Pearson Edexcel

Mark Scheme

Mock Paper Set 2

Pearson Edexcel GCE Further Mathematics
Further Core 2 Paper 9FM0_02

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Mock paper
Publications Code 9FM0_02_Mock paper_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Core Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1	Any two of $\begin{array}{lll} \mathbf{i}: & 10-\lambda = 17+5\mu & (1) \\ \mathbf{j}: & \lambda = 1-\mu & (2) \\ \mathbf{k}: & -9+2\lambda = 3+3\mu & (3) \end{array}$	M1	1.1b
	Solve any two simultaneous equations e.g. (3) – 2(2) gives: $-9 = 1+5\mu \Rightarrow \mu = -2$ e.g. (2) + (1) gives: $10 = 18+4\mu \Rightarrow \mu = -2$	dM1	1.1b
	$\lambda = 3, \mu = -2$	A1	1.1b
	Checks the unused equation e.g. $\lambda = 3$: LHS $= 10-\lambda = 10-3 = 7$ $\mu = -2$: RHS $= 17+5\mu = 17-10 = 7$ therefore the lines intersect or substitutes the values of λ and μ into the relevant equation and draws the conclusion that the lines intersect	B1	2.1
	$\mathbf{r} = \begin{pmatrix} 10 \\ 0 \\ -9 \end{pmatrix} + 3\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 17 \\ 1 \\ 3 \end{pmatrix} - 2\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$	M1	1.1b
	Intersect at $\mathbf{r} = \begin{pmatrix} 7 \\ 3 \\ -3 \end{pmatrix}$ or $\mathbf{r} = 7\mathbf{i}+3\mathbf{j}-3\mathbf{k}$	A1	2.2a
		(6)	

(6 marks)

Notes:

M1: Writes down any two correct equations

dM1: Solve two equations simultaneously to find a value for μ or λ

A1: Correct values for μ and λ

B1: Shows that the values of μ and λ give the same coordinates or point of intersection and draws the conclusion that the **lines intersect**

M1: Substitutes μ and λ into a relevant equation

A1: Deduces the point of intersection.

Question	Scheme	Marks	AOs
2(a)	$f'(x) = \alpha(1+x)^{-1}$ $\quad f''(x) = \beta(1+x)^{-2}$ $f'''(x) = \gamma(1+x)^{-3}$ $\quad f^{iv}(x) = \delta(1+x)^{-4}$	M1	2.1
	$f'(x) = (1+x)^{-1}$ $\quad f''(x) = -1(1+x)^{-2}$ $f'''(x) = 2(1+x)^{-3}$ $\quad f^{iv}(x) = -6(1+x)^{-4}$	A1	1.1b
	$f'(0) = (1+0)^{-1} = 1$ $\quad f''(0) = -1(1+0)^{-2} = -1$ $f'''(0) = 2(1+0)^{-3} = 2$ $\quad f^{iv}(0) = -6(1+0)^{-4} = -6$	M1	1.1b
	$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2} + \frac{f'''(0)x^3}{6} + \frac{f^{iv}(0)x^4}{24}$	M1	2.5
	$f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$ *	A1*	1.1b
		(5)	
(b)	$g(x) = \ln\left(\frac{1+2x}{(1-2x)^2}\right) = \ln(1+2x) - 2\ln(1-2x)$	B1	3.1a
	$\ln(1-2x) = (-2x) - \frac{(-2x)^2}{2} + \frac{(-2x)^3}{3} - \frac{(-2x)^4}{4}$ $\ln(1+2x) = (2x) - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \frac{(2x)^4}{4}$	M1	2.2a
	$g(x) = \left[2x - 2x^2 + \frac{8}{3}x^3 - 4x^4\right] - 2\left[-2x - 2x^2 - \frac{8}{3}x^3 - 4x^4\right]$ $= 6x + 2x^2 + 8x^3 + 4x^4$	A1	1.1b
		(3)	
		(8 marks)	

Notes:

(a)

M1: Differentiates four times to achieve the required form

A1: All four derivatives correct

M1: Evaluates $f'(0)$, $f''(0)$, $f'''(0)$ and $f^{iv}(0)$

M1: Correct use of Maclaurin series up to the term in x^4

A1*: Correct solution, with no errors seen

(b)

B1: Writes $g(x)$ as linear ln terms

M1: Deduces the series expansions for $\ln(1+2x)$ and $\ln(1-2x)$. Do NOT allow a restart, they must use the expansion in part (a).

A1: Correct series expansion.

Question	Scheme	Marks	AOs
3(a) (i)	$\lvert z-2\mathrm{i}\rvert = 2$	B1	1.1b
(ii)	$\arg(z+2) = \frac{\pi}{4}$	B1	1.1b
		(2)	
(b)	$\{z \in \mathbb{C}: \lvert z-2\mathrm{i}\rvert = 2\} \cap \left\{z \in \mathbb{C}: \arg(z+2) = \frac{\pi}{4}\right\}$	B1ft	2.5
		(1)	
(c)	Solves $x^2+(y-2)^2=4$ and $y=x+2$ to reach $x=\ldots$ or $y=\ldots$ Alternatively uses Pythagoras to find the length of triangle $\sqrt{2}$ and uses to reach $x=\ldots$ or $y=\ldots$	M1	3.1a
	Finds a complete coordinate or complex number	dM1	1.1b
	$z = \sqrt{2}+\left(2+\sqrt{2}\right)\mathrm{i}$ and $z = -\sqrt{2}+\left(2-\sqrt{2}\right)\mathrm{i}$	A1	1.1b
		(3)	
			(6 marks)

Notes:

(a) (i)
B1: Correct circle locus seen
(a) (ii)
B1: Correct half-line locus seen

(b)
B1ft: Follow through their equations with set notation

(c)
M1: Identifies a suitable strategy for finding an *x* or *y* coordinate of a point of intersection. An attempt at solving $x^2+(y\pm 2)^2=4$ or 2 and $y=\pm x\pm 2$ or uses Pythagoras to find the length of triangle $\sqrt{2}$ and uses to reach $x=\ldots$ or $y=\ldots$

dM1: Finds a complete coordinate, by substitution into $y=\pm x\pm 2$ or if uses $x^2+(y\pm 2)^2=4$ must reject the incorrect coordinate.

A1: Correct complex numbers.

Question	Scheme	Marks	AOs
4(a)	$I = e^{\int 0.4\,dt} = e^{0.4t} \implies Ve^{0.4t} = \int \cos\left(\frac{t}{2}\right)\,dt$	M1	3.1b
	$Ve^{0.4t} = 2\sin\left(\frac{t}{2}\right) + c$	A1	1.1b
	$V = 10, t = 0 \Rightarrow c = 10$	M1	3.4
	$t = 8 \implies V = e^{-0.4\times 8}\left[2\sin\left(\frac{1}{2}\times 8\right) + 10\right]$	M1	1.1b
	Value = £345.92	A1	2.2b
		(5)	
(b)	e.g. Does not take into account any accidents, wear and tear etc e.g. Does not take into account mileage e.g. The value of the car tends to 0 which may be unrealistic	B1	3.5b
		(1)	
			(6 marks)

Notes:

(a)
M1: A complete method to find the integrating factor and then solve the differential equation
A1: Correct solution in any form
M1: Uses the model and the initial condition to find the constant of integration
M1: Substitutes $t = 8$ and finds a value for V
A1: Correct value of the car in £'s (allow awrt £345 or £346)

(b)
B1: Suggests a valid limitation of the model

Question	Scheme	Marks	AOs
5(a)	$\sum_{r=1}^{n} r(r+1) = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r = \frac{n}{6}(2n+1)(n+1) + \frac{n}{2}(n+1)$	M1 A1	2.1 1.1b
	$= \frac{n}{6}(n+1)[(2n+1)+3]$ or $\frac{n}{3}(n+1)\left[\frac{2n+1}{2}+\frac{3}{2}\right]$	dM1	1.1b
	$= \frac{n}{3}(n+1)(n+2)$	A1	1.1b
		(4)	
(b)	$\log 3^2 + 2\log 3^3 + 3\log 3^4 + \cdots + 11\log 3^{12}$	M1	3.1a
	$(1\times 2)\log 3 + (2\times 3)\log 3 + (3\times 4)\log 3 + \cdots + (11\times 12)\log 3$ $= \log 3\,[(1\times 2)+(2\times 3)+(3\times 4)+\cdots+(11\times 12)]$	M1	3.1a
	$= \log 3 \times \sum_{r=1}^{11} r(r+1) \; = \frac{11}{3}(11+1)(11+2)\log 3$	M1	1.1b
	$= 572\log 3$ o.e.	A1	1.1b
		(4)	
		(8 marks)	

Notes:

(a)

M1: Multiplies out the brackets and uses at least one correct standard series formula

A1: Fully correct expression

dM1: Attempts to factorise out either $\frac{n}{6}$ or $\frac{n}{3}$ having used at least one standard series formula. Dependent on previous method mark and having n in each term.

A1: Correct answer

(b)

M1: Realising the powers of 3 in the log terms.

M1: Uses the log power law and factorises out $\log 3$ to achieve a series of the form $\sum \; r(r+1)$

M1: Uses $\log 3 \times$ their answer to part (a) with $n = 11$

A1: Correct answer

Question	Scheme	Marks	AOs
6(a)	$\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 12 \\ 15 \end{pmatrix} = 16 + 36 - 60$	M1	1.1b
	$\cos\theta = \dfrac{-8}{\sqrt{2^2+3^2+(-4)^2}\sqrt{8^2+12^2+15^2}}$	M1	3.1b
	Acute angle between the sides of the tent is 86°	A1	3.2a
		(3)	
(b)	$2(6) + 3(7) - 4(8) = 1$ and $8(6) + 12(7) + 15(8) = 252$	M1	3.4
	Point *P* lies on both planes therefore lies on the straight line	A1	2.4
		(2)	
(c)	Attempts the scalar product between the direction of the rope and the normal to side *ABCD* of the tent and uses trigonometry to find an angle	M1	3.1b
	$\left(\begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix} - \begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix}\right) \cdot \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = 18$ or $\left(\begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix}\right) \cdot \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = -18$	M1 A1	1.1b 1.1b
	$\cos\alpha = \dfrac{18}{\sqrt{2^2+3^2+(-4)^2}\sqrt{10^2+10^2+8^2}}$ $\therefore\ \theta = 90 - \arccos\left(\dfrac{18}{\sqrt{29}\sqrt{264}}\right)$ or $\theta = \arcsin\left(\dfrac{18}{\sqrt{29}\sqrt{264}}\right)$	M1	1.1b
	Acute angle between tent side and rope is 12	A1	3.2a
		(5)	

(10 marks)

Notes:

(a)

M1: Calculates the scalar product between the two normal vectors

M1: Applies the scalar product formula between the two normal vectors to find a value for cos θ

A1: Identifies the correct angle by linking the angle between the normals and the angle between the planes.

(b)

M1: Substitutes point *P* into each equation of the plane and shows that that each plane is satisfied

A1: Comments that therefore point *P* lies on the straight line.

(c)

M1: Realises the scalar product between the line and the normal to the plane is needed and uses trigonometry to find an angle

M1: Calculates the scalar product between $\pm\left(\begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix} - \begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix}\right)$ and $\pm\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$

A1: $\begin{pmatrix} 10 \\ 10 \\ 8 \end{pmatrix} . \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = 18$ or $\begin{pmatrix} 10 \\ 10 \\ 8 \end{pmatrix} . \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} = -18$ or $\begin{pmatrix} -10 \\ -10 \\ -8 \end{pmatrix} . \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} = 18$ or $\begin{pmatrix} -10 \\ -10 \\ -8 \end{pmatrix} . \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = -18$

M1: A fully complete and correct method for obtaining the acute angle **A1:** Awrt 12° . Do not isw. Withhold this mark if extra answers are given.

Question	Scheme	Marks	AOs
7 (a)	$\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$	B1	1.2
	$\dfrac{e^x - e^{-x}}{e^x + e^{-x}} = y \Rightarrow e^x - e^{-x} = y(e^x + e^{-x}) = ye^x + ye^{-x}$ $\Rightarrow e^x(1-y) = e^{-x}(1+y) \Rightarrow e^{2x} = \dfrac{1+y}{1-y}$ or $\dfrac{e^y - e^{-y}}{\boldsymbol{e}^y + e^{-y}} = x \Rightarrow e^y - e^{-y} = x(e^y + e^{-y}) = xe^y + xe^{-y}$ $\Rightarrow e^y(1-x) = e^{-y}(1+x) \Rightarrow e^{2y} = \dfrac{1+x}{1-x}$	M1	2.1
	$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$*	A1*	1.1b
		(3)	
(b)	$x = \frac{1}{2}\ln\left(\frac{1+0.8}{1-0.8}\right) = \frac{1}{2}\ln(9) = \ln 3$	B1	3.1a
	$V = \pi\int_0^{\ln 3} \tanh^2 x \, dx$	B1	1.1b
	$= \{\pi\}\int_0^{\ln 3}(1 - \text{sech}^2 x)\, dx = [x - \tanh x]_0^{\ln 3} = [\ln 3 - 0.8] - [0]$	M1	3.1a
	$= \pi[\ln 3 - 0.8]$	A1	1.1b
	Volume cylinder = $\pi \times 0.8^2 \times' \ln 3\,'$ or $= \pi\int_0^{\ln 3} 0.8^2 \, dx$	M1	1.1b
	Volume = cylinder – solid of revolution	M1	3.1a
	Volume = $\pi\left[\frac{4}{5} - \frac{9}{25}\ln 3\right]$	A1	1.1b
		(7)	
		(10 marks)	

Notes:

(a)
B1: Correct expression for $\tanh x$
M1: Sets equal to y and rearranges to get $e^{2x} = f(y)$. Alternatively switches x and y and rearranges to get $e^{2y} = f(x)$.
A1*: Achieves the printed answer with no errors seen

(b)
B1: $x = \ln 3$
B1: Correct expression for the volume of the curve including π, dx can be implied later work and follow through upper limit
M1: Uses the identity $\tanh^2 x = 1 - \text{sech}^2 x$ to complete the integration and correct use of limits
A1: Correct volume of the curve
M1: A correct method to find the volume of the required cylinder
M1: A correct method to find the required volume.
A1: Correct volume in the required form

Question	Scheme	Marks	AOs
8(a)	$\mathbf{MM}^{\mathrm{T}} = \begin{pmatrix} 1 & 4 & -1 \\ 3 & 0 & p \\ q & r & s \end{pmatrix}\begin{pmatrix} 1 & 3 & q \\ 4 & 0 & r \\ -1 & p & s \end{pmatrix} = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix} \Rightarrow 3 - p = 0$	M1	2.1
	$p = 3$ *	A1*	1.1b
		(2)	
(b)	$k = 18$	B1	2.2a
		(1)	
(c)	$\mathbf{M}^{-1} = \frac{1}{18}\begin{pmatrix} 1 & 3 & q \\ 4 & 0 & r \\ -1 & 3 & s \end{pmatrix}$	B1ft	2.2a
		(1)	
(d)	Finds any two equations involving q, r and s from $q + 4r - s = 0,\ 3q + 3s = 0,\ q^2 + r^2 + s^2 = 18$	M1	3.1a
	All three correct equations $q + 4r - s = 0,\ 3q + 3s = 0,\ q^2 + r^2 + s^2 = 18$	A1	1.1b
	$s = -q, r = -\frac{1}{2}q \Rightarrow q^2 + \left(-\frac{1}{2}q\right)^2 + (-q)^2 = 18 \Rightarrow q = 2\sqrt{2}$ or $q = -2r, s = 2r \Rightarrow (-2r)^2 + r^2 + (2r)^2 = 18 \Rightarrow r = -\sqrt{2}$ or $q = -s, r = \frac{1}{2}s \Rightarrow (-s)^2 + \left(\frac{1}{2}s\right)^2 + s^2 = 18 \Rightarrow s = -2\sqrt{2}$	M1 A1	3.1a 1.1b
	$q = 2\sqrt{2}$, $r = -\sqrt{2}$ and $s = -2\sqrt{2}$ only	M1 A1	1.1b 2.2a
		(6)	
			(10 marks)

Notes:

(a)
M1: Sets $\mathbf{MM}^{\mathrm{T}} = k\mathbf{I}$ and finds a value for p
A1*: Correct value for p

(b)
B1: Correct value for k

(c)
B1ft: Deduces $\mathbf{M}^{-1}$, follow through on their value of k

(d)
M1: Uses $\mathbf{MM}^{\mathrm{T}} = k\mathbf{I}$ with $p = 3$ and their value of k to find at least two equations involving at least two of the constants q, r and s
A1: All three correct equations
M1: A complete method to solve the equations to find a value for either q, r or s
A1: A correct constant

M1: Finds the other two constants **A1:** Deduces all three correct constants
Alternative method using $\begin{pmatrix} 1 & 3 & q \\ 4 & 0 & r \\ -1 & 3 & s \end{pmatrix}\begin{pmatrix} 1 & 4 & -1 \\ 3 & 0 & 3 \\ q & r & s \end{pmatrix}=\begin{pmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{pmatrix}$ **M1:** First row times first column $1+9+q^2=18 \quad \Rightarrow \quad q=\ldots$ **A1:** Correct value for *q***,** $q=2\sqrt{2} \qquad (q>0)$ **M1:** First row time second column $4+qr=0 \quad \Rightarrow \quad r=\ldots$ **A1:** Correct value for *r***,** $r=-\frac{4}{2\sqrt{2}}=-\sqrt{2}$ **M1:** First row time third column $-1+9+qs=0 \quad \Rightarrow \quad s=\ldots$ **A1:** Deduces the correct value for *s***,** $s=-\frac{8}{2\sqrt{2}}=-2\sqrt{2}$

Question	Scheme	Marks	AOs
9(a)	$e^{2i\theta} + e^{-2i\theta} = (\cos 2\theta + i\sin 2\theta) + (\cos(-2\theta) + i\sin(-2\theta))$ $= (\cos 2\theta + i\sin 2\theta) + (\cos 2\theta - i\sin 2\theta) = \ ...$	M1	2.1
	$= 2\cos 2\theta$ *	A1*	1.1b
		(2)	
(b)	$C + iS = 1 + \frac{1}{4}e^{2i\theta} + \frac{1}{16}e^{4i\theta} + \ ...$	M1	2.1
	$C + iS = \dfrac{1}{1 - \frac{1}{4}e^{2i\theta}}$	M1	1.1b
	$C + iS = \frac{4}{4 - e^{2i\theta}}$	A1	1.1b
		(3)	
(c)	$C + iS = \dfrac{4}{4 - e^{2i\theta}} \times \dfrac{4 - e^{-2i\theta}}{4 - e^{-2i\theta}}$	M1	2.1
	$= \dfrac{16 - 4e^{-2i\theta}}{17 - 4(e^{2i\theta} + e^{-2i\theta})} = \dfrac{16 - 4(\cos 2\theta - i\sin 2\theta)}{17 - 8\cos 2\theta}$	M1	1.1b
	$C = \text{Re}\left(\dfrac{16 - 4(\cos 2\theta - i\sin 2\theta)}{17 - 8\cos 2\theta}\right) = \dfrac{16 - 4\cos(2\theta)}{17 - 8\cos(2\theta)}$ *	A1*	2.2a
		(3)	
(d)	$S = \text{Im}\left(\dfrac{16 - 4(\cos 2\theta - i\sin 2\theta)}{17 - 8\cos 2\theta}\right) = \dfrac{4\sin(2\theta)}{17 - 8\cos(2\theta)}$	B1ft	2.2a
	$\frac{4\sin(2\theta)}{17 - 8\cos(2\theta)} = 0 \Rightarrow 4\sin(2\theta) = 0 \Rightarrow \theta = \cdots$	M1	3.1a
	$\theta = 0, \frac{\pi}{2}, \pi$	A1	1.1b
		(3)	
			(11 marks)

Notes:

(a)
M1: Uses the modulus-argument form (Euler's formula)
A1*: Achieves the printed answer, with no errors seen

(b)
M1: Writes $C + iS$ as a series $1 + \alpha e^{2i\theta} + \beta e^{4i\theta}$
M1: Use the sum to infinity of a GP on their series
A1: Correct answer

(c)
M1: Multiplies top and bottom of the answer to (b) by $4 - e^{-2i\theta}$
M1: Replaces $e^{-2i\theta}$ with $\cos 2\theta - i\sin 2\theta$ and $e^{2i\theta} + e^{-2i\theta} = 2\cos 2\theta$
A1*: Achieves the printed answer, with no errors seen
A1: Deduces the correct expression for S

(d)

B1ft: Deduces the correct imaginary part from their $C + iS$
M1: Sets $S = 0$ and uses trigonometry to find a value for θ
A1: All three correct values for θ