationary by the electric field between two parallel horizontal metal plates which are 5.0 mm apart. The top plate is at a negative potential of −580 V relative to the lower plate which is earthed.

a Calculate the charge on the droplet and state its sign.

b State how many electrons are responsible for the droplet's charge.

Solution

a $Q = \dfrac{mgd}{V_P} = \dfrac{3.8\times10^{-15}\times9.81\times5.0\times10^{-3}}{580} = 3.2\times10^{-19}\,\text{C}$

The droplet is positively charged.

b 2

Measurement of the mass of an oil droplet

When no electric field is present, an oil droplet falls at its terminal speed according to its weight. The greater the weight of an oil droplet, the greater its terminal speed. Millikan realised the drag force on the droplet due to air resistance is equal and opposite to the weight of the droplet when it falls at terminal speed. The resultant force on it is zero and so its acceleration is zero. By measuring the terminal speed of the droplet, he was able to calculate the mass of the droplet as explained below.

Link

Terminal speed was looked at in Topic 9.3 of *AS Physics A*.

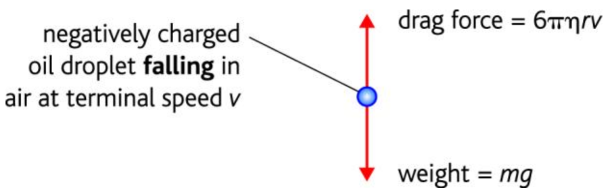


Figure 3 *An oil droplet falling at terminal speed*

Figure 3 shows the forces acting on the droplet as it falls at its terminal speed. Assuming the droplet is a sphere of radius *r*,

- its volume = $\frac{4}{3}\pi r^3$ therefore its mass m = its volume ($\frac{4}{3}\pi r^3$) × its density (ρ) = $\frac{4}{3}\pi r^3\rho$.

  Therefore, its weight $mg = \frac{4}{3}\pi r^3\rho g$

- the drag force, $F_D = 6\pi\eta rv$ according to Stokes' law, where η is the viscosity of air. The drag force is always in the opposite direction to its velocity.

So, $\frac{4}{3}\pi r^3\rho g = 6\pi\eta rv$

Rearranging this equation gives $r^2 = \dfrac{9\eta v}{2\rho g}$

The equation above shows the radius r of the droplet and hence its mass can be calculated if its terminal speed v is measured and the values of the oil density ρ and the viscosity of air η are known.

Link

Density was looked at in Topic 11.1 of *AS Physics A*.

Worked example

density of the oil = 960 kg m^{-3}, viscosity of air = 1.8 × 10^{-5} N s m^{-2}

A charged oil droplet was held stationary between oppositely charged metal plates 5.0 mm apart when the pd between the plates was 490 V. When the pd was switched off, the droplet fell at a terminal speed of 1.3 × 10^{-4} m s^{-1}. Calculate:

a the mass of the droplet

b the charge of the droplet.

Solution

a $r^2 = \frac{9\eta v}{2\rho g} = \frac{9 \times 1.8 \times 10^{-5} \times 1.3 \times 10^{-4}}{2 \times 960 \times 9.81}$ = 1.12 × 10^{-12} m^2 hence r = 1.06 × 10^{-6} m

$m = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi(1.06 \times 10^{-6})^3 \times 960 = 4.75 \times 10^{-15}$ kg

b $Q = \frac{mgd}{V_P} = \frac{4.75 \times 10^{-15} \times 9.81 \times 5.0 \times 10^{-3}}{490}$ = 4.8 × 10^{-19} C

Notes

1 When the drop is held stationary by the electric field and the field is switched off, the droplet falls and reaches its terminal speed in a fraction of a second. Its initial acceleration is g as it is due to gravity only as the drag force is initially zero. As the droplet gains speed, the drag force increases so the resultant force (and hence the acceleration) decreases to zero.

2 When the electric field is on and the droplet is moving vertically, it moves at a constant speed that depends on its weight, the drag force and the electric force. Figure 4 shows the forces acting on a droplet that is made to move up or down by changing the electric force.

- In (a) the electric force is greater than the droplet weight so the droplet moves upwards.
- In (b) the electric force is less than the weight so the droplet falls.
- In (c) the electric force is downwards so the droplet falls.

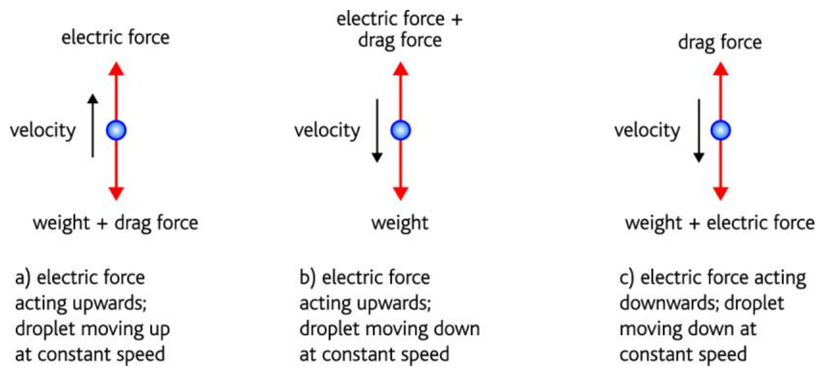


Figure 4 *Using the electric field*

AQA Examiner's tip

When describing the forces acting on the droplet, do not confuse 'electric force' with the term 'electric field'. The electric force is the force due to the electric field.

Summary questions

$g = 9.81\,\text{m}\,\text{s}^{-2}$, $e = 1.6 \times 10^{-19}\,\text{C}$

1 A charged oil droplet of mass 2.60×10^{-15} kg is held stationary by the electric field between two parallel horizontal metal plates which are 6.00 mm apart. The top plate is at a positive potential of 320 V relative to the lower plate which is earthed.

a Calculate the charge on the droplet and state its sign.

b State how many electrons are responsible for the droplet's charge.

2 A charged oil droplet of radius 1.10×10^{-6} m is held stationary in a uniform electric field of strength $8.20 \times 10^{4}\,\text{V}\,\text{m}^{-1}$ which act vertically upwards.

a Show that the mass of the droplet is 5.35×10^{-15} kg.

b **i** Calculate the charge on the droplet and state its sign.

ii State how many electrons are responsible for the droplet's charge.

density of oil = $960\,\text{kg}\,\text{m}^{-3}$

3 A charged oil droplet was held stationary between oppositely charged metal plates 5.00 mm apart when the pd between the plates was 610 V. When the pd was switched off, the droplet fell at a terminal speed of $1.15 \times 10^{-4}\,\text{m}\,\text{s}^{-1}$. Calculate:

a the mass of the droplet

b the charge of the droplet.

density of the oil = $960\,\text{kg}\,\text{m}^{-3}$, viscosity of air = $1.80 \times 10^{-5}\,\text{N}\,\text{s}\,\text{m}^{-2}$

4 **a** State the general result obtained by Millikan for the charge on a charged oil droplet and explain its significance.

b A charged oil droplet is held stationary in a uniform electric field which is then switched off. Describe and explain the subsequent motion of the droplet.

Chapter 2 Wave–particle duality

2.1 Early theories of light

Learning objectives:

- What was Newton's corpuscular theory of light?
- What did corpuscular theory predict should be observed in Young's double slits experiment?
- What does Young's double slits experiment tell us about the nature of light?

Newton's corpuscular theory of light

Newton imagined a light ray as a stream of tiny particles which he referred to as 'corpuscles'. He developed his ideas to explain reflection and refraction.

- When a light ray is **reflected** by a plane mirror, Newton said the corpuscles bounce off the mirror without loss of speed. To explain the law of reflection, he said that the normal component of velocity, v_N, of each corpuscle is reversed and the component of velocity parallel to the mirror, $v_{\uparrow\uparrow}$ is unchanged. Figure 1 shows the idea. Since the magnitude of normal and parallel components of velocity are unchanged on reflection, it can be shown that the angle of reflection, *r*, is equal to the angle of incidence, *i*.

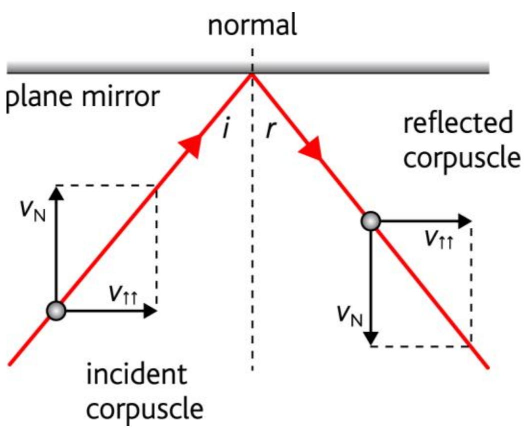


Figure 1 *Reflection of light according to Newton*

- When a light ray is **refracted** on passing from air into a transparent substance, Newton said the corpuscles are attracted into the substance so they travel faster in the substance than in air. To explain the law of refraction, he said that the component of velocity perpendicular to the boundary of each corpuscle is increased as the corpuscle crosses the boundary into the substance and the component of velocity parallel to the boundary is unchanged. Figure 2 shows the idea.

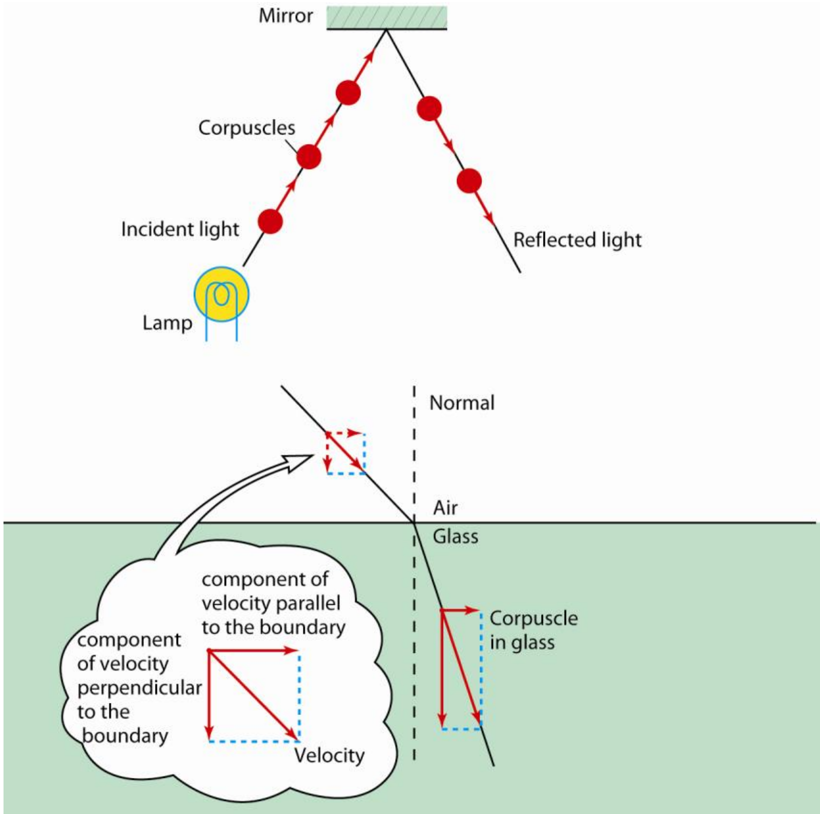


Figure 2 *Refraction of light according to Newton*

Link

Refraction was looked at in Topic 13.1 of *AS Physics A*.

The wave theory of light put forward by Huygens at roughly the same time also explained reflection and refraction of light. Huygens' explanation of reflection and refraction assumed light waves travel **slower** in a transparent substance than in air. The wave theory of light was rejected by most scientists in favour of Newton's corpuscular theory because:

- It was not possible to measure the speed of light in air or in a transparent substance at that time so there was no experimental evidence showing whether light travels faster or slower in a transparent substance than in air.
- Newton had a much stronger scientific reputation than Huygens so most scientists thought Newton's theory was correct.
- The wave theory of light was considered in terms of longitudinal waves so could not explain polarisation of light.

The significance of Young's double slits experiment

Newton's corpuscular theory was the accepted theory of light for over 150 years until Thomas Young in 1803 showed that light passed through double slits produces an interference pattern. Since interference is a wave property, Young's experiment challenged the accepted theory that light consists of corpuscles. Figure 3 represents light waves from a single slit S passing through double slits where the waves spread out because they undergo diffraction as they pass through the slits. The double slits act as coherent sources of waves so the wave fronts from each of the slits produce an interference pattern where they overlap, producing parallel equally spaced bright and dark fringes.

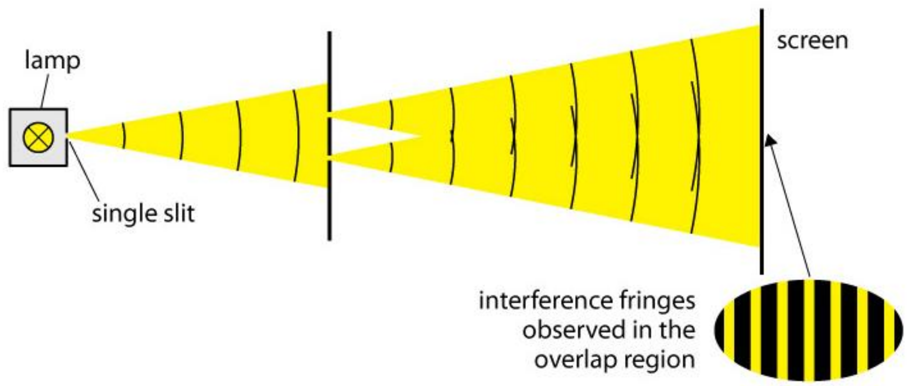


Figure 3 *Young's double slits*

- Each bright fringe is formed where light waves from each slit arrive in phase so they reinforce one another.
- Each dark fringe is formed where light waves from each slit arrive 180° out of phase and therefore cancel each other out.

The number of fringes observed depends on the slit spacing and the width of each slit. The further the slits are from each other (i.e. the larger the slit spacing), the smaller the fringe spacing is. The narrower the slits, the greater the amount of diffraction so the overlap region is greater. Therefore, more fringes are observed using widely spaced slits compared closely spaced slits of the same width.

Newton's corpuscular theory of light predicted that corpuscles would pass through each slit so two bright fringes would be seen corresponding to light passing through each slit. Figure 4 shows the idea. Clearly, the corpuscular theory could not explain the observed interference patterns whereas wave theory could.

Link

Interference was looked at in Topic 13.4 of *AS Physics A*.

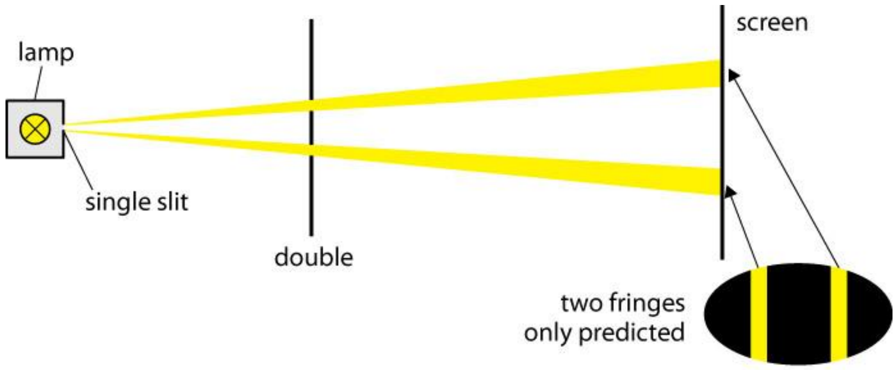


Figure 4 *The corpuscular theory prediction*

Young's double slits experiment showed that light undergoes interference which is a property of waves. However, the wave theory of light was not accepted even after Young demonstrated interference of light for the reasons stated earlier. This was until it was shown experimentally that light in water travels slower than light in air. With this experimental evidence, scientists realised that light is a waveform and it must be transverse rather than longitudinal in order to explain polarisation.

Summary questions

1 State two differences between Newton's theory of light and that of Huygens.

2 Explain with the aid of a diagram how Newton explained the refraction of a light ray when the light ray passes from air into glass.

3 **a** Describe the fringe pattern observed in Young's double slits experiment and explain why it could not be explained using Newton's theory of light.

b Use the wave theory of light to explain the formation of the interference fringes.

4 **a** State one reason why Newton's theory of light was accepted in preference to Huygens' theory.

b Give one reason why the wave theory of light was not accepted immediately after Young first demonstrated that interference is a property of light.

2.2 The discovery of electromagnetic waves

Learning objectives:

- What are electromagnetic waves?
- What did Maxwell prove about the speed of electromagnetic waves?
- How were radio waves first produced and detected?

Maxwell's theory of electromagnetic waves

The wave theory of light developed into the theory of electromagnetic waves. This was as a result of theoretical work by James Clark Maxwell who showed mathematically in 1865 that a changing current in a wire creates waves of changing electric and magnetic fields that radiate from the wire. Maxwell showed that the waves are transverse in nature and that the electric waves are in phase with and perpendicular to the magnetic waves as shown in Figure 1.

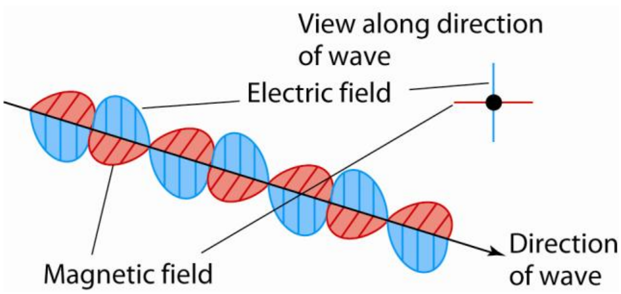


Figure 1 *Electromagnetic waves*

In effect, an alternating current in a wire creates an alternating magnetic field which generates an alternating electric field which generates an alternating magnetic field further from the wire which generates an alternating electric field which generates an alternating magnetic field further yet further from the wire and so on. Maxwell knew that the strength of the electric field depends on the permittivity of free space, ε_0, and he knew that the magnetic field strength depends on the equivalent magnetic constant, the permeability of free space, μ_0. He showed mathematically that the speed of electromagnetic waves in free space, c, is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

The speed of light in free space can therefore be calculated using the values of ε_0 and μ_0 determined from separate electric field and magnetic field measurements. Given $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ and $\mu_o = 4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$, prove for yourself using the given values of ε_0 and μ_0 that the above formula gives $3.0 \times 10^8\,\mathrm{m\,s^{-1}}$ for the speed of light in free space.

Maxwell's mathematical theory showed that light consists of electromagnetic waves and it also predicted the existence of electromagnetic waves outside the boundaries of the then known spectrum which was from infrared to ultraviolet radiation. Many years later, the separate discoveries of X-rays and of radio waves confirmed the correctness of Maxwell's predictions.

AQA Examiner's tip

If you are asked to describe an electromagnetic wave, a diagram is helpful as well as a written description. Remember that the electric and magnetic waves are in phase, perpendicular to each other and perpendicular to the direction in which the wave is travelling.

Notes

The information in these notes is background information about μ_0. This background information is **not** part of the specification for this option. For information about ε_0, see *A2 Physics A* Topic 5.2.

1 The magnetic flux density B inside a long current-carrying solenoid is uniform and is proportional to the current I and to the number of turns per unit length n of the solenoid. The permeability of free space, μ_0, may be defined as the magnetic flux density per unit current ÷ the number of turns per unit length of the solenoid. In other words, μ_0 is the constant of proportionality in the equation $B = \mu_0 nI$.

2 From the above definition of μ_0, it can then be shown that the magnetic flux density B at distance d from a long straight wire carrying current I is given by

$$B = \frac{\mu_0 I}{2\pi d}$$

For two such parallel wires X and Y at distance d apart carrying currents I_X and I_Y, using $F = BIL$ gives the force per unit length on each wire due to their magnetic interaction

$$\frac{F}{L}\ (= BI) = \frac{\mu_0 I_X I_Y}{2\pi d}$$

The wires attract if the currents are in the same direction and repel if the currents are in opposite directions, as shown in Figure 2.

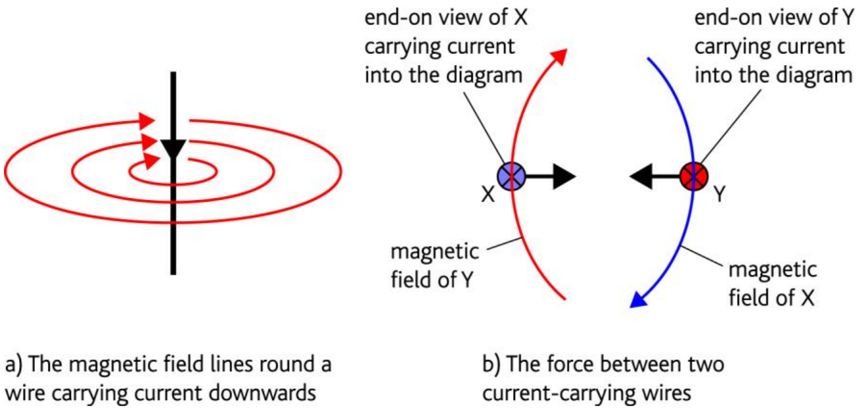


Figure 2 *The magnetic force between two current carrying wires*

The ampere is defined as that current in two parallel wires, one metre apart, which gives a force per unit length on each wire of 2×10^{-7} N m^{-1}. Substituting this value of F/L into the above equation with $I_1 = I_2 = 1$ A and $d = 1$ m therefore gives

$$\mu_0 = 2\pi \times 2 \times 10^{-7} = 4\pi \times 10^{-7}\ \text{T m A}^{-1}$$

Hertz's discovery of radio waves

Heinrich Hertz was the first person to discover how to produce and detect radio waves. More than twenty years after Maxwell predicted the existence of radio waves, Hertz showed that such waves are produced when high voltage sparks jump across an air gap and he showed they could be detected using a wire loop with a small gap in it. This is shown in Figure 3.

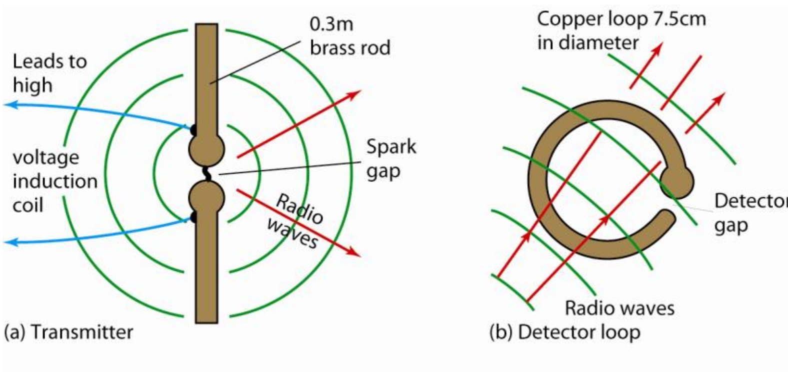


Figure 3 *Hertz's discovery of radio waves*

- The radio waves produced by the spark gap transmitter spread out from the spark gap and pass through the detector loop.
- The waves passing through the detector loop cause a voltage to be induced in the detector loop which makes sparks jump across the detector gap.

Hertz showed that the detector sparks stopped when a metal sheet was placed between the transmitter and the detector thus showing that radio waves do not pass through metal. He found that the radio waves are reflected by a metal sheet and discovered that a concave metal sheet placed behind the transmitter made the detector sparks stronger because it reflected radio waves travelling away from the detector back towards the detector. He also discovered that insulators do not stop radio waves and he showed that the radio waves he produced are polarised.

Note

The induced voltage in the detector loop is due to the oscillating magnetic field of the radio waves. As the waves travel across the loop, the oscillating magnetic field causes oscillating changes in the magnetic flux passing through the loop which causes an alternating pd to be induced in the loop.

Measuring the wavelength of the radio waves

Hertz produced stationary radio waves by using a flat metal sheet to reflect the waves back towards the transmitter, as shown in Figure 4. When he moved the detector along the line between the transmitter and the flat metal sheet, he found that the sparks were not produced at certain detector positions which he realised were the nodes of the stationary wave pattern. He obtained a measurement of 33 cm for the distance between adjacent nodes and so calculated the wavelength of the radio waves as 66 cm (= 2 × the distance between adjacent nodes).

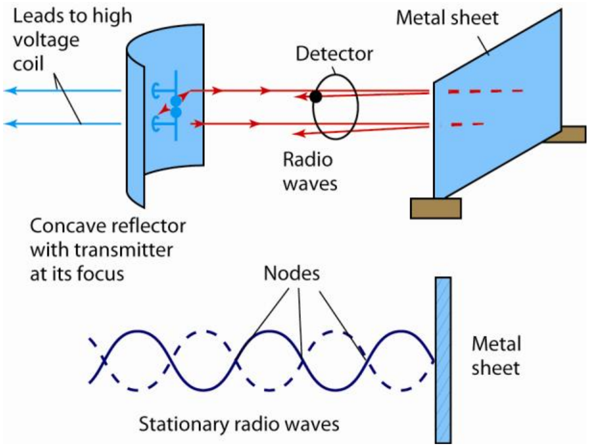


Figure 4 *Measuring the wavelength of radio waves*

Demonstrating the transverse nature of radio waves

Hertz developed a 'dipole' detector consisting of two metal rods aligned with each other at the centre of curvature of a concave reflector, as shown in Figure 5. The reflector focuses the radio waves onto the rods such that the oscillating electric field of the radio waves creates an alternating pd between the two rods (as they are parallel to the electric field), causing sparks at the spark gap.

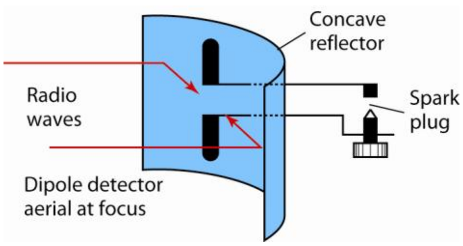
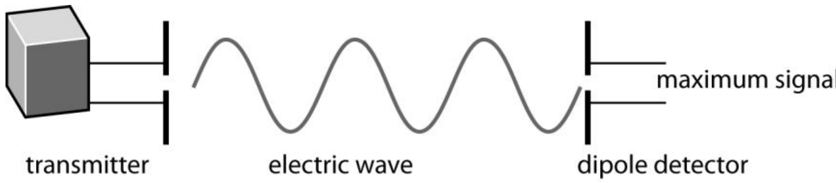
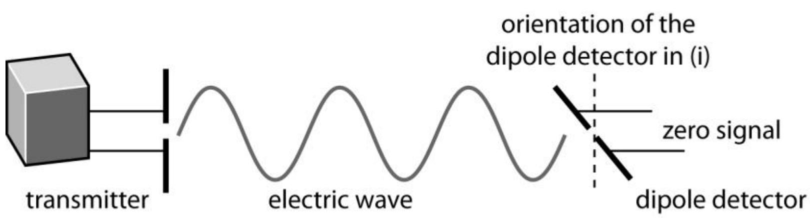


Figure 5 *A dipole detector – construction*

Hertz discovered that when the reflector and the dipole were parallel to the transmitter spark gap, a strong signal was obtained. When the dipole and reflector were rotated gradually from this position, the detector signal decreased and became zero at an angle of rotation of 90° from the initial position. Hertz concluded that the radio waves from the transmitter were polarised and the zero signal was because the dipole had been turned until it was perpendicular to the plane of polarisation of the oscillating electric field which therefore could not produce a pd between the rods.



i) Maximum signal: detector dipole parallel to the transmitter rods



ii) Zero signal: detector dipole perpendicular to the transmitter rods

Figure 6 *A dipole detector – polarisation demonstration*

AQA Examiner's tip

The radio waves from the transmitter are polarised when they are created. The dipole detector needs to be parallel to the electric field oscillations for maximum signal strength. The plane of a loop detector needs to be perpendicular to the magnetic field oscillations (which would mean it too is parallel to the electric field oscillations) for maximum signal strength.

Links

Polarisation was looked at in Topic 12.1 of *AS Physics A*.

Stationary waves were looked at in Topic 12.5 of *AS Physics A*.

Summary questions

1 With the aid of a diagram, state what is meant by an electromagnetic wave.

2 Explain the significance of the equation $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$ derived by Maxwell for the speed of electromagnetic waves in free space.

3 Describe how Hertz set up a stationary wave pattern of radio waves and explain how he measured the wavelength of the radio waves from the stationary wave pattern.

4 Hertz discovered that the strength of the radio signal from a radio wave transmitter varies according to the orientation of the detector. Explain this effect and state the conclusion drawn by Hertz about the radio waves from the transmitter.

2.3 The development of the photon theory of light

Learning objectives:

- Why is wave theory unable to explain photoelectricity?
- What is meant by the stopping potential in the photoelectric effect?
- What was the significance of Einstein's explanation of photoelectricity?

The discovery of photoelectricity

Metals emit electrons when supplied with sufficient energy. Thermionic emission involves supplying the required energy by heating the metal. Another way of supplying the energy is by means of photoelectric emission which involves illuminating the metal with light above a certain frequency. Figure 1 shows a simple demonstration of photoelectric emission from a metal plate. The electroscope is given an initial negative charge which causes the gold leaf to rise. When ultraviolet radiation is directed at the zinc plate, the leaf gradually falls as electrons are emitted from the zinc plate. These emitted electrons are referred to as photoelectrons. The leaf stops falling if the ultraviolet lamp is switched off and resumes its fall when the lamp is switched on again.

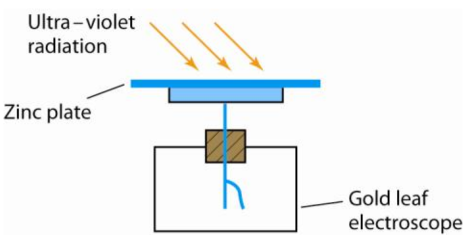


Figure 1 *Demonstrating photoelectricity*

Photoelectric emission was first discovered by Hertz when he was investigating radio waves using a spark gap detector. He observed that the sparks were much stronger when ultraviolet radiation was directed at the spark gap contacts. Further investigations showed the following, for any given metal:

- No photoelectrons are emitted if the frequency of the incident light is below a certain value known as the **threshold frequency**. This is the minimum frequency of light that can cause photoelectric emission from the metal. For some metals, the threshold frequency is in the frequency range for visible light whereas for other metals it is in the ultraviolet range.

(Note: references below to light in this context cover visible and ultraviolet light.)

- Photoelectric emission occurs at the instant that light of a suitably high frequency is incident on the metal surface.
- The photoelectrons have a range of kinetic energies from zero up to a maximum value that depends on the type of metal and the frequency of the incident light.
- The number of photoelectrons emitted from the metal surface per second is proportional to the intensity of the incident radiation (i.e. the light energy per second incident on the surface).

The more intense the radiation is, the greater the number of photoelectrons leaving the metal each second.

The wave theory of light failed to explain the above observations. According to wave theory, light of any frequency should cause photoelectric emission. Wave theory predicted that the lower the frequency of the light, the longer the time taken by electrons in the metal to gain sufficient kinetic energy to escape from the metal. So the wave theory could not account for the existence of the threshold frequency and it could not explain the instant emission of photoelectrons or their maximum kinetic energy.

Einstein's explanation of photoelectric emission

The photoelectric effect could not be explained for more than ten years until Einstein, in 1905, published the **photon theory** of light and used it to explain photoelectricity. Before Einstein developed the theory, the idea of energy 'quanta' as packets of energy had been used by Planck to explain the continuous spectrum of thermal radiation emitted by an object at a constant temperature. Wave theory predicts incorrectly that the intensity of the spectrum should become infinite at smaller and smaller wavelengths. Planck solved the problem by introducing the idea that the energy of a vibrating atom can only be in multiples of a basic amount or 'quantum' of energy. To explain photoelectricity, Einstein applied this 'quantum' idea to electromagnetic radiation which he said consists of wave packets of electromagnetic energy which he referred to as **photons**, each carrying energy given by $E = hf$, where f is the frequency of the radiation and h is the Planck constant introduced earlier by Planck.

Einstein knew that a conduction electron in a metal needs to have a minimum amount of energy, referred to as the **work function** ϕ of the metal, to escape from the surface of the metal. To explain photoelectricity, he assumed that in order for a conduction electron to escape, it needs to:

- absorb a single photon and therefore gain energy hf
- use energy equal to the work function ϕ of the metal to escape.

Since the mean kinetic energy of a conduction electron in a metal at room temperature is negligible compared with the work function of the metal, it follows that the electron can only escape if the energy it gains from a photon is greater than or equal to the work function of the metal. In other words, $hf \geq \phi$. The frequency of the incident radiation $f \geq \phi/h$ for photoelectric emission to occur. So, the threshold frequency of the incident radiation, $f_0 = \phi/h$.

Notes

1 Einstein assumed a conduction electron absorbs a single photon and that the energy of a photon can only be transferred to one electron, not be spread between several electrons.

2 Planck's theory was based on the idea that the quantum of energy of a vibrating atom is proportional to the frequency and he introduced h as the constant of proportionality in the equation $E = hf$ for the quantum of energy of an atom vibrating at frequency f. Note that knowledge of the thermal radiation spectrum, Planck's explanation and the origin of h are not part of this specification and is introduced here to explain why h is referred to as the Planck constant not the Einstein constant.

3 The threshold frequency is the minimum frequency of incident radiation on the metal surface that will cause photoelectric emission. The corresponding wavelength, the threshold wavelength $\lambda_0 = c/f_0$, is therefore the maximum wavelength that will cause photoelectric emission from the metal surface.

AQA Examiner's tip

Remember that photoelectric emission is a one-to-one process in which one electron absorbs one photon.

Stopping potential

Einstein used the photon theory to explain why, when a metal surface is illuminated by monochromatic radiation of frequency greater than the threshold frequency, the photoelectrons emitted have a maximum amount of kinetic energy that depends on the metal. He assumed that each photoelectron must have absorbed one and only one photon so that its kinetic energy after escaping is equal to the energy it gained, (hf), less the energy it used to escape. As the work function of the metal is the minimum energy an electron needs to escape, the maximum kinetic energy E_{Kmax} of a photoelectron from the metal is given by

$$\boldsymbol{E_{\text{Kmax}} = hf - \phi}$$

In practice, the maximum kinetic energy of the emitted photoelectrons can be measured by making the metal plate or 'cathode' increasingly positive with respect to the metal anode used to collect the photoelectrons. The photoelectrons need to do extra work to move away from the cathode if it is positively charged. As a result, the maximum kinetic energy of the photoelectrons is reduced by an amount equal to eV where V is the potential of the cathode relative to the metal anode. In other words, $E_{\text{Kmax}} = hf - \phi - eV$ for a cathode at potential $+V$.

The number of photoelectrons emitted per second decreases as the cathode potential is made increasingly positive. At a certain potential referred to as the **stopping potential**, V_S, photoelectric emission is stopped because the maximum kinetic energy has been reduced to zero. As the work done by a photoelectron in moving through a potential difference V_S is equal to eV_S

$$E_{\text{Kmax}} = hf - \phi - eV_S = 0$$

The stopping potential is therefore given by

$$eV_S = hf - \phi$$

The stopping potential for different frequencies of incident light can be measured and plotted as a graph of stopping potential against frequency. The result for any metal is a straight line graph in agreement with the equation above rearranged as $V_S = \frac{hf}{e} - \frac{\phi}{e}$

Comparison with the general equation for a straight line graph $y = mx + c$ therefore gives:

- h/e for the gradient m,
- $-\phi/e$ for the y-intercept,
- $f_0 = \phi/h$ for the x-intercept.

For different metals:

- the gradient is always the same ($= h/e$)
- the greater the work function, the further the intercepts are from the origin.

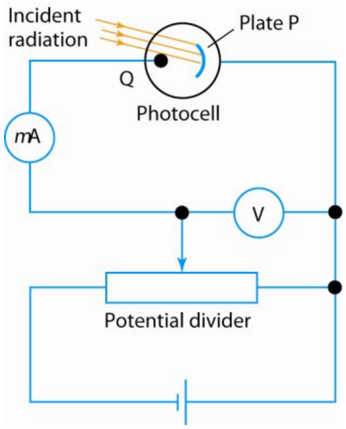


Figure 2 (a) *Stopping potential*

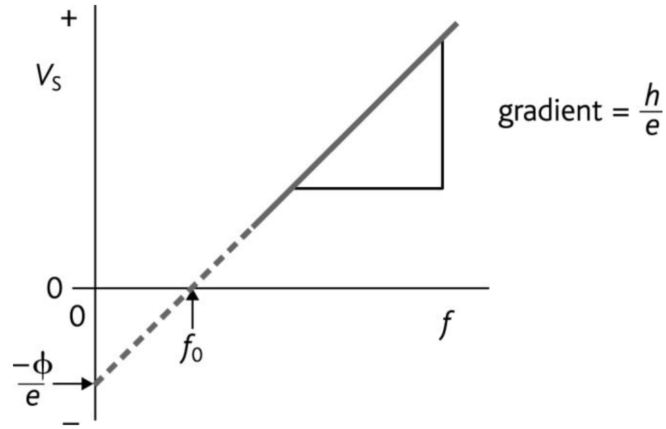


Figure 2(b) *Investigating photoelectricity*

Figure 2(a) shows how the stopping potential can be measured for different frequencies of light using a vacuum photocell. Light of different frequencies can be obtained by passing white light from a filament lamp through different monochromatic filters. With one of the filters in the path of the incident radiation, the potential divider is adjusted initially so the pd across the photocell is zero. The microammeter can be used to measure the current through the photocell due to photoelectric emission from metal plate P (the cathode). When the potential divider is adjusted to make P increasingly positive relative to the collecting terminal Q (the anode), the microammeter reading decreases and becomes zero when the potential of plate P is equal to the stopping potential for that frequency of light. The procedure is then repeated for other values of light frequency. A graph of stopping potential against frequency can then be plotted, as shown in Figure 2(b). The first measurements obtained in this way gave results and a graph as above that confirmed the correctness of Einstein's explanation and thus confirmed Einstein's photon theory of light.

Einstein was awarded the 1921 Nobel Prize for physics for the photon theory of light which he put forward in 1905 although it was not confirmed experimentally until ten years later.

Link

The vacuum photocell was looked at in Topic 3.2 of *AS Physics A*.

Worked example

$e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m s^{-1}

Monochromatic light of wavelength 560 nm incident on a metal surface in a vacuum photocell causes a current through the cell due to photoelectric emission from the metal cathode. The emission is stopped by applying a positive potential of 1.30 V to the cathode relative to the anode. Calculate:

a the work function of the metal cathode in electron volts

b the maximum kinetic energy of the emitted photoelectrons when the cathode is at zero potential.

Solution

a The frequency of the incident light $f = c/\lambda = 3.00 \times 10^{8}/560 \times 10^{-9} = 5.36 \times 10^{14}$ Hz

Rearranging $eV_S = hf - \phi$ gives

$\phi = hf - eV_S = (6.63 \times 10^{-34} \times 5.36 \times 10^{14}) - (1.6 \times 10^{-19} \times 1.30) = 1.47 \times 10^{-19}$ J = 0.92 eV

b At zero potential, the maximum kinetic energy = $hf - \phi = eV_S = 2.08 \times 10^{-19}$ J

AQA Examiner's tip

Remember that the work function is the *minimum* energy an electron needs to escape from the metal.

The significance of Einstein's photon theory

Einstein showed that light consists of photons which are wave packets of electromagnetic radiation, each carrying energy hf, where f is the frequency of the radiation. The photon is the least quantity or 'quantum' of electromagnetic radiation and may be considered as a massless particle. It has a dual 'wave–particle' nature in that its particle-like nature is observed in the photoelectric effect and its wave-like nature is observed in diffraction and interference experiments such as Young's double slits experiment.

Summary questions

$e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m s^{-1}

1 Light of wavelength 535 nm is directed at a metal surface that has a work function of 1.85 eV.

Calculate:

a the energy of a photon of wavelength 535 nm

b the maximum kinetic energy of the emitted photoelectrons.

2 When a metal surface is illuminated with light of wavelength 410 nm, photoelectrons are emitted with a maximum kinetic energy of 2.10×10^{-19} J.

Calculate:

a the energy of a photon of this wavelength

b the work function of the metal surface

c the stopping potential for this surface illuminated by light of wavelength 410 nm.

3 A certain metal at zero potential emits photoelectrons when it is illuminated by blue light but not when it is illuminated with red light. Explain why photoelectric emission from this metal takes place with blue light but not with red light.

4 **a** State one experimental observation in the photoelectric effect that cannot be explained using the wave theory of light.

b Describe how the observation in **a** is explained using the photon theory of light.

2.4 Matter waves

Learning objectives:

- Do matter particles have a dual wave–particle nature?
- Can matter particles be diffracted?
- Why is an electron microscope more powerful in terms of magnification than an optical microscope?

Wave–particle duality

If light has a particle-like nature as well as a wave-like nature, do matter particles have a wave-like nature as well as a particle-like nature? In other words, do matter particles have a dual wave–particle nature? Louis de Broglie in 1925 suggested they do. He put forward the hypothesis that all matter particles have a wave-like nature. He said the particle momentum mv is linked to its wavelength λ by the equation

$mv \times \lambda = h$ where h is the Planck constant

De Broglie arrived at this equation after successfully explaining one of the laws of thermal radiation by using the idea of photons as 'atoms of light'. Although photons are massless, in his explanation he supposed a photon of energy hf to have an equivalent mass m given by $mc^2 = hf$ and therefore a momentum $mc = hf/c = h/\lambda$ where λ is its wavelength.

De Broglie's theory of matter waves and his equation 'momentum × wavelength = h' remained a hypothesis for several years until the experimental discovery that electrons in a beam were diffracted when they pass through a very thin metal foil. Figure 1 shows the arrangement.

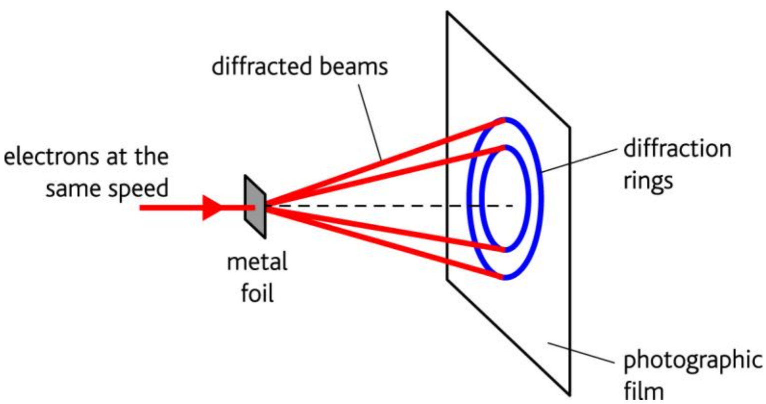


Figure 1 *Diffraction of electrons*

Photographs of the diffraction pattern showed concentric rings, similar to those obtained using X-rays. Since X-ray diffraction was already a well-established experimental technique for investigating crystal structures, it was realised that similar observations with electrons instead of X-rays meant that electrons can also be diffracted and therefore they have a wave-like nature. So de Broglie's hypothesis was thus confirmed by experiment. Particles do have a wave-like nature.

The correctness of de Broglie's equation was also confirmed as the angles of diffraction were observed to:

- increase when the speed of the electrons was decreased, and
- decrease when the speed was increased.

This is because an increase (or decrease) in the speed of the electrons would increase (or decrease) their momentum. This would therefore reduce (or increase) the de Broglie wavelength in accordance with the rearranged form of the de Broglie equation $\lambda = h/mv$, causing the angles of diffraction of the diffracted electrons to decrease (or increase).

For different electron speeds, the angle of diffraction for each ring was measured and used to calculate the de Broglie wavelength of the electrons. The results showed that the de Broglie wavelength of the electrons is inversely proportional to their speed, in accordance with the de Broglie equation.

Notes

1 From X-ray experiments it was known that a metal consists of lots of tiny crystals called 'grains' packed together. The regular array of atoms in each grain causes the X-rays to be diffracted at certain angles only to the incident beam. Because the grains in a metal are orientated in random directions, the diffracted X-rays form a pattern of rings on the photographic film. The same effect is observed with a beam of monoenergetic electrons. In addition, with electrons, the diffraction rings can be made smaller or larger by altering the speed of the electrons.

2 For electrons produced by thermionic emission, the speed v of the electrons depends on the anode potential V_A in accordance with the equation $\frac{1}{2}mv^2 = eV_A$, assuming $v << c$, the speed of light in free space. Multiplying both sides of this equation by $2m$ gives $m^2v^2 = 2meV_A$

Taking the square root of both sides of this equation gives an equation for the momentum of each electron in terms of the anode potential: $mv = (2meV_A)^{1/2}$

Using the de Broglie equation 'momentum × wavelength = h' therefore gives the de Broglie wavelength λ of an electron in terms of the anode potential V_A,

$$\lambda = \frac{h}{\sqrt{(2meV_A)}}$$

Worked example

$e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.11 \times 10^{-31}$ kg

Calculate the de Broglie wavelength of an electron in a beam produced by thermionic emission and accelerated from rest through a pd of 3600 V.

Solution

$$\lambda = \frac{h}{\sqrt{2meV_A}} = \frac{6.63\times10^{-34}}{2\times9.11\times10^{-31}\times1.6\times10^{-19}\times3600} = 2.05 \times 10^{-11}\ \text{m}$$

AQA Examiner's tip

Be careful not to confuse the symbol *v* for speed (or velocity) and the symbol for potential difference *V*.

Electron microscopes

An electron microscope makes use of the particle nature of the electron because it uses electric and/or magnetic fields to control electrons and it makes use of the wave nature of the electron to obtain detailed images. To form an image of an atom, the electrons need to have a de Broglie wavelength of 0.1 nm which is an 'order of magnitude' value of the diameter of an atom. The above equation gives an anode potential of about 150 V for electrons to have a de Broglie wavelength of 0.1 nm. However, as we shall see below, other factors such as lens aberrations in the travelling electron microscope are more significant in determining the detail in an 'electron microscope' image.

The transmission electron microscope (TEM)

The transmission electron microscope consists of an evacuated tube in which a beam of electrons is directed at a thin sample, as shown in Figure 2. Some of the electrons are scattered by the structures in the sample as they pass through the sample (e.g. grain boundaries in a thin metal sample). Electromagnetic coils acting as 'magnetic lenses' focus the scattered electrons onto a fluorescent screen at the end of the tube to form a magnified image of the sample structure.

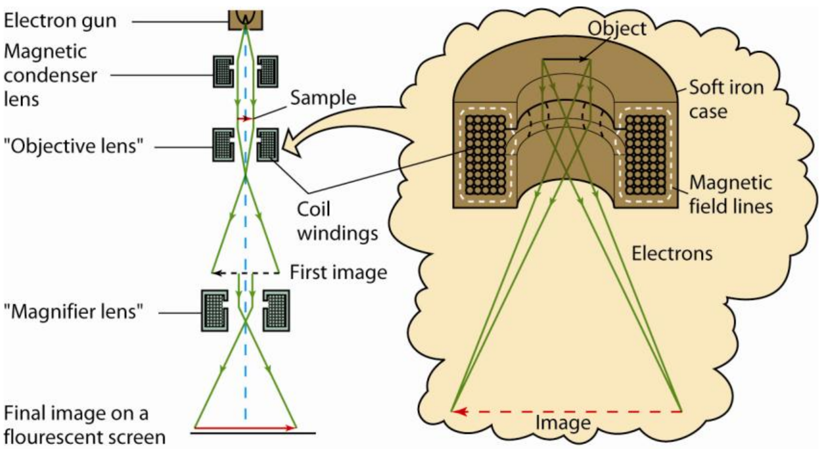
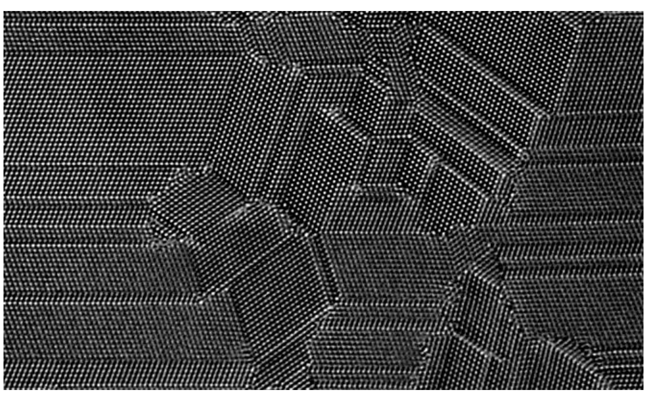


Figure 2 *The transmission electron microscope*



CREDIT: C. Song, National Center of Electron Microscopy, USA

Figure 3 *A TEM image giving an atomic resolution image of a nanocrystalline film of silicon*

1 The 'electron gun' produces electrons by thermionic emission from a heated filament and accelerates them through a hole in a metal anode at constant pd relative to the filament. The electrons emerge through the hole in the anode at the same speed that depends on the anode potential (relative to the filament).

2 The magnetic condenser lens produces a magnetic field that forces the electrons into a parallel beam directed at a very thin sample.

3 The objective lens deflects the scattered electrons so they form an enlarged inverted ‘first’ image of the sample.

4 The magnifier lens focuses the electrons from the central area of the first image to form a magnified final image on the screen.

The amount of detail in a TEM image (and in an optical microscope image) is determined by the resolving power of the microscope. This is the least separation between two objects in the image that can just be seen apart. The resolving power of a microscope depends on how much diffraction occurs when the electrons (or light in the case of an optical microscope) scattered by the sample pass through the objective lens. As with single slit diffraction, the smaller the wavelength of the waves, the less the amount of diffraction and hence the greater the resolving power.

In an electron microscope, the resolving power can therefore be increased by increasing the anode pd which increases the speed of the electrons and therefore reduces their de Broglie wavelength. Increasing the anode pd also enlarges the image on the screen so a larger and more detailed image is seen.

Note

In an optical microscope, a more detailed image is seen if blue light is used instead of any other colour. This is because blue light has a smaller wavelength than any other colour.

Link

Single slit diffraction was looked at in Topic 13.6 of *AS Physics A*.

The limitations on the amount of detail seen in a TEM image are due to two main factors:

- **Sample thickness**: electrons passing through the sample suffer a slight loss of speed which increases the de Broglie wavelength slightly and so reduces the resolving power.
- **Lens aberrations**: the magnetic field in the outer and inner parts of the lens gap may focus electrons from a given point to different positions on the screen instead of to the same position, causing the image to be blurred. Also, the electrons scattered from a given point on the sample may have slightly different speeds due to the process of thermionic emission and also due to passing through different thicknesses of the sample and so they would be focused differently on the screen.

The scanning tunnelling microscope (STM)

In the STM, a fine-tipped metal probe scans across a small area of a surface under investigation at a height of no more than about 1 nm above the surface. The probe is at a constant negative potential of about −1 volt relative to the surface, as shown in Figure 4. Because the gap between the tip and the surface is so small, there is a small but finite probability that electrons can ‘tunnel’ across the gap. The tip must be at a negative potential relative to the surface to ensure the electrons only tunnel across the gap in one direction (i.e. from the tip to the surface under test).

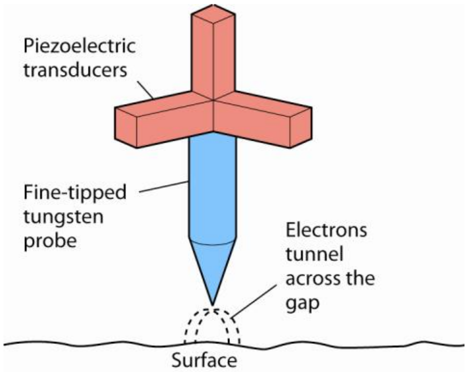


Figure 4 *The scanning tunnelling microscope*

The probe's scanning movement is controlled by two piezoelectric transducers which move it along successive lines parallel to each other. A third piezoelectric transducer is used to adjust the gap width between the tip and the surface. The tunnelling current increases if the gap is made smaller and decreases if the gap is made larger. As the tip scans across the surface, if it moves near a raised atom the gap width decreases and the tunnelling current increases.

- In constant height mode, the tunnelling current is recorded as the tip scans across the surface in a fixed plane. As it does so, the tunnelling current is recorded and used to map the height of the surface on a computer screen.
- In constant current mode, the gap width is kept constant by feeding back changes in the tunnelling current to the piezoelectric transducer that controls the tip height. If the gap between the tip and the surface decreases due to a raised atom, the tunnelling current increases which causes the tip to be raised until the tunnelling current and the gap width are the same as before. The signal to the transducer is recorded and used to map the height of the surface on a computer screen.

In either mode, the image or 'map' of the surface shows the peaks and dips in the surface due to individual atoms or groups of atoms on the surface. If the initial gap width is too large, the tunnelling current will be negligible. If the initial gap width is too small, the tip might be damaged by collisions with raised atoms on the surface. The probe and the surface are normally in a vacuum to prevent contamination of the surface under investigation.

The wave nature of the electron is the reason why electrons can cross the gap. They have insufficient kinetic energy to overcome the potential barrier caused by the work function of the metal tip. However, their de Broglie wavelength is sufficiently long to stretch across the narrow gap, giving the electrons a finite probability of crossing the gap. In effect, the amplitude of the electron wave decreases exponentially in the gap as shown in Figure 5. The gap is sufficiently small so:

- the amplitude is finite on the other side of the gap
- small changes in the gap width produce measurable changes in the number of electrons per second crossing the gap.

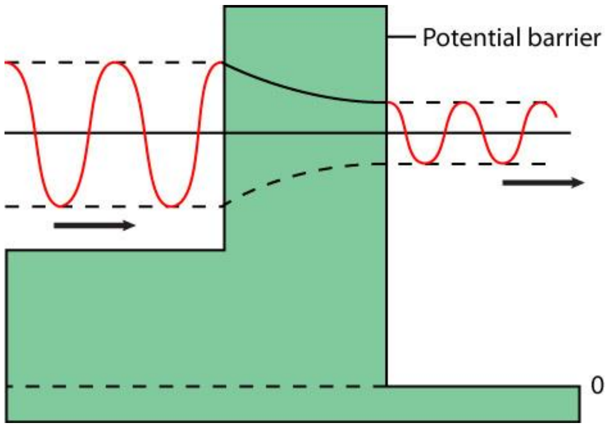


Figure 5 *Electron waves*

How science works

Piezoelectricity

Piezoelectricity is a property of certain materials which produce a pd when stretched or compressed and conversely undergo a tiny change of length when a pd is applied to them. This tiny change of length is used to move the tip of an STM by very small distances.

Summary questions

$e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.11 \times 10^{-31}$ kg, $m_p = 1.67 \times 10^{-27}$ kg

1 Calculate the de Broglie wavelength of:

a an electron moving at a speed of 3.2×10^6 m s^{-1}

b a proton moving at the same speed.

2 Calculate the speed and de Broglie wavelength of an electron in an electron beam that has been accelerated from rest through a pd of 2800 V.

3 Thinking about a transmission electron microscope, state and explain how the image of a thin sample would change if:

a the anode pd was increased

b the sample was moved so that the beam passed through a thicker part of the sample.

4 **a** What is meant by the term ‘matter waves’?

b Figure 6 shows a graph of how the tunnelling current in an STM changed when the tip moved along a straight line at constant height. Use the graph to describe and explain how the height of the surface under the tip changed as the tip moved along the line.

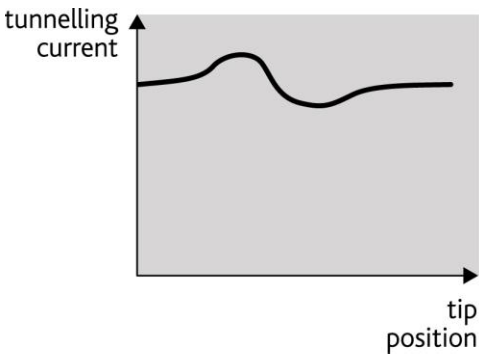


Figure 6

Chapter 3 Special relativity

3.1 About motion

Learning objectives:

- What do we mean by absolute motion and relative motion?
- What experimental evidence is there that all motion is relative?
- Why do we think the speed of light does not depend on the speed of the observer?

Absolute motion?

Galileo and Newton based their theories and laws of motion on the assumption that motion can always be detected. You can usually tell if an object is moving towards or away from you but can you tell if you are in motion or if the object is in motion? Galileo and Newton thought of space and time as absolute quantities that do not depend on the motion of any observer and are therefore the same throughout the universe. For example, a racing car that travels a distance of 10 000 metres in a straight line in 100 seconds according to timing devices at the trackside should record the same timing on an on-board timer.

Now imagine a 22nd century rocket race over a distance of 30 million kilometres. Light takes 100 seconds to travel this distance so you would expect a rocket travelling at half the speed of light to take 200 seconds. The rocket crew and the race officials would also expect a time of 200 s if, like Newton and Galileo, they thought in terms of absolute motion. However, according to Einstein's theory of special relativity, the rocket crew's on-board 'clock' would show they arrive at the finishing line 173 seconds after passing the starting line. The race officials at the starting or finishing line would still record 200 seconds on their clocks. Which timing is correct? In this chapter we will look at:

- why scientists began to question the concept of absolute motion
- how Einstein redefined our concept of motion
- experimental evidence that supports Einstein's theory of special relativity.

After Maxwell published his theory of electromagnetic waves (see Topic 2.2), many physicists thought that the waves are vibrations in an invisible substance which they called ether (or 'aether') and which they supposed exists throughout the universe. According to this hypothesis, the Earth must be moving through the ether and electromagnetic waves are vibrations in the ether. Light was thought to travel at a fixed speed relative to the ether. Detection of the ether was thought to be possible as a result of comparing the time taken by light to travel the same distance in different directions. The same idea applies to sound travelling through the air.

Imagine a very long ship with a hooter in the middle that sends out a short blast of sound on a wind-free day. If the ship is moving forwards, a person in the stern (the back end) will hear the sound before a person in the bow (the front end) because the ship's forward motion moves the person in the stern towards the sound whereas it moves the person in the bow away from the sound. By comparing when the sound is heard at each end, it is possible to tell if the ship is moving forwards or backwards or not at all.

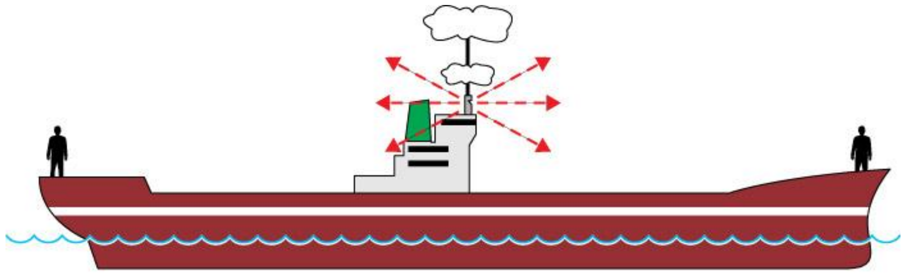


Figure 1 *At sea*

Now consider light instead of sound and the Earth as a ‘ship’ moving through space. The Earth moves on its orbit through space at a speed of about 30 km s^{-1} which is 0.01% of the speed of light so the differences in the travel time of light over a given distance in different directions would be very difficult to detect. This would not be so if the Earth travelled much faster so let us imagine the Earth’s speed is 10% of the speed of light.

Light travels 300 m in 1 microsecond. Suppose a light pulse travels from its source to a mirror 300 m away and is then reflected back to the source, as shown in Figure 2.

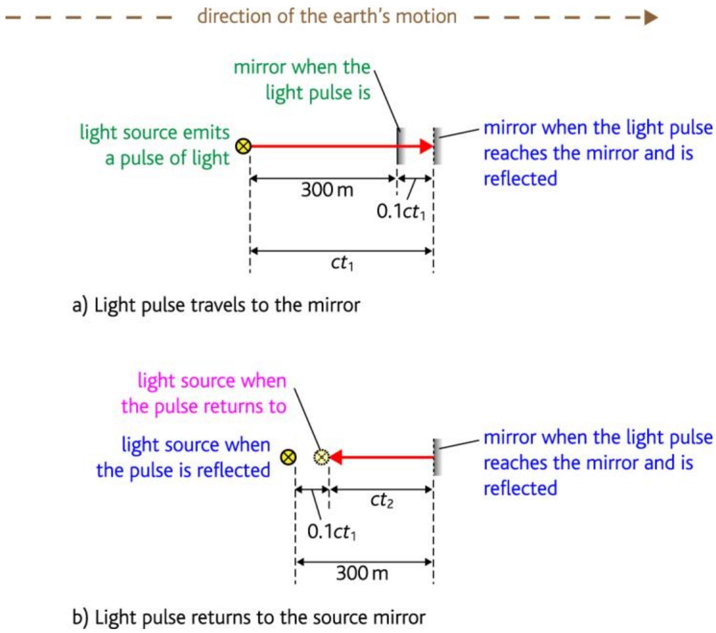


Figure 2 *Using the ether hypothesis*

According to pre-Einstein dynamics:

- If the line between the source and the mirror is parallel to the Earth’s motion, in the time taken t_1 by the light pulse to reach the mirror, the pulse travels a distance ct_1. This distance is equal to 300 metres plus the distance travelled by the mirror in that time which is $0.1ct_1$. Hence $ct_1 = 300 + 0.1ct_1$ which gives $0.9ct_1 = 300$. This means that $t_1 = 300$ metres $\div 0.9c =$ 1.111 microseconds.

  A similar argument for the return journey time, t_2, gives $ct_2 = 300 - 0.1ct_2$ since the light pulse is moving towards the light source. Hence $t_2 = 300$ metres $\div 1.1c = 0.909$ microseconds. So the total journey time of the light pulse would be 2.020 microseconds.

- If the line between the source and the mirror is perpendicular to the Earth's motion, light from the source would need to travel more than 300 m on each part of its journey. Its total journey time would be 2.010 microseconds, as explained in Note 1 below.

So the light pulse would take 0.01 microseconds longer to travel along the parallel path than along the perpendicular path. For light of frequency 6.0×10^{14} Hz, a difference in travel time of 0.01 microseconds corresponds to 6 million cycles (= 6.0×10^{14} Hz × 0.01 microseconds). If the same analysis is applied to the Earth moving through space at its correct speed of 0.01% of the speed of light, the difference in time travel would correspond to 6 cycles for the same distance of 300 m. A distance of 1.0 m would therefore correspond to 0.02 cycles. With this type of analysis in mind, physicists before Einstein realised that the phenomenon of light interference could be used to detect such very small differences. They confidently expected that the Earth's absolute motion through the ether could be detected.

Notes

1 Consider a pulse of light emitted from a source in a direction perpendicular to the Earth's motion towards a mirror at distance *d* then reflected back to the source. Because the source is moved by the Earth's motion while the light pulse is in transit, the pulse must travel as shown in Figure 3 from the position of the source when the light is emitted, S_1, to the mirror at position M_2 then back to the source at position S_3. $M_2S_2 = d$

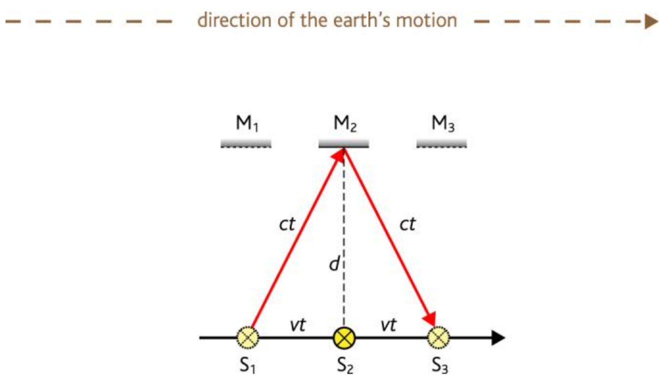


Figure 3 *Light travelling sideways to the Earth's motion*

Let *t* represent the time taken by the light source to travel from the source to the mirror (so light takes a time 2*t* to travel from the source to the mirror and back). In the time taken by the light to travel to the mirror:

- the light source moves a distance *vt* in the direction of the Earth's motion from S1 to S2, and
- the light travels a distance *ct* along the path S_1M_2.

Applying Pythagoras' theorem to triangle $S_1S_2M_2$ therefore gives $(ct)^2 = (vt)^2 + d^2$

Rearranging this equation gives $t = \dfrac{d}{\sqrt{(c^2 - v^2)}}$

Using this formula for $d = 300$ m and $v = 0.1c$ (10% of the speed of light) gives $t = 1.005$ microseconds. The total time taken $2t = 2.010$ microseconds.

2 The specification does not require the detailed analysis above. The key point to bear in mind is that the 'ether theory' predicted that absolute motion can be detected by a difference in the time taken for light to travel the same distance parallel and perpendicular to the Earth's motion.

Maxwell's hypothesis

An experiment to detect the ether was put forward by Maxwell in 1878. He suggested that a beam of light could be split into two perpendicular beams which could then be brought together again by reflection to produce an interference pattern. Interference would occur where the waves and crests of one beam overlapped with those of the other beam. According to Maxwell, rotating the whole apparatus horizontally through 90° would swap the directions of the beams and shift the interference fringes.

Maxwell's idea can be demonstrated using 3 cm microwaves as shown in Figure 4. The whole apparatus would need to be on a table that can be turned.

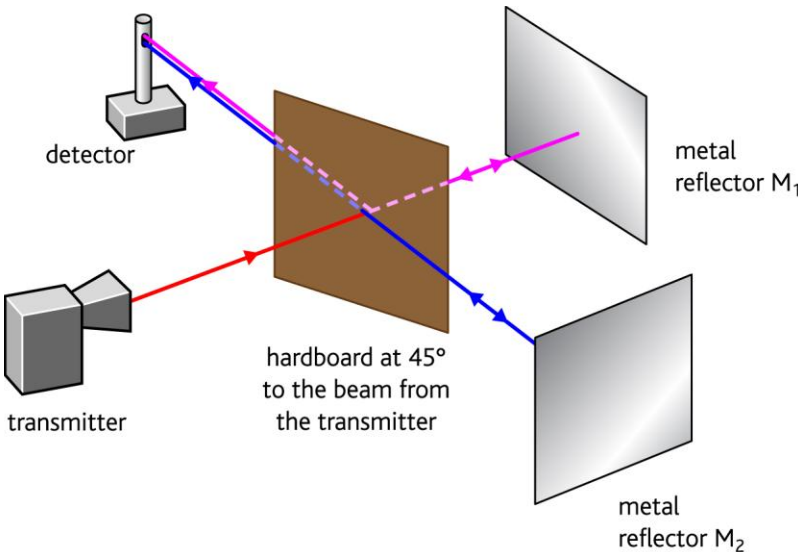


Figure 4 *Using microwaves*

The beam from the transmitter is split by the hardboard into a beam that passes through the hardboard to metal plate M_1 and a beam travelling towards metal plate M_2 due to partial reflection from the hardboard. Both beams are reflected back towards the hardboard where the beam from M_1 is partially reflected to the detector and the beam from M_2 passes straight through to the detector. The two beams reach the detector with a phase difference that depends on the difference in their path lengths. If either M_1 or M_2 are moved gradually away from the hardboard, the detector signal rises and falls repeatedly.

- When the detector signal is a maximum, the two beams arrive at the detector in phase and therefore reinforce.
- When the detector signal is a minimum, the two beams arrive at the detector out of phase by 180° and therefore cancel each other out.

To test Maxwell's idea, the whole apparatus would need to be turned through 90°. This would cause the detector signal to change if the 'ether hypothesis' is correct. Unfortunately for Maxwell, microwaves (and radio waves) were future discoveries so he could not test his idea. If the above test is carried out, no such change is detected.

Note

Movement by half a wavelength of either mirror from a position where the detector signal is a minimum takes the mirror to the next 'minimum signal' position. This is because the change of the path length of the beam that reflects from that mirror is 1 full wavelength. Hence the wavelength can be measured accurately by moving one of the mirrors though a known number of 'minimum signal' position.

The Michelson–Morley experiment

Could the ether hypothesis be tested with light using Maxwell's idea? Michelson and Morley were two American physicists who designed an experimental 'interferometer' apparatus for the purpose of testing the ether hypothesis. Figure 5 shows how their interferometer works.

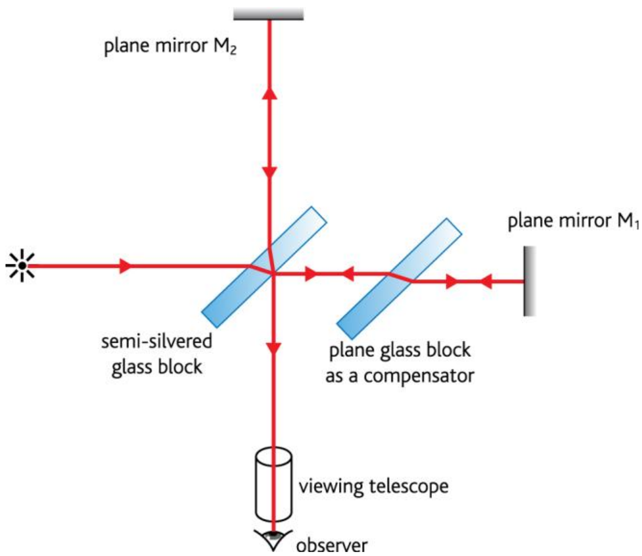


Figure 5 *The Michelson–Morley interferometer*

The light beam from the light source is split into two beams at the back surface of the semi-silvered glass block. One of the two beams continues towards M_1, reflects back to the glass block where it partially reflects into the viewing telescope. The other beam is due to partial reflection at the glass block so it travels towards mirror M_2 where it reflects back to the glass block and travels through it to the telescope. The compensator is present to ensure both beams travel through the same thickness of glass otherwise the two wave trains would not overlap.

An observer looking through the telescope sees a pattern of interference fringes because of the difference in the path lengths of the two beams.

- A bright fringe is where the two beams arrive in phase with each other.
- A dark fringe is where they arrive out of phase by 180° with each other.

Suppose the apparatus is initially aligned so the M_1 beam travels parallel to the direction of the Earth's motion and the M_2 beam travels perpendicular to the direction of the Earth's motion. Turning the apparatus through 90° in a horizontal plane would swap the beam directions relative to the Earth's motion. If the ether hypothesis is correct, this would cause the difference in the travel times of the two beams to reverse, resulting in a noticeable shift in the interference fringes.

Michelson and Morley had predicted using the ether theory that the fringes would shift by about 0.4 of a fringe width and they knew their apparatus was sensitive enough to detect a 0.05 fringe shift. However, they were unable to detect the predicted fringe shift. This 'null' result effectively finished the 'ether' theory off although some physicists tried to maintain the theory by supposing the Earth dragged the ether along with it but there was no astronomical evidence for this supposition. Physicists were presented with a major problem. Galileo's laws of dynamics and Newton's laws of motion didn't seem to work for light!

AQA Examiner's tip

Make sure you can explain why an interference pattern is observed and remember the pattern was expected to shift when the apparatus was turned through 90° – but it didn't!

Summary questions

1 **a** What is meant by:
- **i** absolute rest?
- **ii** absolute time?

b What was the purpose of the Michelson–Morley experiment?

2 In the Michelson–Morley experiment, state and explain the function of:

a the two plane mirrors

b the compensator glass block.

3 **a** Explain the formation of a dark fringe in the fringe pattern observed when looking through the telescope in the Michelson–Morley experiment.

b Describe what observation was expected when the apparatus was rotated through 90°.

4 The Michelson–Morley experiment produced a null result.

a State what the result was.

b **i** Explain why it was referred to as a null result.
- **ii** What was the significance of the null result?

3.2 Einstein's theory of special relativity

Learning objectives:

- What assumptions did Einstein make in his theory of special relativity?
- What do we mean by length contraction, time dilation and relativistic mass?
- Why do we believe that nothing can travel faster than light?

Relative motion

If we can't detect absolute motion, then perhaps absolute rest doesn't exist and all motion is relative. It seems absurd to suggest to a fellow passenger on a train or plane journey that your destination is in motion and moving towards you. Travellers can usually tell they are moving even if they can't see the ground outside because they can sense changes in their velocity. But sometimes your senses can deceive you – for example, in a train waiting at a station, you may have momentarily thought you were starting to move when an adjacent train starts to move.

In general, to detect the relative motion of an object, you need to observe the object's position against your **frame of reference** (i.e. markers that are fixed relative to your own position). Anyone in the adjacent train may think you are moving – until their own train starts to creak and rattle as it moves. Your position would be changing in the other person's frame of reference. If two trains glide perfectly smoothly past each other in darkness, could the passengers tell if their own train is moving? Yes, according to most physicists before Einstein. But no evidence was found for absolute motion from the Michelson and Morley experiment – a 'null' result which confounded the scientific community for many years.

Einstein's theory

Einstein put forward his theory of special relativity in 1905, the same year as he put forward his photon theory of light. He rejected the idea of absolute motion and started from two key ideas.

- The **laws of physics** should be the same in all **inertial** frames of reference which are frames of reference that move at constant velocity relative to each other. For example, an object released at rest in an inertial frame of reference (e.g. a passenger jet moving at constant velocity) will remain at rest in accordance with Newton's first law of motion. However, an object released at rest in an accelerating frame of reference (e.g. a passenger jet taking off or a rotating platform) will not remain at rest.
- The **speed of light** in free space, *c*, is **invariant** which means it is always the same and is independent of the motion of the light source and the motion of any observer.

In other words, in developing his theory of special relativity, Einstein started from two postulates or 'fundamental statements':

Physical laws have the same form in all inertial frames of reference.

The speed of light in free space is invariant.

Einstein realised that the equations representing physical laws be in the same form in any inertial frame of reference. As outlined in the notes below, he knew that this could not be achieved using the ideas of absolute space and time even for a simple equation such as $s = ct$ for the distance, *s*, travelled by a light pulse in time t. So he worked out mathematically how to do it by changing or 'transforming' the distance and time coordinates from any inertial frame of reference to any other. We will look at some of the consequences of these transformations such as time dilation and length contraction.

AQA Examiner's tip

Be sure you know what an *inertial frame of reference* is and what the word *invariant* means.

Time dilation

A moving clock runs more slowly that a stationary clock.

Einstein showed that if the time interval between two events measured by an observer at rest relative to the events is t_0, (called 'the proper time'), an observer moving at speed v relative to the events would measure a longer time interval, t, given by

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = t_0\left(1 - \frac{v^2}{c^2}\right)^{-0.5}$$

The time between the events is stretched out or 'dilated' according to the moving observer. Particle beam experiments provide direct evidence of time dilation.

For example, the half-life of muons at rest is 1.5 μs. A beam of muons travelling at 99.6% of the speed of light ($v = 0.996c$) would therefore decay to 50% of its initial intensity in a distance of 450 m (= $0.996c \times 1.5$ μs) in their own frame of reference.

However, experimental measurements in the Earth's frame of reference found that such a beam moving at this speed would take about 5000 m to decay to 50% of its initial intensity. Given $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$, the time taken to travel 5000 m at a speed of $0.996c$ is 17 μs (= 5000 m ÷ $0.996c$). This agrees with Einstein's time dilation formula which gives a dilated time of 17 μs for a proper time of 1.5 μs (i.e. over 11 times longer than the half-life of stationary muons).

Note

The measurements were made using muons created in the upper atmosphere as a result of cosmic rays from space colliding with the nuclei of atoms in the atmosphere. A detector 2000 m below the top of the atmosphere recorded an intensity of about 80% of the initial intensity. A further 2000 m would cause a decrease to about 60% (≈ 80% of 80%.) To decrease to about 50%, a further 1000 m would be needed (about 5000 m in total).

Worked example

$c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$

The half-life of charged π mesons at rest is 18 ns. Calculate the half-life of charged mesons moving at a speed of $0.95c$.

Solution

$t_0 = 18$ ns

$$\frac{v}{c} = 0.95$$

Therefore

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{18}{\sqrt{\left(1 - 0.95^2\right)}} = 3.2 \times 18 = 58\,\text{ns}$$

The twins paradox

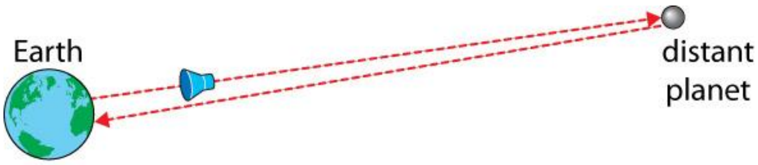


Figure 1 *A paradox*

An astronaut aged 25 says goodbye to his twin and travels at 0.95*c* in a spaceship to a distant planet, arriving there 5 years later. After spending a few weeks there, the astronaut returns to Earth at the same speed on a journey that takes another 5 years. The astronaut is 35 years old on his return. Use Einstein's time dilation formula to prove for yourself that 16 years on Earth elapse in the 5 years the astronaut, travelling at 0.95*c*, takes to travel each way. So his 'stay-at-home' twin is aged 57 when the astronaut returns!

Note that the proper time for each part of the journey was 5 years. This is the time between departure and arrival and only the astronaut and the other space crew were present at both events.

You can make up some strange scenarios round the above journey – if the astronaut was a parent of a child aged 4 on departure from Earth, the child would be a year older than the astronaut on return.

It could be argued that the Earth travels away from the spaceship at 0.95*c* and therefore the 'Earth' twin is the one who ages. However, this apparent paradox can be resolved on the grounds that the twin who travels to the distant planet has to accelerate and decelerate and therefore the twins' 'journeys' are not equivalent. Therefore the twin who has travelled to the distant planet was not in the inertial frame and so came back younger than his 'stay at home' brother.

Length contraction

A rod moving in the same direction as its length appears shorter than when it is stationary.

Einstein showed that the length *L of* a rod moving in the same direction as its length is given by

$$L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)} = L_0\left(1 - \frac{v^2}{c^2}\right)^{0.5}$$

where L_0, the **proper length** of the rod, is the length measured by an observer at rest relative to the rod.

Let's return to the charged π mesons travelling at 0.9995*c* in a particle beam. Remember their half-life of 18 ns when at rest is stretched to 570 ns in the laboratory by time dilation and they travel a laboratory distance of 171 m in this time. Imagine travelling alongside them at the same speed. The laboratory distance of 171 m would appear contracted to 5.4 m $(171 \times (1 - 0.995^2)^{1/2})$. This is the distance travelled by the charged π mesons in their frame of reference and is equal to how far they travel in one 'proper time' half-life of 18 ns.

Worked example

$c = 3.00 \times 10^8\ \mathrm{m\,s^{-1}}$

A spaceship moving at a speed of 0.99*c* takes 2.3 seconds to fly past a planet and one of its moons. Calculate:

a the distance travelled by the spaceship in its own frame of reference in this time

b the distance from the planet to the moon in their frame of reference.

Solution

a Distance = speed × time = 0.99*c* × 2.3 = $0.99 \times 3.0 \times 10^8 \times 2.3 = 6.8 \times 10^8$ m

b The distance from the planet to the Moon in their own frame of reference is the proper distance (L_0).

The distance travelled by the spaceship is the contracted distance (L).

Rearranging the length contraction formula to find L_0 therefore gives

$$L_0 = \frac{L}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{6.8\times10^8}{\sqrt{\left(1 - 0.99^2\right)}} = 7.1 \times 6.8 \times 10^8 = 4.8 \times 10^9\ \mathrm{m}$$

Relativistic mass

The mass of an object increases with speed.

By considering the law of conservation of momentum in different inertial frames of reference, Einstein showed that the mass *m* of an object depends on its speed *v* in accordance with the equation

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where m_0 is the rest mass or 'proper mass' of the object (i.e. its mass at zero speed).

The equation above shows that:

- the mass increases with increasing speed
- as the speed increases towards the speed of light, the equation shows that the mass becomes ever larger.

Experimental evidence for relativistic mass was obtained from electron beam experiments soon after Einstein published the theory of special relativity. The specific charge of the electron *e*/*m* was measured for electrons moving at different speeds. For example, at a speed of 0.69*c*, *e*/*m* was measured as $1.28 \times 10^{11}\ \mathrm{C\,kg^{-1}}$ which can be compared with the accepted value of

$1.76 \times 10^{11}\,\mathrm{C\,kg^{-1}}$ for the 'rest' value, e/m_0. Prove for yourself using the relativistic mass formula that the value of e/m at $0.69c$ is indeed $1.28 \times 10^{11}\,\mathrm{C\,kg^{-1}}$.

The cosmic speed limit

Figure 2 shows how the mass of an object varies with its speed in accordance with the relativistic mass equation. The graph shows that:

- at speed $v << c$, $m = m_0$
- at speed $v \rightarrow c$, m increases gradually to about $2m_0$ at $v \approx 0.9c$ and then increases sharply and tends to infinity as v approaches c.

Einstein's relativistic mass formula means that no material object can ever reach the speed of light as its mass would become infinite and therefore can never travel faster than light. Thus the speed of light in free space, c, is the ultimate cosmic speed limit.

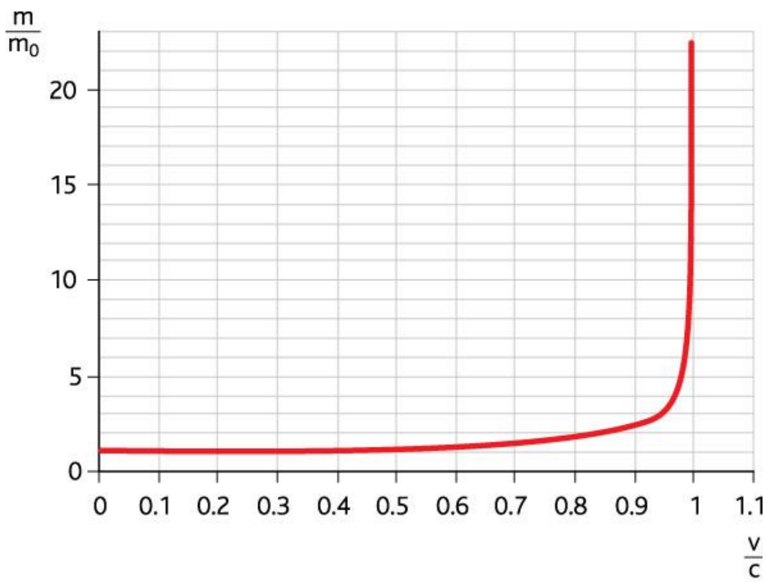


Figure 2 *Relativistic mass v speed*

Mass and energy

Increasing the speed of an object increases its kinetic energy. So Einstein's relativistic mass formula tells us that the mass of an object increases if it gains kinetic energy. In his theory of special relativity, Einstein went further and proved that transferring energy in any form:

- to an object increases its mass
- from an object decreases its mass.

He showed that energy E and mass m are equivalent (interchangeable) on a scale given by his now-famous equation

$$E = mc^2$$

Since the value of $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$, then 1 kg of mass is equivalent to 9.0×10^{16} J ($= 1 \times (3.0 \times 10^8)^2$).

In terms of the rest mass m_0 of an object, the above equation may be written as

$$E = \frac{m_0 c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

At zero speed, $v = 0$, therefore $\boldsymbol{E = m_0c^2}$ represents the **rest energy** of the object.

At speed v, the difference between its total energy E and its rest energy E_0 represents its energy due to its speed (i.e. its kinetic energy). Therefore,

its kinetic energy $\boldsymbol{E_k = mc^2 - m_0c^2}$

For example, if an object is travelling at a speed $v = 0.99c$, the relativistic mass formula gives

mass $m = \frac{m_0}{\sqrt{(1-0.99^2)}}$ = $7.1m_0$ so its kinetic energy $E_k = 7.1m_0c^2 - m_0c^2 = 6.1m_0c^2$

Notes

1 At speeds $v << c$, the above kinetic energy formula $E_k = mc^2 - m_0c^2 \to \frac{1}{2}mv^2$ as $\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \to \left(1 - \frac{v^2}{2c^2}\right)$ as $v \to c$. You don't need to know this for the option specification but it's helpful to know how the formula $E_k = \frac{1}{2}mv^2$ fits in.

2 If a charged particle of charge Q is accelerated from rest through a potential difference V to a certain speed, the work done on it is $W = QV$. Its kinetic energy after being accelerated is therefore equal to QV. Therefore its total energy $E = mc^2 = m_0c^2 + QV$.

3 The rest energy values in MeV of some particles will be supplied in an exam in the data booklet. These values are obtained by inserting mass values in kilograms and the value of the speed of light c into the rest energy formula $\boldsymbol{E = m_0c^2}$ to obtain the rest energy value in joules and then converting into MeV using the conversion factor 1 MeV = 1.6×10^{-13} J.

Worked example

$e = 1.6 \times 10^{-19}$ C, electron rest mass $m_0 = 9.1 \times 10^{-31}$ kg

An electron is accelerated from rest through a pd of 6.0 MV. Calculate:

a its gain of kinetic energy

b its mass in terms of its rest mass

c the ratio of its speed to the speed of light c.

Solution

a Its gain of kinetic energy = $QV = 1.6 \times 10^{-19} \times 6.0 \times 10^6 = 9.6 \times 10^{-15}$ J

b Its total energy = $mc^2 = m_0c^2 + QV$

Therefore $\frac{m}{m_0} = 1 + \frac{QV}{m_0c^2} = 1 + \frac{9.6\times10^{-13}}{9.1\times10^{-31}\times(3.0\times10^8)^2} = 1 + 11.7 = 12.7$

c Using the relativistic mass equation gives

$$12.7m_0 = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

Cancelling m_0 from both sides gives

$$12.7^2 = \frac{1}{\left(1-\frac{v^2}{c^2}\right)}$$

Rearranging gives

$$\left(1-\frac{v^2}{c^2}\right) = \frac{1}{12.7^2} = 6.2\times10^{-3}$$

Hence $\frac{v^2}{c^2} = 1 - 6.2 \times 10^{-3} = 0.9937$

Therefore $\frac{v}{c} = 0.997$

How science works

About Einstein

Einstein has become a byword for the ultimate in brain power. Although he displayed remarkable talent in maths and physics at school, Einstein was not rated very highly by his physics professors at university. He upset his university tutors by asking too many awkward questions and his unusual talents were dismissed. He secured a position as a patent officer in the Swiss Patent Office. The job was not too demanding and he had time to think about fundamental issues in physics. 'Thought' experiments have always been important in physics. Galileo and Newton developed their ideas about motion by developing thought experiments from their observations. In 1905, Einstein introduced the two key theories of modern physics, namely quantum theory and relativity theory. In his later theory of general relativity, he predicted that gravity can bend light. When this was confirmed by astronomical observations of the 1918 solar eclipse, Einstein became an overnight celebrity worldwide as 'the scientist who knew how to bend light'.

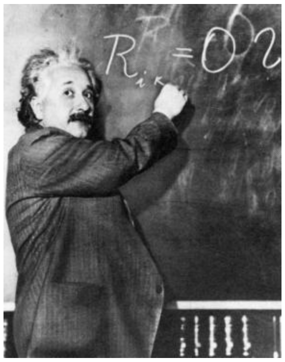


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Figure 3 *Einstein on tour*

How Einstein worked out the rules for relativity

The information below is provided to give a deeper understanding of the topic and is **not** required by the specification.

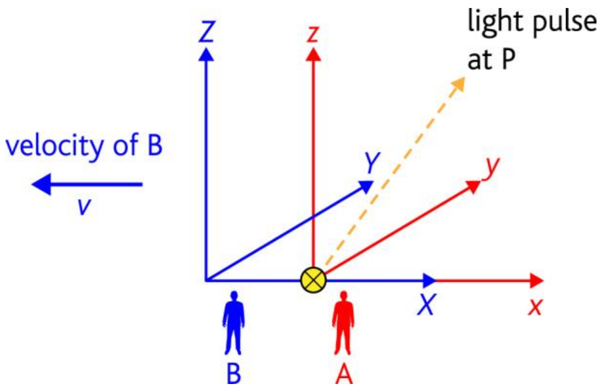

Figure 4 *Observers at work*

Consider the distance travelled by a light pulse in a certain time after being emitted from a light source, as shown in Figure 4, at the instant the axes of the frames of reference of two observers coincide with each other.

Observer A's frame of reference (in red in Figure 4) has the light source stationary at its origin. The equation $s = ct$ gives the distance, s, travelled by a light pulse in time t. The distance, s, travelled by the light pulse to a point P with coordinates (x, y, z) is given by

$$s^2 = x^2 + y^2 + z^2 \text{ (using Pythagoras theorem).}$$

Therefore, according to observer A, the time taken by a light pulse to travel from the light source to P is given by

$$c^2t^2 = x^2 + y^2 + z^2$$

Observer B and the corresponding frame of reference (in blue in Figure 4) is travelling at velocity v relative to the light source in the negative x-direction.

In the observer B's frame of reference, the time taken T by a light pulse to travel from the light source to P is given by

$$c^2T^2 = X^2 + Y^2 + Z^2$$

where (X, Y, Z) are the coordinates of P in the observer B's frame of reference.

Not too difficult so far…

In pre-Einstein dynamics, space and time are absolute quantities so $T = t$, $X = x + vt$, $Y = y$ and $Z = z$. However, substituting these transformations into the equation above gives

$$c^2t^2 = (x + vt)^2 + y^2 + z^2$$

which is not the same as the corresponding equation for the frame of reference of observer A.

Still with it?

Einstein showed that there is a unique transformation solution given by

$$X = \beta(x + vt),\ T = \beta\left(t + \frac{vx}{c^2}\right),\ Y = y \text{ and } Z = z \text{ where } \beta = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Don't even try!

Now you know why Einstein was considered to be a genius! But *remember* you can *forget* this information for your examination – but not the consequences such as time dilation, length contraction and relativistic mass!

Summary questions

$e = 1.6 \times 10^{-19}$ C, electron rest mass $m_0 = 9.1 \times 10^{-31}$ kg, $c = 3.00 \times 10^{8}\ \mathrm{m\,s^{-1}}$

1 **a** State what is meant by an inertial frame of reference.

b Give an example of a frame of reference that is:

i inertial

ii non-inertial.

c Einstein put forward two postulates, one of which is that the speed of light in free space is invariant.

i What is meant by the word 'invariant' in this context?

ii State the other postulate put forward by Einstein.

2 A beam of particles travelling at a speed of 0.98c travels in a straight line between two stationary detectors which are 200 m apart.

Calculate:

a **i** the time taken by a particle in the beam to travel from one detector to the other in the frame of reference of the detectors

ii the proper time between a particle passing the detectors particles

b the distance between the detectors in the frame of reference of the particles.

3 An electron is accelerated from rest through a pd V to a speed 0.95c. Calculate:

a the mass of the electron at this speed

b its kinetic energy at this speed

c the pd, V, through which it was accelerated.

4 **a** In terms of the speed of light in free space, c, calculate the speed that a particle must have in order for its mass to be 100 times its rest mass.

b The kinetic energy of a particle can be increased by any amount but its speed cannot exceed the speed of light. Explain the apparent contradiction in this statement.

Answers

1.1

4 3.75×10^{7} m s^{-1}

1.2

4 9.0×10^{-4} V m^{-1}

1.3

1 **a** 3.39×10^{7} m s^{-1} **b** 1.75×10^{11} C kg^{-1}

2 1.80×10^{11} C kg^{-1}

3 1.79×10^{11} C kg^{-1}

4 The value was many times larger than the largest known specific charge which was that of the hydrogen ion. The magnitude of the charge of the electron was not known at the time. However it was realised the electron either has much less mass than the hydrogen ion or it has much more charge.

1.4

1 **a** 4.78×10^{-19} C **b** 3

2 **b** **i** 6.40×10^{-19} C, positive **ii** 4

3 **a** 3.96×10^{-15} kg **b** 3.18×10^{-19} C

2.3

1 **a** 3.72×10^{-19} J **b** 7.60×10^{-20} J

2 **a** 4.85×10^{-19} J **b** 1.61×10^{-19} J **c** + 1.31 V

2.4

1 **a** 2.27×10^{-10} m **b** 1.24×10^{-13} m

2 3.14×10^{6} m s^{-1}, 2.32×10^{-11} m

3.2

2 **a** **i** 680 ns **ii** 135 ns, **b** 40 m

3 **a** 2.9×10^{-30} kg **b** 1.8×10^{-13} J **c** 1.1(3) MV

4 **a** 0.99995c

Additional examination-style questions

1 **(a)** Describe, in terms of electric and magnetic fields, the nature of electromagnetic waves travelling in a vacuum. You may wish to draw a labelled diagram. *(3 marks)*

(b) Electrons are emitted from a metal plate when monochromatic light is incident on it, provided that the frequency of the light is greater than or equal to a threshold value.
You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer.

(i) How did Einstein explain this effect?

(ii) Discuss the significance of Einstein's explanation. *(4 marks)*

AQA 2006

2 **Figure 1** shows an electron gun in an evacuated tube. Electrons emitted by *thermionic emission* from the metal filament are attracted to the metal anode which is at a fixed potential, V, relative to the filament. Some of the electrons pass though a small hole in the anode to form a beam which is directed into a uniform magnetic field.

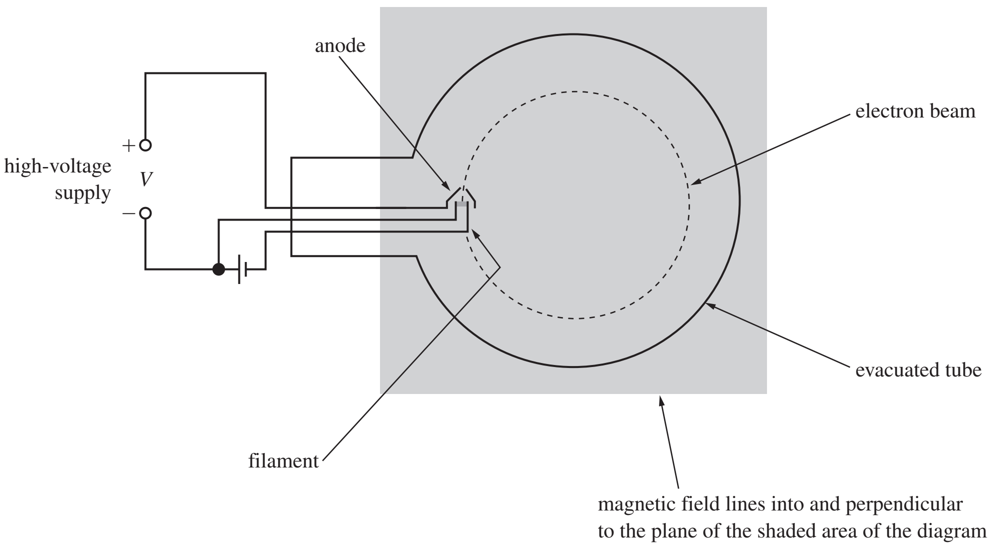

Figure 1

(a) **(i)** Explain what is meant by thermionic emission.

(ii) Show that the speed, v, of the electrons in the beam is given by

$$v = \frac{2eV^{\frac{1}{2}}}{m},$$

where m is the mass of the electron and e is the charge of the electron. *(3 marks)*

(b) The beam of electrons travels through the field in a circular path at constant speed.

(i) Explain why the electrons travel at constant speed in the magnetic field.

(ii) Show that the radius, r, of the circular path of the beam in the field is given by

$$r = \frac{2mV^{\frac{1}{2}}}{B^2e},$$

where B is the magnetic flux density and V is the pd between the anode and the filament.

(iii) The arrangement described above was used to measure the specific charge of the electron, e/m. Use the following data to calculate e/m.

$B = 3.1$ mT
$r = 25$ mm
$V = 530$ V *(7 marks)*

AQA 2006

3 π mesons, travelling in a straight line at a speed of 0.95 c, pass two detectors 34 m apart, as shown in **Figure 2**.

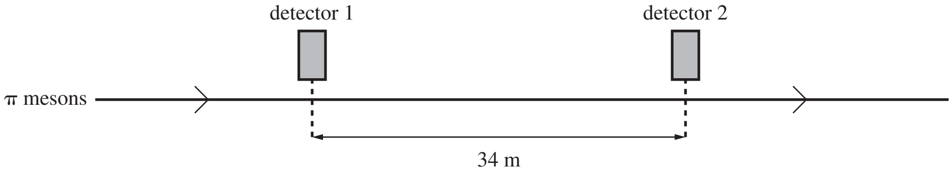

Figure 2

(a) Calculate the time taken, in the frame of reference of the detectors, for a π meson to travel between the two detectors.

(b) π mesons are unstable and decay with a half-life of 18 ns when at rest. Show that approximately 75% of the π mesons passing the first detector decay before they reach the second detector. *(5 marks)*

AQA 2006

4 A narrow beam of electrons, all with the same kinetic energy, is directed between two horizontal deflecting plates, X and Y, in a vacuum tube, as shown in **Figure 3**.

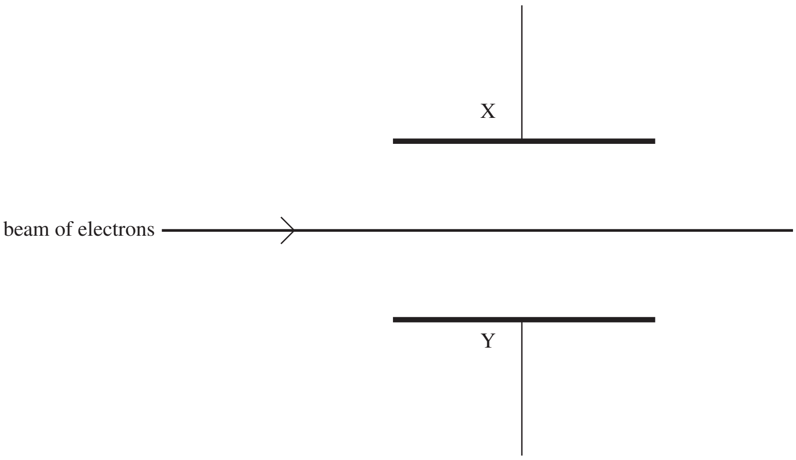

Figure 3

(a) State and explain the effect on the electron beam of applying a constant pd between X and Y, with X negative relative to Y. *(2 marks)*

(b) With a constant pd, V, between X and Y, a uniform magnetic field is applied perpendicular to the plane of the diagram between the plates. The magnetic flux density is adjusted to a certain value B_0, so that the beam is undeflected.

(i) Explain why the beam is undeflected at this value of the magnetic flux density.

(ii) Show that the speed, v, of the electrons in the beam is given by

$$v = \frac{V}{B_0 d},$$

where d is the perpendicular distance between plates X and Y. *(4 marks)*

(c) Electrons are accelerated from rest through a pd of 3800V to a speed of 3.7×10^7 m s^{-1}. Use this information to calculate the specific charge e/m of the electron. *(3 marks)*

AQA 2007

5 **Figure 4** shows a radio wave transmitter T and a detector D. The detector consists of a metal loop connected to a suitable meter.

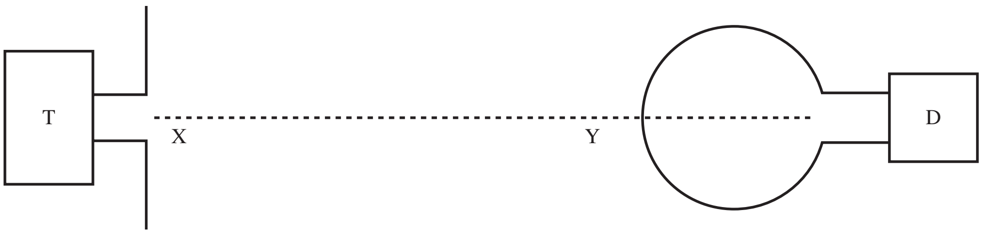

Figure 4

(a) Explain why radio waves from the transmitter induce an alternating emf in the metal loop. You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer. *(3 marks)*

(b) When the metal loop is rotated through 90° about the line XY, the detector signal falls to zero. Explain why the signal decreases and why it falls to zero. *(3 marks)*

AQA 2007

6 **Figure 5** represents the Michelson-Morley interferometer.

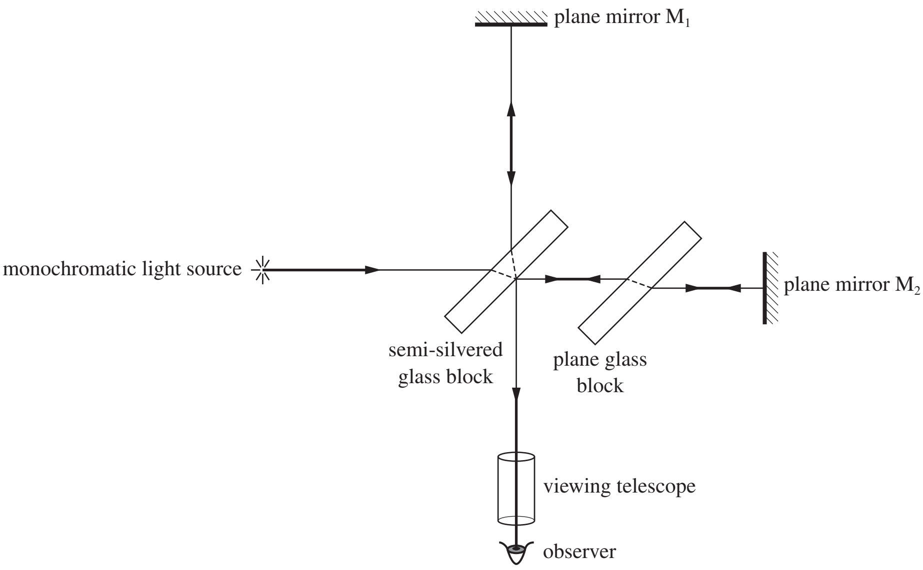

Figure 5

(a) **(i)** What was the principal purpose for which Michelson and Morley designed this apparatus?

(ii) Explain why interference fringes are observed through the telescope. *(3 marks)*

(b) Michelson and Morley expected the interference fringes would shift when the apparatus was rotated through 90°.

(i) Why was it thought that a fringe shift would be observed?

(ii) What conclusion did Michelson and Morley draw from the observation that the fringes did not shift? *(3 marks)*

AQA 2007

7 **Figure 6** shows a charged oil droplet between two oppositely-charged horizontal parallel plates X and Y which are 6.0 mm apart.

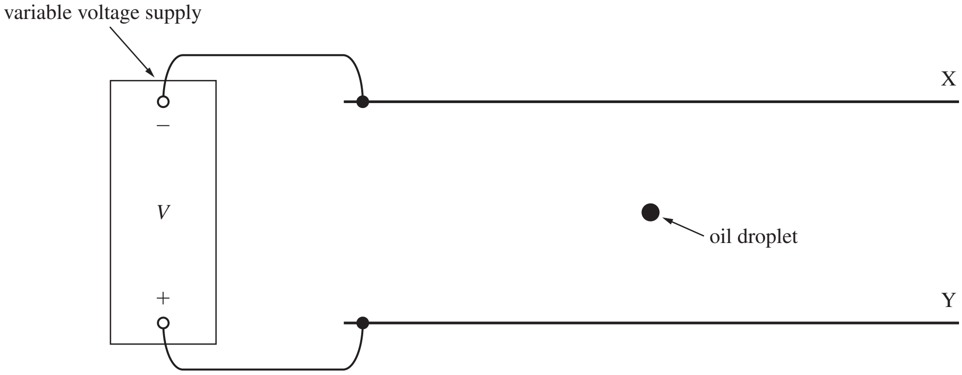

Figure 6

(a) When the potential difference between the two plates is zero, the droplet falls vertically at a steady speed of 7.8×10^{-5} m s^{-1}.

density of oil droplet = 960 kg m^{-3}
viscosity of air = 1.8×10^{-5} N s m^{-2}

(i) Explain why the droplet falls at a steady speed.

(ii) Show that the radius of the droplet is 8.2×10^{-7}m.

(iii) Show that the mass of the droplet is 2.2×10^{-15} kg. *(6 marks)*

(b) The potential difference between X and Y is adjusted until the droplet becomes stationary.

(i) Explain why the droplet becomes stationary.

(ii) The droplet is stationary when the potential difference is 410 V. Show that the charge of the droplet is 3.2×10^{-19} C .

(iii) Discuss the significance of this result and the results of similar tests on other charged droplets. *(5 marks)*

AQA 2008

8 A particle has a rest mass of 1.9×10^{-28} kg. Calculate

(i) the speed of the particle at which its mass would be 9.5×10^{-28} kg,

(ii) the kinetic energy, in J, of the particle at this speed. *(6 marks)*

AQA 2008

Answers to examination-style questions

Answers	Marks	Examiner's tips
1 (a) • Diagram/description of electric wave and magnetic wave in phase. • Diagram/description/statement that electric wave is at 90º to the magnetic wave. • Diagram/description/statement that direction of propagation/travel is perpendicular to both waves.	**3**	It is a good idea to use a diagram, but very important that it is fully labelled. A 3D diagram is tricky to draw, but the perpendicular nature of the electric and magnetic parts can be indicated or described.
(b) (i) • (Conduction) electron (in the metal) absorbs a photon and gains energy *hf*. • Work function of a metal is the minimum energy needed by an electron to escape from the metal (surface). • An electron can only escape if $hf \geq$ work function.	**max 2**	One electron absorbs one photon; it is essential to stress the work function is a **minimum** energy.
(ii) • The photon is the quantum of e-m radiation/light. • Classical wave theory could not explain threshold frequency. • Classical wave theory was replaced by the photon theory. • [*or* photons can behave as waves or particles][*or* photons have a dual wave/particle nature].	**max 2**	The overall significance of Einstein's explanation is the the photon model became accepted. It needs to explained in detail how this arises from the failure of the wave model.
2 (a) (i) • Emission of (conduction) electrons from a heated metal (surface) or filament/cathode. • Work done on electron = eV	**2**	
(ii) Gain of kinetic energy (or $\frac{1}{2}mv^2$) = eV; rearrange to give required equation.	**1**	It is often useful to start this kind of explanation from a "word equation" rather than just jump into symbols to make it clear.

Answers to examination-style questions

Answers	Marks	Examiner's tips
(b) (i) • Work done = force × distance moved in direction of force • Force (due to magnetic field) is at right angles to the direction of motion/velocity [or no movement in the direction of the magnetic force ∴ no work done] • Electrons do not collide with atoms **Alternative for first and second marks:** • (magnetic) force has no component along direction of motion • No acceleration along direction of motion or acceleration perpendicular to velocity]	**max 2**	
(ii) $r = \frac{mv}{Be}$ or $\left(Bev = \frac{mv^2}{r}\right)$ $v^2 = \frac{2eV}{m}$ $\therefore r^2 \left(= \frac{m^2v^2}{B^2e^2}\right) = \frac{m^2}{B^2e^2} \times \frac{2eV}{m}$ giving $\frac{2mV}{B^2e}$	**3**	Starting from a known equation, show as many steps as possible.
(iii) (Re-arranging gives) $\frac{e}{m} = \frac{2V}{B^2r^2}$ $\frac{e}{m} = \frac{2 \times 530}{(3.1 \times 10^{-3})^2 \times (25 \times 10^{-3})^2}$ $= 1.7(6) \times 10^{11}$ C kg^{-1}	**2**	Be careful to change the numbers to base units. [You could work out e/m from the data sheet to see if it gives the same value.]

Answers to examination-style questions

Answers	Marks	Examiner's tips
3 (i) $t = \left(\frac{\text{distance}}{\text{speed}} = \frac{34}{0.95 \times 3.0 \times 10^8}\right)$ $= 1.1(9) \times 10^{-7}$ (s)	1	The measurements are in the rest frame of the detectors, so the time calculated is the dilated time, *t*.
(ii) • Use of $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ • where t_0=18 ns, and *t* is the half-life in the detectors' frame of reference. • $\therefore\ t = \frac{18 \times 10^{-9}}{\sqrt{1 - 0.95^2}}$ $= 57.(6) \times 10^{-9}$ s • Time taken for π meson to pass from one detector to the other = 2 half-lives (approx) (in the detectors' frame of reference). • 2 half-lives correspond to a reduction to 25%, so 75% of the π mesons passing the first detector do not reach the second detector. **Alternatives for first three marks:** 1. Use of $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ where t_0 = 18 ns $\therefore\ t = \frac{18 \times 10^{-9}}{\sqrt{1 - 0.95^2}}$ $= 57.(6) \times 10^{-9}$ s Journey time in detector frame (= 2*t*) = 2 × 57.6 ns (≈ 2 half-lives) 2. Use of $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ where *t* =119 ns $\therefore\ t_0 = 119\sqrt{(1 - 0.95^2)} = 37$ ns journey time in rest frame = 2 × 18 ns (which is 2 half-lives)	4	The simplest method is to calculate the half-life in the frame of the mesons, then show how many half-lives this. Each half-life is a further 50% reduction, so two half-lives is 50% of 50%, i.e. 25% remaining.
4 (a) • The beam deflects towards Y • because each electron is acted on by an electric force towards Y (or is attracted to Y or repelled by X).	2	
(b) (i) • Each electron is acted on by a magnetic force in the opposite direction to the electric force. • When $B = B_0$ the magnetic force is equal (and opposite) to the electric force.	2	

Answers to examination-style questions

Answers	Marks	Examiner's tips
(ii) • Magnetic force = Bev, electric force $\frac{eV}{d}$ • $B_0ev = \frac{eV}{d}$ (at $B = B_0$) • $\left(\therefore v = \frac{V}{B_0d}\right)$	2	
(c) • Work done on electron (or change of potential energy of electron) = eV_A (where V_A = 3800 V). • ∴(kinetic energy of electron), $\frac{1}{2}mv^2 = eV_A$ • (rearranging this equation gives) $\frac{e}{m}\left(=\frac{v^2}{2V_A}\right) = \frac{(3.7 \times 10^7)^2}{2 \times 3800}$ $= 1.8 \times 10^{11}$ C kg^{-1}	3	
5 (a) • Radio wave is an electromagnetic wave/ includes a magnetic (or electric) wave. • Magnetic flux (or field or wave) through the loop changes as the waves pass the loop. • Induced emf is due to changing magnetic flux through the loop. • Induced emf is alternating because flux (or field or wave) alternates. *Alternatively:* • Electric wave passes the loop. • Electrons in loop forced to oscillate by electric wave. • Movement of electrons causes an induced emf.	max 3	
(b) • Radio waves from T are polarised. • Magnetic flux through loop decreases as it is rotated (or component of magnetic flux density perpendicular to loop decreases). • At 90°, no magnetic flux passes through loop, so induced emf is zero.	3	

Answers to examination-style questions

Answers	Marks	Examiner's tips
6 (a) (i) To see if they could detect the ether (or absolute motion of the Earth through space or absolute rest). (ii) • Light reaches the observer from the light source via each mirror. • There is a phase difference between the two light beams. • Bright fringes are seen where the two light beams are in phase (or dark fringes are seen where the two light beams are out of phase by 180º).	**max 3 for (a)(i) and (a)(ii)**	
(b) (i) • Earth's motion through space was thought to affect the speed of light (along each arm of the apparatus). • The distance travelled by each beam of light did not change. • The difference in the time taken by light to travel along each arm would change. • The phase difference between the two lights beams would change. (ii) • Earth's motion through space does not affect the speed of light (or ether does not exist, or absolute motion does not exist, or all motion is relative, or absolute rest).	**max 3 for (b)(i) and (b)(ii)**	
7 (a) (i) • Drag (or viscous) force acts upwards on droplet. • Drag (or viscous) force increases with speed. • At this speed, drag (or viscous) force (+ upthrust) = weight of droplet (or force of gravity on it). • No resultant force so acceleration is zero (and therefore velocity (or speed) is constant).	**max 3**	Use your AS mechanics here. The key is that forces produce acceleration, so zero resultant force is no acceleration.
(ii) • Viscous force $= 6\pi\eta rv$ weight (or mg) $= \frac{4}{3}\pi r^3 g\rho$ $\therefore \frac{4}{3}\pi r^3 g\rho = 6\pi\eta rv$ • $r^2 \left(= \frac{9\eta v}{2\rho g}\right)$ $= \frac{9 \times 1.8 \times 10^{-5} \times 7.8 \times 10^{-5}}{2 \times 960 \times 9.81}$ $(= 6.7 \times 10^{-13}\ \text{m}^2)$ (which gives $r = 8.2 \times 10^{-7}$ m)	**2**	This is a very important derivation to learn. Balance up the weight with the viscous force from Stokes' law.

Answers to examination-style questions

Answers	Marks	Examiner's tips
(iii) Mass, $m \left(= \frac{4}{3}\pi r^3 \rho\right)$ $= \frac{4}{3}\pi \times (8.2 \times 10^{-7})^3 \times 960$ $(= 2.2 \times 10^{-15}\ \text{kg})$ *Alternatively:* $m\left(= \frac{6\pi\eta r v}{g}\right)$ $= \frac{6\pi \times 1.8 \times 10^{-5} \times 8.2 \times 10^{-7} \times 7.8 \times 10^{-5}}{9.81}$ $(= 2.2 \times 10^{-15}\ \text{kg})$	1	Even if you couldn't do the previous part, you can use the radius that has been given to work out the mass of the spherical drop.
(b) (i) • Electric force acts upwards and slows droplet. • Electric force depends on/varies with speed. • Pd adjusted until electric force = weight so droplet becomes stationary (or droplet becomes stationary when electric force = weight)	max 2	
(ii) (electric force = weight) $\frac{QV}{d} = mg$ $Q = \frac{mgd}{V}$ $= \frac{2.2 \times 10^{-15} \times 9.81 \times 6.0 \times 10^{-3}}{410}$ $(= 3.2 \times 10^{-19}\ \text{C})$	2	You need to know about electric fields to complete this part of the question.
(iii) • Droplet charge is always a whole number $\times 1.6 \times 10^{-19}$ C *or* • 1.6×10^{-19} C is the basic quantum of charge (or the charge of the electron).	1	

Answers to examination-style questions

Answers	Marks	Examiner's tips
8 (a) $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ gives $9.5 \times 10^{-28} = \frac{1.9 \times 10^{-28}}{\sqrt{1 - \frac{v^2}{c^2}}}$ $\therefore \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{9.5}{1.9} = 5.0$ $\frac{v}{c} = 0.98 \times 3.0 \times 10^8 = 2.94 \times 10^8$ m s^{-1} $v = 2.94 \times 10^8$ m s^{-1} **Alternative for (a)** $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ gives $\frac{v}{c} = \sqrt{1 - \frac{{m_0}^2}{m^2}}$ Correct substitution of m, m_0 and c gives $v = 2.94 \times 10^8$ m s^{-1}	**4**	Always work in terms of $\frac{v}{c}$ until the final part of the question. Putting in a value of c too early makes it much more difficult. Remember you are expecting a value close to the speed of light.
(b) E_K $(= (m - m_0)c^2)$ $= (9.5 \times 10^{-28} - 1.9 \times 10^{-28}) \times (3 \times 10^8)^2$ $= 6.8(4) \times 10^{-11}$ J	**2**	Don't use $E_k = \frac{1}{2}mv^2$!

Nelson Thornes is responsible for the solution(s) given and they may not constitute the only possible solution(s).