Pearson Edexcel

Mark Scheme

Mock Paper Set 2

Pearson Edexcel GCE Further Mathematics
Further Core 1 Paper 9FM0_01

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Mock paper
Publications Code 9FM0_01_Mock paper_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Core Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1(a)	$z = \pm 4$	B1	1.1b
	$z = -3 \pm 2\text{i}$	B1	1.1b
		(2)	
(b)	$a = 16$	B1	1.1a
	A complete method to find *b* and *c*	M1	3.1a
	$b = 6$ and $c = 13$	A1	1.1b
		(3)	
			(5 marks)

Notes:

(a)
B1: Correct values only
B1: Correct values only

(b)
B1: Correct value
M1: Uses the complex roots or multiplies out and compares coefficients
A1: Correct values only

Question	Scheme	Marks	AOs
2	Using $z = x + y\text{i}$ and $z^* = x - y\text{i}$ $(x + y\text{i})(x - y\text{i}) + 3\text{i}(x + y\text{i}) = p + 9\text{i}$ $\Rightarrow\ x^2 + y^2 + 3x\text{i} - 3y = p + 9\text{i}$	M1	3.1a
	$x = 3$	B1	1.1b
	Equate real parts $3^2 + y^2 - 3y = p$	M1	1.1b
	Complete method to find the value of y	M1	3.1a
	$z = 3 + \frac{3}{2}\text{i}$	A1	2.2a
		(5)	
			(5 marks)

Notes:

M1: Substitutes $z = x + y\text{i}$ and $z^* = x - y\text{i}$ into the equation, or vice versa.
B1: Equate imaginary parts $3x = 9 \Rightarrow x = 3$
M1: Equating real parts to get a 3TQ in y. May be unsimplified
M1: Any valid method to obtain a value for y e.g. $-\frac{b}{2a}$ **or** complete the square **or** discriminant = 0
$\left(\text{You may see } p = \frac{27}{4}\right)$
A1: Correct complex number

Question	Scheme	Marks	AOs
3 (a)	$\det \mathbf{A} = k^2 + 2(1-k) = (k-1)^2 + 1$ or uses quadratic formula/discriminant = $(-2)^2 - 4(1)(2)$	M1	2.1
	$(k-1)^2 + 1 \geq 1$ or discriminant = $-4 < 0$ therefore **A** is non-singular for all values of *k*.	A1	2.4
		(2)	
(b)	$\begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix}\begin{pmatrix} a \\ 2a \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \end{pmatrix} \Rightarrow$ at least one equation $ka - 4a = 7$ $(1-k)a + 2ak = -3$	M1	3.1a
	Solves simultaneously to find a value for either *a* or *k* e.g $a + 2ak = -3 \Rightarrow ak = -3 - a$ $\Rightarrow -3 - a - 4a = 7 \Rightarrow a = \ldots$	M1	1.1b
	$a = -2, k = \frac{1}{2}$	A1	1.1b
		(3)	
(c)	$\begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix}\begin{pmatrix} x \\ 2x \end{pmatrix} = \begin{pmatrix} X \\ 2X \end{pmatrix} \Rightarrow$ at least one equation $kx - 4x = X$ $(1-k)x + 2kx = 2X$	M1	3.1a
	$2(kx - 4x) = (1-k)x + 2xk \Rightarrow k = \ldots$	M1	1.1b
	$k = 9$	A1	1.1b
		(3)	
			(8 marks)

Notes:

(a)

M1: Finds the determinant and chooses an appropriate method to show that the resulting quadratic has no real roots.

A1: Complete process to show discriminant > 0 and draws the conclusion that **A** is **non-singular** for **all values of *k*.**

(b)

M1: Translates the problem into a matrix multiplication to obtain at least one equation.

M1: Solves simultaneously to find a value for either *a* or *k*

A1: Correct values for both *a* and *k*

(c)

M1: Translates the problem into a matrix multiplication to obtain at least one equation.

M1: Solves simultaneously to find a value of *k*

A1: Correct value for *k*

Question	Scheme		Marks	AOs
4(a)	$\sin y = x \implies \cos y \dfrac{dy}{dx} = 1$	$\sin y = x \implies \dfrac{dx}{dy} = \cos y$	M1	1.1b
	$\sin^2 y + \cos^2 y = 1 \Rightarrow \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$		M1	2.1
	$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1 - x^2}}$ * cso		A1*	1.1b
			(3)	
(b)	$f(x) = \frac{3x+2}{\sqrt{4-x^2}} = \frac{3x}{\sqrt{4-x^2}} + \frac{2}{\sqrt{4-x^2}}$		M1	3.1a
	$\displaystyle\int \frac{3x}{\sqrt{4-x^2}}\,dx = \beta\sqrt{4-x^2}$		M1	1.1b
	$\displaystyle\int \frac{2}{\sqrt{4-x^2}}\,dx = \alpha \arcsin\left(\frac{x}{2}\right)$		M1	1.1b
	$\displaystyle\int \frac{3x+2}{\sqrt{4-x^2}} = -3\sqrt{4-x^2} + 2\arcsin\left(\frac{x}{2}\right) \{+c\}$		A1	1.1b
	Mean $= \frac{1}{\sqrt{2}-0}\left[-3\sqrt{4-x^2} + 2\arcsin\left(\frac{x}{2}\right)\right]_0^{\sqrt{2}}$ $= \frac{\sqrt{2}}{2}\left[\left(-3\sqrt{4-(\sqrt{2})^2} + 2\arcsin\left(\frac{\sqrt{2}}{2}\right)\right) - \left(-3\sqrt{4-(0)^2} + 2\arcsin\left(\frac{0}{2}\right)\right)\right]$		M1	2.1
	$= \frac{\pi\sqrt{2}}{4} + 3\sqrt{2} - 3$		A1	2.2a
			(6)	
			(9 marks)	

Notes:

(a)

M1: Finds x in terms of y and differentiates

M1: Uses the trig identity $\sin^2 y + \cos^2 y = 1$ to express $\cos y$ in terms of x

A1*: Correctly achieves the required answer. cso

(b)

M1: Splitting the fraction into two separate expressions

M1: Integrates the first fraction into the required form

M1: Integrates the second fraction into the required form

A1: Correct integration of f(x)

M1: Applies the correct method to find the mean value over the required interval

A1: Correct answer

Question	Scheme	Marks	AOs
5(a)	$m^2 + 3m + 2 = 0 \Rightarrow m = -2, -1$	M1	1.1b
	$R = A\mathrm{e}^{-t} + B\mathrm{e}^{-2t}$	A1	1.1b
	$R = \lambda t + \mu \Rightarrow \frac{\mathrm{d}R}{\mathrm{d}t} = \lambda \Rightarrow \frac{\mathrm{d}^2R}{\mathrm{d}t^2} = 0$ $\Rightarrow 0 + 3\lambda + 2(\lambda t + \mu) = 4t \Rightarrow \lambda = 2, \mu = -3$	M1	3.1b
	$R = PI + CF = A\mathrm{e}^{-t} + B\mathrm{e}^{-2t} + 2t - 3$	M1 A1	1.1b 1.1b
	$t = 0, R = 20 \Rightarrow 20 = A + B - 3$	M1	3.4
	$\frac{\mathrm{d}R}{\mathrm{d}t} = -A\mathrm{e}^{-t} - 2B\mathrm{e}^{-2t} + 2, t = 0, \frac{\mathrm{d}R}{\mathrm{d}t} = 5 \Rightarrow -A - 2B + 2 = 5$	M1	3.4
	$R = 49\mathrm{e}^{-t} - 26\mathrm{e}^{-2t} + 2t - 3$	A1	1.1b
	$t = 10 \Rightarrow R = 17 \Rightarrow 1700$ rabbits	A1	3.2a
		(9)	
(b)	The population will keep increasing ($R \rightarrow 2t - 3$) which is unrealistic.	B1	3.5b
		(1)	
			(10 marks)

Notes:

(a)

M1: Forms and solves the auxiliary equation

A1: Correct complementary function

M1: A complete method to find the particular integral. Look for the correct from of the PI $= \lambda t + \mu$, differentiates twice and substituted into the differential equation to find the values of λ *and* μ

M1: Adding CF and PI

A1: Correct general solution for R

M1: Uses the information from the model $t = 0, R = 20$ to find an equation for the constants.

M1: Uses the information from the model $t = 0, \frac{\mathrm{d}R}{\mathrm{d}t} = 5$ to find another equation for the constants.

A1: Correct particular solution

A1: Correct answer only

(b)

B1: An appropriate limitation

Question	Scheme	Marks	AOs
6(i)	$n=1$, LHS $= 2\times 4 = 8$, RHS $= \frac{2}{3}(1)(1+1)(1+5) = 8$ So the result is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ $$\sum_{r=1}^{k} 2r(r+3) = \frac{2}{3}k(k+1)(k+5)$$	M1	2.4
	$$\sum_{r=1}^{k+1} 2r(r+3) = \frac{2}{3}k(k+1)(k+5) + 2(k+1)(k+1+3)$$	M1	1.1b
	$= \frac{2}{3}(k+1)[k(k+5)+3(k+4)]$ or $\frac{2}{3}(k+1)[k^2+8k+12]$	A1	1.1b
	$$= \frac{2}{3}(k+1)(k+2)(k+6)$$	A1	2.1
	If **true for $\boldsymbol{n=k}$** then **true for $\boldsymbol{n=k+1}$** and as it is **true for $\boldsymbol{n=1}$** the statement is **true for all** (positive integers) $\boldsymbol{n}$	A1	2.4
		(6)	
(ii)(a)	$n = 1, \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^1 = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}; \begin{pmatrix}1+4(1) & -8(1)\\ 2(1) & 1-4(1)\end{pmatrix} = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$ So the result is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ $\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^k = \begin{pmatrix}1+4k & -8k\\ 2k & 1-4k\end{pmatrix}$	M1	2.4
	$$\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^{k+1} = \begin{pmatrix}1+4k & -8k\\ 2k & 1-4k\end{pmatrix}\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$$ or $$\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^{k+1} = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}\begin{pmatrix}1+4k & -8k\\ 2k & 1-4k\end{pmatrix}$$	M1	1.1b
	$\begin{pmatrix}1+4k & -8k\\ 2k & 1-4k\end{pmatrix}\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$ $= \begin{pmatrix}5(1+4k)-16k & -8(1+4k)-3(-8k)\\ 5(2k)+2(1-4k) & -8(2k)-3(1-4k)\end{pmatrix}$ or $\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}\begin{pmatrix}1+4k & -8k\\ 2k & 1-4k\end{pmatrix}$ $= \begin{pmatrix}5(1+4k)-8(2k) & 5(-8k)-8(1-4k)\\ 2(1+4k)-3(2k) & 2(-8k)-3(1-4k)\end{pmatrix}$	A1	1.1b
	$$\begin{pmatrix}1+4(k+1) & -8(k+1)\\ 2(k+1) & 1-4(k+1)\end{pmatrix}$$	A1	2.1
	If **true for $\boldsymbol{n=k}$** then **true for $\boldsymbol{n=k+1}$** and as it is **true for $\boldsymbol{n=1}$** the statement is **true for all** (positive integers) $\boldsymbol{n}$	A1	2.4
		(6)	

(ii)(b)	**Either** $\det(\mathbf{M}^n) = (1+4n)(1-4n) - (-8n)(2n) = 1$ **or** $\det(\mathbf{M}) = -15 + 16 = 1$	M1	2.1
	Either $\det(\mathbf{M}^n) = 1$ **or** $\det(\mathbf{M}^n) = (\det\mathbf{M})^n = 1$ Therefore $\det(\mathbf{M}^n)$ is **independent of *n***	A1	2.4
		(2)	
			(14 marks)

Notes:

(i)

B1: Shows that the result holds for $n = 1$. Must see substitution in the RHS and LHS and reach 8

M1: Makes a statement that assumes the result is true for some value of *n*

M1: Set up sum with assumed formula $2(k+1)(k+1+3)$

A1: Achieves a correct expression with a factor of $\frac{2}{3}(k+1)$

A1: Reaches a correct simplified expression with **no errors and the correct unsimplified expression with a factor of $\frac{2}{3}(k+1)$ seen previously.**

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution

(ii) (a)

B1: Shows that the result holds for $n = 1$. Must see substitution in the RHS and reach $\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$

M1: Makes a statement that assumes the result is true for some value of *n*

M1: Set up a matrix multiplication of the assumed result multiplied by the original matrix, either way round

A1: Achieves a correct unsimplified matrix

A1: Reaches a correct simplified matrix with **no errors and the correct unsimplified matrix seen previously.**

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution.

(ii) (b)

M1: Finds the determinant of matrix $\mathbf{M}^n$ or matrix **M**

A1: Shows that $\det(\mathbf{M}^n) = 1$, therefore independent of *n*

Question	Scheme	Marks	AOs
7(a)	Replaces $x^2+y^2 \rightarrow r^2$ **and** $x \rightarrow r\cos\theta$ **and** $y \rightarrow r\sin\theta$	M1	2.1
	$r^4 = 6r^2\cos\theta\sin\theta$ or $r^2 = 6\cos\theta\sin\theta$	A1	1.1b
	$r^2 = 3\sin 2\theta$ * cso	A1*	1.1b
		(3)	
(b)	**Alternative 1 for the first three marks** $y^2 = r^2\sin^2\theta = 3\sin 2\theta\sin^2\theta$ $2y\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \alpha\cos 2\theta\sin^2\theta + \beta\sin 2\theta\sin\theta\cos\theta$	M1	3.1a
	$2y\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = 6\cos 2\theta\sin^2\theta + 6\sin 2\theta\sin\theta\cos\theta$	A1	1.1b
	$6\cos 2\theta\sin^2\theta + 6\sin 2\theta\sin\theta\cos\theta = 0$ $6\cos 2\theta\sin^2\theta + 12\sin^2\theta\cos^2\theta = 0$ $6\sin^2\theta(\cos 2\theta + 2\cos^2\theta) = 0$ $6\sin^2\theta(4\cos^2\theta - 1) = 0$ $\sin^2\theta = 0 \implies \theta = \ldots$ or $\cos^2\theta = \frac{1}{4} \implies \theta = \ldots$	M1	3.1a
	Alternative 2 for the first three marks $y = r\sin\theta = \sqrt{3}\sin\theta(\sin 2\theta)^{\frac{1}{2}}$ $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \alpha\cos\theta(\sin 2\theta)^{\frac{1}{2}} + \beta(\sin 2\theta)^{-\frac{1}{2}}\cos 2\theta\cos\theta$	M1	3.1a
	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \sqrt{3}\cos\theta(\sin 2\theta)^{\frac{1}{2}} + \sqrt{3}(\sin 2\theta)^{-\frac{1}{2}}\cos 2\theta\cos\theta$	A1	1.1b
	$\sqrt{3}\cos\theta(\sin 2\theta)^{\frac{1}{2}} + \sqrt{3}(\sin 2\theta)^{-\frac{1}{2}}\cos 2\theta\cos\theta = 0$ $\sqrt{3}\cos\theta\sin 2\theta + \sqrt{3}\cos 2\theta\cos\theta = 0$ $\sqrt{3}\cos\theta \times 2\sin\theta\cos\theta + \sqrt{3}(2\cos^2\theta - 1)\cos\theta = 0$ $\sqrt{3}\sin\theta(4\cos^2\theta - 1) = 0$ $\sin\theta = 0 \implies \theta = \ldots$ or $\cos^2\theta = \frac{1}{4} \implies \theta = \ldots$	M1	3.1a
	Alternative 3 for the first three marks $(x^2+y^2)^2 = 6xy \Rightarrow x^4 + 2x^2y^2 + y^4 = 6xy$ $4x^3 + \alpha xy^2 + \beta x^2y\dfrac{\mathrm{d}y}{\mathrm{d}x} + 4y^3\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6y + 6x\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1	3.1a
	$4x^3 + 4xy^2 + 4x^2y\dfrac{\mathrm{d}y}{\mathrm{d}x} + 4y^3\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6y + 6x\dfrac{\mathrm{d}y}{\mathrm{d}x}$	A1	1.1b
	$4x^3 + 4xy^2 + 4x^2y(0) + 4y^3(0) = 6y + 6x(0)$ $4x^3 + 4xy^2 = 6y$ $4r^3\cos^3\theta + 4r\cos\theta \times r^2\sin^2\theta = 6r\sin\theta$ $4r^3\cos\theta[\cos^2\theta + \sin^2\theta] = 6r\sin\theta$ $4(3\sin 2\theta)\cos\theta - 6\sin\theta = 0$ $24\sin\theta\cos^2\theta - 6\sin\theta = 0$ $6\sin\theta\ [4\cos^2\theta - 1]$ $\sin\theta = 0 \implies \theta = \ldots$ or $\cos^2\theta = \frac{1}{4} \implies \theta = \ldots$	M1	3.1a

	For the final 6 marks		
	Deduces that $\theta = \frac{\pi}{3}$ is required	A1	2.2a
	Area bounded by the curve = $\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\sin 2\theta \; \mathrm{d}\theta = [\delta \cos 2\theta]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \; ...$ Note: $\delta = -\frac{3}{4}$	M1	1.1b
	$\left\{-\frac{3}{4}\cos\pi - \left(-\frac{3}{4}\cos\frac{2\pi}{3}\right)\right\} = \frac{3}{8}$	A1	1.1b
	Area of triangle = $\frac{1}{2}xy = \frac{1}{2}r^2 \sin\theta\cos\theta$ $= \frac{1}{2} \times 3\sin\frac{2\pi}{3}\sin\frac{\pi}{3}\cos\frac{\pi}{3} = \ldots$	M1	1.1b
	Finds the required area = area of triangle – area bounded by the curve $\left\{= \frac{9}{16} - \frac{3}{8}\right\}$	ddM1	3.1a
	$= \frac{3}{16}$	A1	1.1b
		(9)	
			(12 marks)

Notes:

(a)
M1: Replaces all Cartesian variables with polar variables
A1: A correct unsimplified polar equation
A1*: Correct polar equation achieved from correct work.

(b) Alternatives 1 and 2
M1: Writes y^2 or y in terms of trig and differentiates (implicitly) to the correct form
A1: Fully correct differentiation
M1: Uses correct trig work to solve $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$ to achieve a value for θ

(b) Alternative 3
M1: Differentiates implicitly the Cartesian equation to the correct form
A1: Fully correct differentiation
M1: Uses correct polar equation and correct trig work to solve $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ to achieve a value for θ

(b) For the last 6 marks
A1: Deduces that $\theta = \frac{\pi}{3}$
M1: Applies area $= \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} r^2 \, \mathrm{d}\theta$, integrates to a correct form and applies their lower limit and upper limits of $\frac{\pi}{2}$ correctly
A1: Correct area under the curve
M1: Correct method to find the area of triangle = $\frac{1}{2}xy$
ddM1: Dependent of previous two method marks. Fully correct method to find the required area. Finds the required area = area of triangle – area bounded by the curve
A1: Correct answer

Question	Scheme	Marks	AOs
8(a)	$-7.5 + 2(0) + \lambda(1.5) = 0 \Rightarrow \lambda = \cdots$	M1	3.3
	$\lambda = 5$ *	A1*	1.1b
		(2)	
(b)	Solves $m^2 + 2m + 5 = 0 \Rightarrow m = ..$	M1	3.1b
	$m = -1 \pm 2\mathrm{i}$	A1ft	1.1b
	$x = \mathrm{e}^{-t}(A\cos 2t + B\sin 2t)$	A1	1.1b
	Using the initial conditions, $x = 1.5, t = 0$ to find a constant $1.5 = \mathrm{e}^0(A\cos 0 + B\sin 0) \Rightarrow A = ...\{1.5\}$	M1	3.4
	$\frac{dx}{dt} = -\mathrm{e}^{-t}(A\cos 2t + B\sin 2t) + \mathrm{e}^{-t}(-2A\sin 2t + 2B\cos 2t)$	M1	1.1b
	Using the initial conditions, $v = 0, t = 0$ to find the second constant $0 = -\mathrm{e}^0(1.5\cos 0 + B\sin 0) + \mathrm{e}^0(-3\sin 0 + 2B\cos 0)$ $\Rightarrow B = ...\{0.75\}$	dM1	3.4
	$x = e^{-t}(1.5\cos 2t + 0.75\sin 2t)$	A1	1.1b
		(7)	
(c)	Substitutes $t = 4.5$ into their equation for x ($x = -0.0117\ldots$)	M1	3.4
	Compares their value of x with 0 and evaluates the model	A1ft	3.5a
		(2)	
(d)	e. g. Take into account air resistance	B1	3.5c
		(1)	
			(12 marks)

Notes:

(a)
M1: Substitutes $\ddot{x} = -7.5$, $\dot{x} = 0$ and $x = 1.5$ into the differential equation to find a value for λ
A1*: Correct solution only

(b)
M1: Forms and solves the auxiliary equation
A1ft: Correct solution to their auxiliary equation. Follow through on their value of λ only.
A1: Correct complementary function
M1: Uses the information from the model $x = 1.5, t = 0$ to find a constant
M1: Differentiates to find an expression for the velocity
dM1: Uses the information from the model, $v = 0, t = 0$ to find another equation for the constants.
A1: Correct equation for displacement

(c)
M1: Substitutes to find a value of x

A1ft: Any suitable comment consistent with their value e.g. a good model since only 1cm out
(d) **B1:** A suitable refinement of the model e.g. air resistance