Question	Scheme	Marks	AOs
1. (a)			
		M1	1.1b
		A1	1.1b
		M1	1.1b
		A1	1.1b
		(4)	
(b)	The algorithm finds the highest common factor of x and y… so y must be a factor of x.	B1 B1 dep	2.4 2.2a
		(2)	
			Total: 6 marks

	a	b	Is $a > b$?	Is $a < b$?	Is $a = b$?	Output
	70	168	No	Yes		
		98			No	
			No	Yes		
		28			No	
			Yes			
	42			No	No	
			Yes			
	14			Yes		
		14			Yes	14

Notes

a1M1 Clear attempt at the first two lines of the table

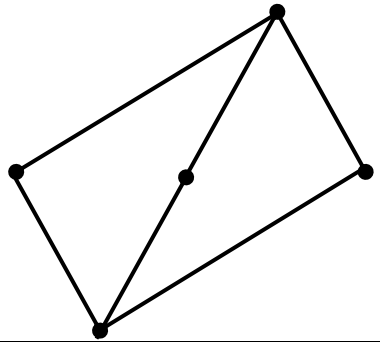
a1A1 First four lines fully correct

a2M1 Attempt to apply algorithm to completion

a2A1 Fully complete answer including output

b1B1 Highest common factor … (oe e.g. x is a multiple of y)

b2B1 dep … of the input values (or "of x and y"; accept "of a and b") – dependent on first B1

Question	Scheme	Marks	AOs
2 (a)	A Hamiltonian cycle is • a sequence of **edges** • that passes through **every vertex** of a graph • **exactly once** • and **returns to its start** vertex.	M1 A1	1.2 2.5
		(2)	
(b)	 or equivalent	B1	1.1b
		(1)	
(c)	The graph does not include a Hamiltonian cycle (so the algorithm cannot be applied)	B1	2.4
		(1)	
			Total: 4 marks

Notes	
a1M1	A good attempt at a definition containing two of the four characteristics.
a1A1	Full, clear definition. Only accept accurate, mathematical language e.g. “vertex”/”node”, not “point” and “arc”/”edge” not “line”.
b1B1	Any equivalent graph with no crossings
c1B1	cao

Question	Scheme	Marks	AOs
3. (a)	(NNA route from A is AFCDEBA): $(x – 5) + (x – 7) + (x – 3) + (x – 5) + (x – 3) + 19 = 5x – 4$ (NNA route from D is DEBFCAD): $(x – 5) + (x – 3) + (x – 2) + (x – 7) + (x + 4) + (x – 2) = 6x – 15$ $(5x – 4) + 2 = 6x – 15$ $x = 13$	M1 M1 A1 M1 A1	1.1b 1.1b 1.1b 2.1 2.2a
		(5)	
(b)	The Nearest Neighbour Algorithm finds an upper bound for the route (or for the Travelling Salesperson Problem) & not necessarily the shortest/optimal route… … so the student's claim is unlikely to be correct.	M1 A1	2.4 2.2b
		(2)	
			Total: 7 marks

Notes

a1M1 Correct NNA route from A **or** D, either as pond letters or in terms of x. The route must return to the initial vertex.

a2M1 Correct NNA route from both A **and** D

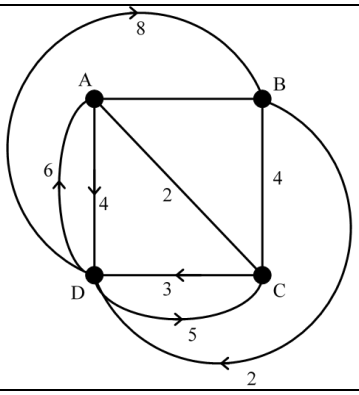
a1A1 Both algebraic expressions for the Nearest Neighbour routes correct

a3M1 Route lengths equated

a2A1 Correct value of x

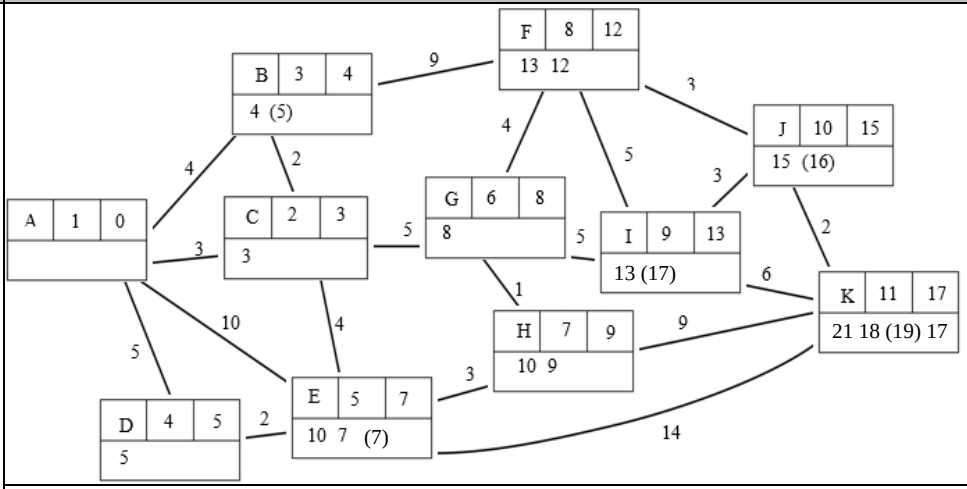
b1M1 Correct statement about the **purpose** of the Nearest Neighbour Algorithm

b1A1 Correct inference that it is unlikely to give the quickest time

Question	Scheme	Marks	AOs
4	Initial tables **A B C D** | **A B C D** **A** - 1 2 4 | **A** A B C D **B** 1 - 4 2 | **B** A B C D **C** 2 4 - 3 | **C** A B C D **D** 6 8 5 - | **D** A B C D		
(a)		B1	1.1b
(b)		**(1)**	
	1st iteration **A B C D** | **A B C D** **A** - 1 2 4 | **A** A **B** C D **B** 1 - **3** 2 | **B** A B **A** D **C** 2 **3** - 3 | **C** A **A** C D **D** 6 **7** 5 - | **D** A **A** C D	M1 A1	1.1b 1.1b
	2nd iteration **A B C D** | **A B C D** **A** - 1 2 **3** | **A** A B C **B** **B** 1 - 3 2 | **B** A B A D **C** 2 3 - 3 | **C** A A C D **D** 6 7 5 - | **D** A A C D	M1 A1 ft	1.1b 1.1b
	3rd iteration **A B C D** | **A B C D** **A** - 1 2 3 | **A** A B C B **B** 1 - 3 2 | **B** A B A D **C** 2 3 - 3 | **C** A A C D **D** 6 7 5 - | **D** A A C **D**	M1 A1 ft	1.1b 1.1b
	4th iteration **A B C D** | **A B C D** **A** - 1 2 3 | **A** A B C B **B** 1 - 3 2 | **B** A B A D **C** 2 3 - 3 | **C** A A C D **D** 6 7 5 - | **D** A A C D	A1	1.1b
		(7)	

	Total: 8 marks

	Notes
a1B1	Fully correct diagram including arrows
b1M1	No change in the first row and first column of both tables with at least one value in the distance table reduced and one value in the route table changed
b1A1	cao
b2M1	No change in the second row and second column of both tables with at least two values in the distance table reduced and two values in the route table changed
b2A1 ft	Correct second iteration follow through from the candidate's first iteration
b3M1	No change in the third row and third column of both tables with at least one value in the distance table reduced and one value in the route table changed
b3A1 ft	Correct third iteration follow through from the candidate's second iteration
b4A1	cso

Question	Scheme	Marks	AOs
5. (a)		M1 ABCD A1 EFGH A1 ft IJK	1.1b 1.1b 1.1b
	Min time: 17 minutes (condone lack of units) ACGFJK	A1 ft A1	2.2a 2.2a
		(5)	
(b)	Add 3 minutes to the weight of every arc incident on G	M1 A1	2.4 2.2a
		(2)	
(c)	The order of an algorithm gives the order of the dominant term in the time needed; other terms will still affect the total time (o.e.)	B1	3.5b
		(1)	
(d)	$0.07\left(\frac{1100\ln 625}{720\ln 250}\right)$ $=$ 0.12469… (seconds)	M1 A1	1.1a 1.1b
		(2)	
		Total: 10 marks	

	Notes
All working values at each node must be correct, including both the value *and* the order for corresponding A marks. The order of labelling must also be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling. Penalise order of labelling only once.	
a1M1 **a1A1** **a2A1ft** **a3A1ft** **a4A1**	A larger value replaced by a smaller value at least once in the working values at E or F or I or K and All values in A, B, C, and D correct All values in E, F, G and H correct All remaining values (I, J, K) correct on follow through and working values in the correct order. Penalise order of labelling only once. Ignore permanent label and final value at H 17 minutes ft their final value cao
b1M1 **b1A1**	Add values to all arcs incident on G … states the value of 3 minutes
c1B1	Idea of dominant term and possible presence of other terms in the time needed
d1M1 **d1A1**	Correct method seen… logarithms in any base awrt 0.12 seconds (condone lack of units)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>6. (a)</td><td>
<table>
<tr><td>Pairings</td><td>Shortest path distances</td><td>Total length of pairings</td></tr>
<tr><td>A and C
E and F</td><td>1+2+1= 4
2+3 = 5</td><td>9</td></tr>
<tr><td>A and E
C and F</td><td>1+2+3 = 6
1+2 = 3</td><td>9</td></tr>
<tr><td>A and F
C and E</td><td>1
1+3 = 4</td><td>5 *</td></tr>
</table>
</td><td>M1
A1
A1 ft</td><td>2.1
1.1b
1.1b</td></tr>
<tr><td></td><td>Repeat AF, CD and DE
Length = 80 + 5 = 85 (km)</td><td>A1
A1</td><td>2.2a
1.1b</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(b)</td><td>Finish at either A or F or E
Shortest path between any pair of these three vertices is AF = 1
So finish at E
Shortest distance = 80 – 1 + 1 = 80 (km)</td><td>M1

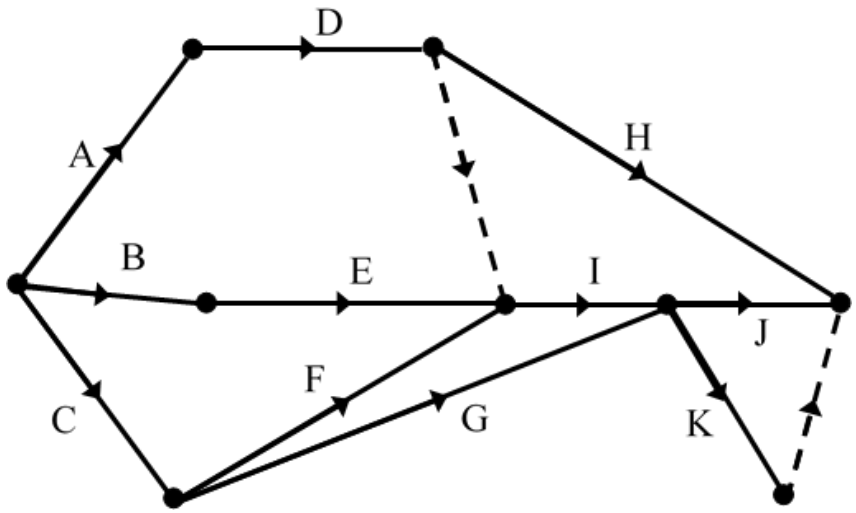
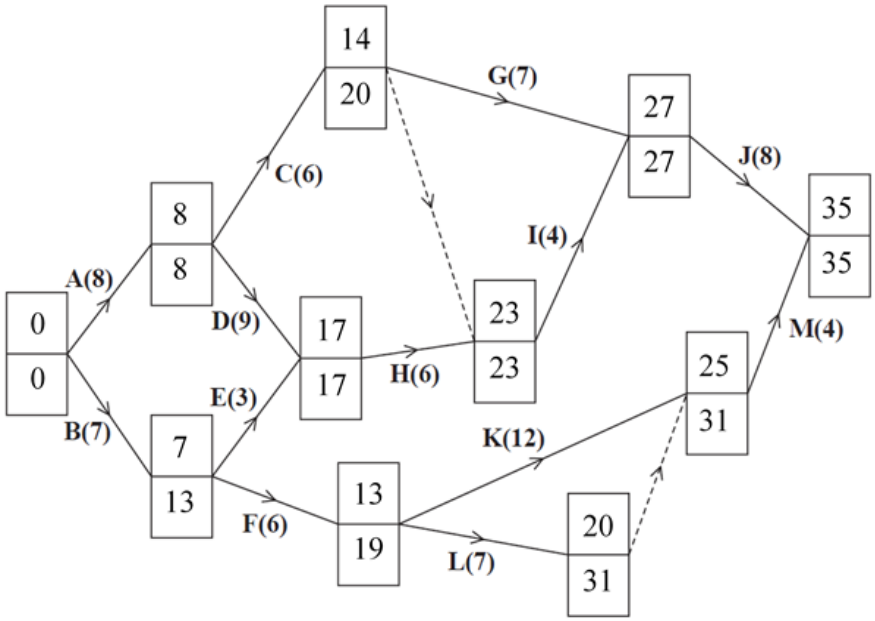
A1
A1</td><td>3.1b

1.1b
2.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(c)</td><td>e.g. DFAFGACBAHBGHFEDHCE</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4">Total: 9 marks</td></tr>
</table>

Notes	
a1M1	Three pairings of correct four odd nodes (A, C, E and F)
a1A1	Calculations for one pairing correct
a2A1	All correct
a3A1	Their repeated arcs
a4A1 ft	80 + their least = their correct sum. Condone lack of units
b1M1	Complete strategy to find finish point or the "80"
b1A1	Correct calculations and "finish at E"
b2A1	cao (80)
c1B1	Must start at D and finish at E, with AF repeated and have length 19 nodes

Question	Scheme	Marks	AOs
7.	Maximise $x + y + z$	B1	3.3
	Subject to $100x + 75y + 200z \leq 2000$ simplifying to $4x + 3y + 8z \leq 80$	B1	3.3
	$x \geq 0.4(x + y + z)$ $3x \geq 2y + 2z$	M1 A1	3.3 1.1b
	$2x + 3y + z \leq 6$ $x,\ y,\ z \geq 0$	M1 A1	3.3 1.1b
			Total: 6 marks

Notes	
B1	Correct objective (must include the word "Maximise")
B1	Cao $4x+3y+8z \leq 80$
M1	x □ 0.4 $(x + y + z)$ where □ is any inequality or equals
A1	Cao $3x \geq 2y+2z$
M1	$\frac{x}{3}+\frac{y}{2}+\frac{z}{6}$ □ 1 where □ is any inequality or equals
A1	Cao $2x+3y+z \leq 6$

Question	Scheme	Marks	AOs
8. (a)		M1 A1 A1 A1 A1	1.1b 1.1b 1.1b 1.1b 1.1b
	A dummy is needed as H depends on D only but I depends on E, F and D. A dummy is needed to ensure unique numbering using the events at each end of the activities.	B1 B1	2.4 2.4
		(7)	
(b)		M1 A1 A1	2.1 1.1b 1.1b
		(3)	
(c)	A, D, H, I, J	B1	2.2a
		(1)	
(d)	G has float = 27–14–7 = 6 so a delay of 10 to G uses this and delays the whole project by 4 (days)	M1 A1 ft	2.4 3.2a
		(2)	
		Total: 13 marks	

Notes	
In (a) condone lack of, or incorrect, numbered events throughout – also 'dealt with correctly' means that the activity starts from the correct event but may not finish at the correct event. Activity on node is M0. Do not penalise the same error twice with the first three A marks, for example, if activity C is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. Penalise lack of, or incorrect, arrows on the dummies only on the first occurrence.	
a1M1	Seven activities (labelled on arc), one start and at least one dummy placed.
a1A1	Activities A, B, C, D, E, F and G dealt with correctly
a2A1	1st dummy (+arrow) and H and I dealt with correctly
a3A1	J, K and 2nd dummy (+ arrow) dealt with correctly. Note that 2nd dummy arrow could go in opposite direction
a4A1	CSO – all arrows present and correctly placed, no additional dummies and one finish.
a1B1	Dependency of activities on different preceding activities to include a contrast between D and at least one of E and F.
a2B1	Unique event numbering. 'So that activities can be defined uniquely' is not sufficient to earn this mark. There must be some mention of describing activities either in terms of the event at each end or in terms of an activities events. However, give bod on statements that imply that an activity begins and ends at the same event.
b1M1	All top boxes complete, values generally increasing from left to right, condone one 'rogue' **AND** all bottom boxes complete, values generally decreasing right to left, condone one 'rogue'
b1A1	cao on top boxes
b2A1	cao on bottom boxes
c1B1	Cao
d1M1	Reasoning including 10 – their float on G
d1A1 ft	correct answer based on follow though

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>9. (a)</td><td>θ values: 12/1 = 12; 5/1 = 5; 7/5 = 1.4
<table>
<tr><td>Basic variable</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>Value</td><td></td></tr>
<tr><td>r</td><td>$-\frac{3}{5}$</td><td>0</td><td>$-\frac{14}{5}$</td><td>1</td><td>0</td><td>$-\frac{2}{5}$</td><td>$\frac{46}{5}$</td><td>$R_1 - 2r_3$</td></tr>
<tr><td>y</td><td>$\frac{6}{5}$</td><td>0</td><td>$-\frac{27}{5}$</td><td>0</td><td>1</td><td>$-\frac{1}{5}$</td><td>$\frac{18}{5}$</td><td>$R_2 - r_3$</td></tr>
<tr><td>t</td><td>$\frac{4}{5}$</td><td>1</td><td>$-\frac{8}{5}$</td><td>0</td><td>0</td><td>$\frac{1}{5}$</td><td>$\frac{7}{5}$</td><td>$R_3/5$</td></tr>
<tr><td>P</td><td>$\frac{78}{5}$</td><td>0</td><td>$-\frac{91}{5}$</td><td>0</td><td>0</td><td>$\frac{12}{5}$</td><td>$\frac{84}{5}$</td><td>$R_4 + 12r_3$</td></tr>
</table></td><td>B1

M1
A1
A1 ft
B1</td><td>2.4

1.1b
1.1b
1.1b
2.4</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(b)</td><td>$P = -\frac{78}{5}x + \frac{91}{5}z - \frac{12}{5}t + \frac{84}{5}$,
$x = 0,\ y = \frac{7}{5},\ z = 0,\ r = \frac{46}{5},\ s = \frac{18}{5},\ t = 0$
$P = \frac{84}{5}$</td><td>B1 ft

B1</td><td>3.4

3.4</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>e.g. Only negative in objective row is in the z column, but all entries in this column are negative</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td><table>
<tr><td>Basic variable</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>u</td><td>a</td><td>Value</td></tr>
<tr><td>r</td><td>1</td><td>2</td><td>6</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>12</td></tr>
<tr><td>s</td><td>2</td><td>1</td><td>7</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>5</td></tr>
<tr><td>t</td><td>4</td><td>5</td><td>8</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>7</td></tr>
<tr><td>a</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>–1</td><td>1</td><td>1</td></tr>
<tr><td>P</td><td>6</td><td>–12</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>A</td><td>–1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>–1</td></tr>
</table></td><td>B1
B1
M1
A1</td><td>3.1a
3.1a
2.1
1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td colspan="4">Total: 12 marks</td></tr>
<tr><td colspan="4">Total for Paper is 75 marks</td></tr>
</table>

Notes	
a1B1	Calculations for theta values shown and correct
a1M1	Correct pivot located, attempt to divide row.
a1A1 ft	The correct row operations used correctly at least once from *their* pivot, column *x*, *z*, *s* or value 'correct'.
a2A1	cao on values (ignore row operations and b.v.)
a2B1	All row operations cao – allow if given in terms of old row 2
b1B1 ft	Two M marks in (a) must have been awarded for these marks. Allow implicit stating of *P* e.g. $P + \frac{39}{2}x - 17y + \frac{3}{2}t = \frac{21}{2}$ as above
b2B1	Their correct values stated for at least *P*, *x*, *y*, *z* from their 'optimal' iteration.
c1B1	e.g.: A new pivot cannot be found as the only negative in the objective row is in the *y* column, but all entries in this column are negative There is no limit to the magnitude of *y* (and so *P* can increase without limit)
d1B1	A surplus and an artificial variable added. (Ignore letters used) e.g. $x \geq 1$ implies that $x - u + a = 1$
d2B1	New objective function row ($A = -a$ implies $A - x + u = -1$)
d1M1	Correct number of rows and columns with three correct rows
d1A1	Cao