Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Mock Paper Set 2

(Time: 1 hour 30 minutes) | Paper Reference **9FM0/01**

Further Mathematics

Advanced

Paper 1: Core Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1.

$$f(z) = z^4 + 6z^3 - 3z^2 - 96z - 208$$

(a) Use your calculator to solve the equation $f(z) = 0$

(2)

(b) Hence, or otherwise, show that $f(z)$ can be written as

$$(z^2 - a)(z^2 + bz + c)$$

where a, b and c are real constants to be determined.

(3)

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Question 1 continued

(Total for Question 1 is 5 marks)

2.

$$zz^* + 3\mathrm{i}z = p + 9\mathrm{i}$$

where z is a complex number and p is a real constant.

Given that this equation has exactly one root, determine the complex number z.

(5)

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Question 2 continued

(Total for Question 2 is 5 marks)

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3.

$$\mathbf{A} = \begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix} \qquad \text{where } k \text{ is a constant}$$

(a) Show that the matrix **A** is non-singular for all values of k.

(2)

A transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ is represented by the matrix **A**.

The point P has position vector $\begin{pmatrix} a \\ 2a \end{pmatrix}$ relative to an origin O.

The point Q has position vector $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$ relative to O.

Given that the point P is mapped onto the point Q under T,

(b) determine the value of a and the value of k.

(3)

Given that, for a different value of k, T maps the line $y = 2x$ onto itself,

(c) determine this value of k.

(3)

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Question 3 continued

Question 3 continued

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Question 3 continued

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4. Given that

$$y = \arcsin x \qquad -1 \leqslant x \leqslant 1$$

(a) show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{1 - x^2}}$$

(3)

Given that

$$\mathrm{f}(x) = \frac{3x + 2}{\sqrt{4 - x^2}}$$

(b) show that the mean value of $\mathrm{f}(x)$ over the interval $\left[0, \sqrt{2}\right]$ is

$$\frac{\pi\sqrt{2}}{4} + A\sqrt{2} - A$$

where A is a constant to be determined.

(6)

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

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5. A population of rabbits on an island is studied over a period of time.
The number of rabbits is modelled by the differential equation

$$\frac{d^2R}{dt^2} + 3\frac{dR}{dt} + 2R = 4t \qquad t \geqslant 0$$

where t is the time, in years, from the start of the study and R is in hundreds of rabbits.

At the start of the study, there are 2 000 rabbits on the island and the number of rabbits is increasing at a rate of 500 rabbits per year.

(a) Determine, according to the model, the number of rabbits that there will be on the island, 10 years after the start of the study.

(9)

(b) Give a limitation of the model.

(1)

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Question 5 continued

Question 5 continued

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Question 5 continued

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6 (i) Prove by induction that for $n \in \mathbb{Z}^+$

$$2 \times 4 + 4 \times 5 + 6 \times 6 + \ldots + 2n(n+3) = \frac{2}{3}n(n+1)(n+5)$$

(6)

(ii) Given that

$$\mathbf{M} = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$$

(a) Prove by induction that for $n \in \mathbb{Z}^+$

$$\mathbf{M}^n = \begin{pmatrix} 1+4n & -8n \\ 2n & 1-4n \end{pmatrix}$$

(6)

(b) Show that $\det(\mathbf{M}^n)$ is independent of n.

(2)

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Question 6 continued

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Question 6 continued

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7. The curve C has Cartesian equation

$$(x^2 + y^2)^2 = 6xy \qquad x > 0, y > 0$$

(a) Show that for $0 < \theta < \frac{\pi}{2}$ the equation for C can be written as the polar equation

$$r^2 = 3\sin 2\theta$$

(3)

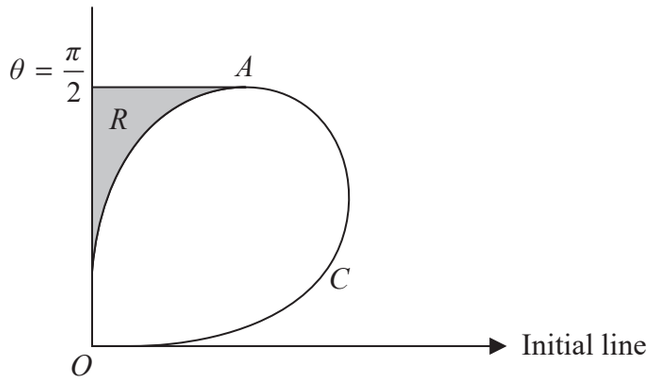

Figure 1

Figure 1 shows a sketch of the curve C. The tangent to C at the point A is parallel to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by C, the tangent to the curve at the point A and the line with equation $\theta = \frac{\pi}{2}$

(b) Use calculus to determine the area of the shaded region R.

(9)

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Question 7 continued

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Question 7 continued

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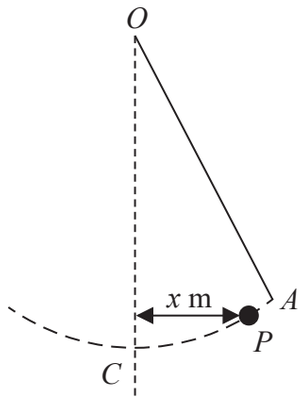

Figure 2

A child plays on a rope swing.
One end of the rope is attached to a tree and the child sits on a large knot at the other end of the rope.
The child swings back and forth in a vertical plane.

The rope is modelled as a light and inextensible string. The child is modelled as a particle.

Figure 2 represents the child and the rope swing. The rope is attached to the tree at the point O and the point C is vertically below O. The point P represents the child.

The horizontal displacement of P from the line OC at time t seconds ($t \geqslant 0$) is x metres, as shown in Figure 2.

The motion of P is modelled by the differential equation

$$\ddot{x} + 2\dot{x} + \lambda x = 0$$

where λ is a positive constant.

The child is initially at rest, at the point A, with a horizontal displacement of 1.5 m from the line OC.

Given that the initial horizontal acceleration of the child is $-7.5\,\text{m s}^{-2}$

(a) show that $\lambda = 5$

(2)

Using the model,

(b) find an expression for the horizontal displacement of the child at time t.

(7)

Given that, when $t = 4.5$, the child is vertically below O,

(c) evaluate the model explaining your reasoning.

(2)

(d) Suggest one refinement for the model.

(1)

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TOTAL FOR PAPER IS 75 MARKS

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