

Mock Paper Set 1

**Answer ALL questions. Write your answers in the spaces provided.**

**1.**



An algorithm is described in Figure 1. The algorithm is run with *a* = 70 and *b* = 168

(*a*) Complete the table in the answer book to show the result obtained at each step when

the algorithm is applied.

**(4)**

When the algorithm is applied using new values for *a* and *b*, where *a* = *x* and *b* = *y* the

output is *y*.

(*b*) Write down a conclusion about the relationship between the values of *x* and *y* in this

case, giving a reason for your answer. You do not need to rerun the algorithm.

**(2)**

**(Total for Question 1 is 6 marks)**

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**2.** (*a*) Define the term “Hamiltonian cycle”.

**(2)**

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(*b*) Redraw the graph in Figure 2 to show that it is planar.

**(1)**

(*c*) Explain why the planarity algorithm cannot be used to show that the graph shown in

Figure 2 is a planar graph.

**(1)**

**(Total for Question 2 is 4 marks)**

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**3.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** | **D** | **E** | **F** |
| **A** | – | 19 | *x* + 4 | *x* – 2 | *x* – 2 | *x* – 5 |
| **B** | 19 | – | *x* + 3 | *x* | *x*–3 | *x* – 2 |
| **C** | *x* + 4 | *x* + 3 | – | *x* – 3 | *x* + 6 | *x* – 7 |
| **D** | *x* – 2 | *x* | *x* – 3 |  –  | *x* – 5 | *x* + 1 |
| **E** | *x* – 2 | *x* – 3 | *x* + 6 | *x* – 5 |  –  | *x* |
| **F** | *x* – 5 | *x* – 2 | *x* – 7 | *x* + 1 | *x* |  –  |

Claudia must check 6 ponds, A, B, C, D, E and F, to see which plant species are growing

in them. The times, in minutes, that she will need to take the direct routes between the

ponds are shown in the table.

Initially, some of the times are only known relative to the time it will take to travel

between B and D, which is shown as *x* minutes in the table. It is known that 7 < *x* < 20

The length of the route obtained when the Nearest Neighbour Algorithm is applied from A

takes 2 minutes less than the length of the route obtained when the Nearest Neighbour

Algorithm is applied from D.

(*a*) Determine the value of *x*.

**(5)**

A student claims that the quickest time required for Claudia to check all 6 ponds, starting

and finishing at A, can be found by substituting the value of *x* found in (a) into the

expression obtained from the Nearest Neighbour Algorithm starting from A.

(*b*) Without any further working, state, giving a reason, whether or not the student’s

claim is correct.

**(2)**

**(Total for Question 3 is 7 marks)**

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**4.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | A | B | C | D |
| A | – | 1 | 2 | 4 |
| B | 1 | – | 4 | 2 |
| C | 2 | 4 | – | 3 |
| D | 6 | 8 | 5 | – |

The distance table shown above represents the paths between four checkpoints, A, B,

C and D, on an orienteering course. The numbers represent the time, in minutes, needed

to run along the corresponding path. Most of the paths have been designated as one‑way

routes, to ease possible congestion.

The initial distance and route tables for the network are given in the answer book.

(*a*) Draw the network using the nodes given in Diagram 1 in the answer book.

**(1)**

(*b*) Use Floyd’s algorithm to find the complete network of shortest distances. Show the

distance and route tables after each iteration.

**(7)**

**(Total for Question 4 is 8 marks)**

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**5.**



Figure 3 represents a network of paths in a park. The number on each arc shows the time

it takes, in minutes, to walk along the corresponding path. Lola wants to walk from A to K

in the shortest possible time.

(*a*) Using Dijkstra’s algorithm,

(i) determine the minimum time for Lola’s route,

(ii) determine the route she should take.

**(5)**

The following day, Lola wants to walk from A to K again. A market at G means that

there will be a 6‑minute delay for anyone passing through G.

(*b*) Explain how the network may be adapted to allow for this change.

**(2)**

Dijkstra’s algorithm has approximate order

*x* ln( *y*)

where *x* is the number of arcs in the network and *y* is the number of nodes.

(*c*) Explain why the order given is only approximate.

**(1)**

A computer takes exactly 0.07 seconds to apply Dijkstra’s algorithm to a network that has

720 arcs and 250 nodes.

(*d*) Find an estimate for the time that the computer will take to apply Dijkstra’s algorithm

to a network of 1100 arcs and 625 nodes.

**(2)**

**(Total for Question 5 is 10 marks)**

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**6.**



Figure 4 models a network of roads that have to be inspected. The number on each arc is

the length, in km, of the corresponding section of road.

Each road must be traversed at least once and the length of the inspection route must be

minimised.

The inspection route must start and finish at the same vertex.

(*a*) Use the route inspection algorithm to find the length of a shortest route. You must

make your method and working clear, stating any arcs that must be repeated.

**(5)**

The road that runs directly from C to D is closed.

George decides to drive along all the remaining roads. He will start at D and wants a

route that minimises the distance he drives.

(*b*) Determine where George should finish his route and the distance he will drive.

You should make your reasoning clear.

**(3)**

(*c)* Write down a possible route for George.

**(1)**

**(Total for Question 6 is 9 marks)**

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**7.** Darren makes three types of muesli, Traditional, Fruity and Nutty, to sell in his café.

Each type of muesli includes dried fruit and nuts.

Let *x*, *y* and *z* be the weight in kg of the amount of Traditional muesli, Fruity muesli and

Nutty muesli he makes respectively.

Each kg of Traditional, Fruity and Nutty muesli contains 100 g, 75 g and 200 g of nuts

respectively.

Darren has 2 kg of nuts available.

He wants to ensure that the amount of Traditional muesli he makes is at least 40% of the total.

He has enough dried fruit to make 3 kg of Traditional muesli or 2 kg of Fruity muesli or

6 kg of Nutty muesli.

Darren wants to make as much muesli as possible.

Express this information as a linear programming problem. You should simplify the

constraints, where possible, leaving your answers with integer coefficients. (You may

assume the non‑negativity constraints.)

**(Total for Question 7 is 6 marks)**

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**8.** Robert is planning a conservation project. The activities he must complete, along with

the immediately preceding activities, are shown in the table below.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Activity | A | B | C | D | E | F | G | H | I | J | K |
| Immediately preceding activities |  –  |  –  |  –  | A | B | C | C | D | D, E, F | G, I | G, I |

(*a*) (i) Draw an activity network for this project using activity on arc and exactly two

dummies.

(ii) Explain why each of the dummies is necessary.

**(7)**

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Another conservation project is modelled by the activity network shown in Figure 5. The

activities are represented by the arcs. The number in brackets on each arc gives the time,

in days, to complete the corresponding activity. The project is to be completed in the

shortest possible time.

(*b*) Add the early event times and the late event times to Diagram 1 in the answer book.

**(3)**

(*c*) Write down the critical activities.

**(1)**

Just as the project is about to start, the project organiser discovers that activity G will

take an additional 10 days to complete.

(*d*) Find the effect of this change on the earliest completion time of the project, giving a

reason for your answer.

**(2)**

**(Total for Question 8 is 13 marks)**

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**9.** The tableau shown below is the initial tableau for a linear programming problem in *x*, *y*

and *z*. The objective is to maximise the profit, *P*.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Basic variable | *x* | *y* | *z* | *r* | *s* | *t* | Value |
| *r* | 1 | 2 |  – 6 | 1 | 0 | 0 | 12 |
| *s* | 2 | 1 |  – 7 | 0 | 1 | 0 | 5 |
| *t* | 4 | 5 |  – 8 | 0 | 0 | 1 | 7 |
| *P* | 6 |  – 12 | 1 | 0 | 0 | 0 | 0 |

(*a*) Perform **one** iteration of the Simplex algorithm to obtain a new tableau, T. Make your

method clear by explaining how you select the pivot and stating the row operations

you use.

**(5)**

(*b*) Write down the profit equation given by T and state the current values of all the

variables.

**(2)**

(*c*) Use T to explain why the Simplex algorithm cannot be used to find the optimal value

of *P* for this problem.

**(1)**

The linear programming problem given above is changed to give the tableau shown below.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Basic variable | *x* | *y* | *z* | *r* | *s* | *t* | Value |
| *r* | 1 | 2 | 6 | 1 | 0 | 0 | 12 |
| *s* | 2 | 1 | 7 | 0 | 1 | 0 | 5 |
| *t* | 4 | 5 | 8 | 0 | 0 | 1 | 7 |
| *P* | 6 |  –12 | 1 | 0 | 0 | 0 | 0 |

The additional constraint *x* ≥ 1 is added to the problem.

(*d*) Without performing any iterations, complete Tableau S in the answer book so that the

two‑stage Simplex method could be applied to solve the problem.

**(4)**

**(Total for Question 9 is 12 marks)**

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**TOTAL FOR PAPER IS 75 MARKs**