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| **Question** | **Scheme** | **Marks** | **AOs** |
| **1 (a)** | H0; recycling plastics and the distance to the nearest recycling point are independentH1; recycling plastics and the distance to the nearest recycling point are not independent | B1 | 1.2 |
|  | Degrees of freedom, $ν=\left(3-1\right)×\left(2-1\right)=2$Therefore critical value = $χ^{2}\left(0.05\right)=5.991$ | B1 | 3.1b |
|  | Test statistic $=\sum\_{}^{}\frac{\left(O\_{i}-E\_{i}\right)^{2}}{E\_{i}}=\frac{\left(64-58.3\right)^{2}}{58.3}+\frac{\left(42-47.7\right)^{2}}{47.7}+\frac{\left(32-29.7\right)^{2}}{29.7}+…=$ | M1 | 1.1b |
|  | $8.0988….. =$awrt 8.10 | A1 | 1.1b |
|  | In critical region, therefore sufficient evidence to reject H0. Data does **not** support Barbara’s belief at the 5% significance level that people recycling plastics is independent of the distance to the nearest recycling point. | A1 | 3.2a |
|  |  |  | **(5)** |
| **(b)** | * Test statistic is halved and
* Critical value stays the same
 | B1 | 2.4 |
|  | This is not in the critical region and therefore Barbara’s original conclusions are not valid, data suggests independence. | dB1 | 2.3 |
|  |  |  | **(2)** |
|  |  |  | **(7)** |

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| **Notes** |
| a) 1st B1 accept ‘no association between …’ & ‘association between…’ in H0 and H1 oe but must mention context 2nd B1 must have ($ν$ = 2) awrt 5.99 for critical value (implies correct degrees of freedom)M1 need to see at least 2 terms if A1 not awarded1st A1 awrt 8.10 (accept *p* = awrt 0.0174)2nd A1 cso must have correct comment in context. |
| b)1st B1 valid reason why test statistic is halved2nd B1 dependent on 1st B1 cao |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **2 (a)** | $$P\left(X=4\right)=\frac{7}{10}×\frac{6}{9}×\frac{5}{8}$$ | M1 | 2.1 |
|  | $ =\frac{210}{720} =\frac{7}{24}$ \* | A1\* | 1.1b |
|  |  |  | **(2)** |
| **(b)** | $$E\left(X\right)=1×\frac{3}{10}+2×\frac{7}{30}+3×\frac{7}{40}+4×\frac{7}{24}=$$$$\frac{59}{24}=2.458\dot{3}$$$$E\left(X^{2}\right)=1^{2}×\frac{3}{10}+2^{2}×\frac{7}{30}+3^{2}×\frac{7}{40}+4^{2}×\frac{7}{24}=$$$$\frac{299}{40}=7.475$$ | M1A1 | 3.1b1.1b |
|  | $Var\left(X\right)=\frac{299}{40}-\left(\frac{59}{24}\right)^{2}=1.43159…$ awrt 1.43 | M1A1 | 1.1b1.1b |
|  |  |  | **(4)** |
| **(c)** | Let rv $T=$ number of tokens drawn$$T\~Geo\left(0.3\right)$$ | B1 | 3.3 |
|  | $$Y=T-1$$ | B1 | 3.1b |
|  | $$Var\left(Y\right)=Var\left(T\right)$$ | M1 | 3.4 |
|  | $$Var\left(Y\right)=\frac{1-p}{p^{2}}=\frac{0.7}{0.3^{2}}=\frac{70}{9}=7.\dot{7}$$ | A1 | 1.1b |
|  |  |  | **(4)** |
|  |  |  | **(10)** |

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| **Notes** |
| a) M1 need $\frac{7}{10}×\frac{6}{9}×…$ either using P(GGG), or P(GGGR or GGGG)A1 \* full correct solution |
| b) 1st M1 correct method for $E\left(X\right)$ or $E\left(X^{2}\right)$ 1st A1 both $E\left(X\right)$ and $E\left(X^{2}\right)$ correct2nd M1 full method for $Var\left(X\right)$ involving attempt at $E\left(X\right)$ and $E\left(X^{2}\right)$ or $E\left(\left(X-μ\right)^{2}\right)$2nd A1 awrt 1.43Correct answer with no working scores 1/4 |
| c) 1st B1 use of Geometric distribution for number of tokens drawn2nd B1 Establishing link between *Y* and *T*1st M1 same variance for *Y* and *T*A1 awrt 7.78 Correct answer with no working scores 2/4 |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **3 (a)(i)** | $P\left(X=9\right)=0.11237…$ awrt 0.112 | B1 | 1.1b |
| **(ii)** | $$P\left(X\geq 9\right)=1-P\left(X\leq 8\right)$$ | M1 | 1.1b |
|  | $=1-0.250228…=0.749771…$ awrt 0.750 | A1 | 1.1b |
|  |  |  | **(3)** |
| **(b)** | Select $X\~Po\left(6\right)$ and H0: $λ=6$ and H1: $λ>6$  | M1 | 3.3 |
|  | [$P\left(X\leq 10\right)=0.957379…$ ] $∴ P\left(X\geq 11\right)=0.04262…$ | M1 | 3.4 |
|  | $X\geq 11$ is critical region | A1 | 1.1b |
|  | Power of test = $P\left(Y\geq 11|λ=10\right)=1-P\left(Y\leq 10|λ=10\right)=$ | dM1A1ft | 3.1b1.1b |
|  | $1-0.583039…=0.41696…$ awrt 0.417 | A1 | 1.1b |
|  |  |  | **(6)** |
| **(c)** | Test is only valid if the teacher can assume that Charlie looks at his mobile phone * independently each time, or
* at a constant rate, or
* randomly

(any one of these) | B1 | 3.5b |
|  |  |  | **(1)** |
|  |  |  | **(10)** |

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| **Notes** |
| a) (i) B1 calculator value awrt 0.112(ii) M1 $1-P\left(X\leq 8\right)$ oe seen or impliedA1 awrt 0.750 (condone 0.75) |
| b) 1st M1 $X\~Po\left(6\right)$ seen and correct hypotheses (allow *μ* for *λ* or $λ=3$ and $λ>3$ )2nd M1 attempt to find critical region using Po(6), upper tail onlyA1 $X\geq 11$ as critical region seen or implied 3rd M1 complete strategy to find power of test, dependent on 2nd M mark2nd A1ft on their critical region (must be using Po(10))3rd A1 awrt 0.417 |
| c) B1 cao must mention context |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **4 (a)** | $H\_{0}:$ Geo(1/3) is a suitable model for the number of rolls to 1st 5 or 6$H\_{1}:$ Geo(1/3) is not a suitable model for the number of rolls to 1st 5 or 6 | B1 | 2.5 |
|  | Assuming $H\_{0}$ is true , expected values are $E\_{i}=100×\frac{1}{3}×\left(\frac{2}{3}\right)^{i-1}$ | M1 | 3.4 |
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| Number of rolls to 1st 5 or 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 or more |
| Expected frequency | 33.3… | 22.2… | 14.8… | 9.87… | 6.58… | 4.38… | 8.77…. |

 | A1A1 | 1.1b1.1b |
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| Number of rolls to 1st 5 or 6 | 1 | 2 | 3 | 4 | 5 | 6 or more |
| Observed frequency | 39 | 17 | 17 | 10 | 3 | 14 |
| Expected frequency | 33.3… | 22.2… | 14.8… | 9.87… | 6.58… | 13.1… |

Combining last two groups | M1 | 2.1 |
|  | $$ν=6-1=5$$ | B1ft | 1.1b |
|  | Critical value, $χ^{2}\left(0.05\right)=11.070$ | B1ft | 1.1a |
|  | Test statistic = $\frac{\left(39-33.3\right)^{2}}{33.3}+\frac{\left(17-22.2\right)^{2}}{22.2}+\frac{\left(17-14.8\right)^{2}}{14.8}+…$ | M1 | 1.1b |
|  | = 4.5181… awrt 4.5 | A1 | 1.1b |
|  | Not in critical region, insufficient evidence to reject $H\_{0}$, no significant evidence at the 5% level that the die is biased. | A1cso | 3.5a |
|  |  |  | **(10)** |
| **(b)(i)** | e.g. Mai’s test measures results against the frequencies of all six possible scores, whereas Desmond’s doesn’t. | B1 | 3.5b |
|  | Mai’s test needs a fixed number of rolls, whereas number of rolls for Desmond’s test is unknown. | B1 | 3.5b |
| **(ii)** | Desmond’s test is likely to collect more data. | B1 | 3.5b |
|  |  |  | **(3)** |
|  |  |  | **(13)** |

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| **Notes** |
| a)1st B1 $H\_{0}$/$H\_{1}$ must include correct value of parameter.1st M1 use of geometric distribution (may be implied)1st A1 at least 3 values correct to 1dp2nd A1 all correct to 1 dp (allow last 2 groups already combined)2nd M1 combining last two groups only2nd B1 ft if not combined groups3rd B1 ft their $ν$3rd M1 at least 2 terms shown (give if test value correct and no working shown)3rd A1 awrt 4.54th A1 cso |
| b) (i) B1 valid comment on limitation of Desmond’s test compared to Mai’sB1 valid comment on number of rolls of die expected compared to Mai’s(ii) B1 valid comment on amount of data collected.  |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **5 (a)(i)** | $$X\~B\left(1000, 0.005\right)$$ | B1cao | 3.3 |
| **(ii)** | $$P\left(X\geq 10\right)=1-P\left(X\leq 9\right)=1-0.96853476…$$ | M1 | 2.1 |
|  |  = 0.03146523…  |  |  |
|  | Approximation $Y\~Po\left(5\right)$$$P\left(Y\geq 10\right)=1-P\left(Y\leq 9\right)=1-0.9681719435…$$ | M1 | 3.3 |
|  |  =0.0318280564… | A1 | 1.1b |
|  | Percentage error $=\frac{0.031828056-0.03146523}{0.03146523}×100$(denominator must be their ‘0.031465…’) | M1 | 1.1b |
|  | =1.1530… awrt 1.15% | A1 | 1.1b |
|  |  |  | **(6)** |
| **(b)** | Size of Ushma’s test = $1-P\left(X\leq 8\right)=1-0.932397…$ | M1 | 2.1 |
|  | $$=0.067602…$$ | A1 | 1.1b |
|  | [Size of test = $P\left(Type I error\right)$]Size of Ushma’s test < Size of Javed’s test (0.08) | B1 | 2.4 |
|  | So implies Ushma’s test is better | dB1ft | 2.2b |
|  |  |  | **(4)** |
|  |  |  | **(10)** |

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| **Notes** |
| a)(i) B1 cao must include correct parameters(ii)1st M1 attempting $P\left(X\geq 10\right)$ using a Binomial distribution2nd M1 attempting $P\left(Y\geq 10\right)$ using Po(5)1st A1 both correct to 3 sfs (awrt 0.0315 and 0.0318 respectively)3rd M1 correct method for percentage error using their probabilities from Bin and Poisson distributions2nd A1 accept awrt 1.14 – 1.16% with working seen (do not accept 0.0115) |
| b) M1 attempt at $P\left(X\geq 9\right)$ using $ X\~B\left(1000, 0.005\right)$ or $X\~Po\left(5\right)$A1 awrt 0.0676 (or 0.0681 if using $X\~Po\left(5\right))$1st B1 valid argument implying smaller size of test is preferable2nd B1 dependent on 1st B1 ft their size of Ushma’s test.  |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **6 (a)(i)** | Use of negative binomial with $r=3 and p=0.2$ | B1 | 3.3 |
|  | Use of $μ=\frac{r}{p}=\frac{3}{0.2}$ | M1 | 3.4 |
|  |  = 15 | A1 | 1.1b |
|  |  |  | **(3)** |
| **(ii)** | Success of each throw is independent of all previous throws (oe) | B1 | 2.4 |
|  | Probability of hitting the bullseye on each throw remains the same (oe) | B1 | 2.4 |
|  |  |  | **(2)** |
| **(b)** | Variance of number of throws required, $σ^{2}=\frac{r\left(1-p\right)}{p^{2}}=\frac{3×0.8}{0.2^{2}}=60$ | B1 | 1.1b |
|  | By CLT need to use $\overbar{X}\~N\left("15",\frac{"60"}{100}\right)$ | M1A1 | 3.1b1.1b |
|  | $P\left(\overbar{X}>16\right)$  | dM1 | 3.4 |
|  |  = 0.09835…. awrt 0.0984 | A1 | 1.1b |
|  |  |  | **(5)** |
|  |  |  | **(10)** |

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| **Notes** |
| a)(i) B1 needs correct distribution and correct parameters(ii) 2xB1 must be valid comments in context. |
| b) 1st M1 use of Normal distribution with their $μ$ and $σ^{2}$1st A1 must be correct values2nd M1 dependent on use of $\overbar{X}\~N\left("15",\frac{"60"}{100}\right)$2nd A1 awrt 0.0984 |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **7 (a)** | $G\_{X}^{'}\left(t\right)=λe^{λ\left(t-1\right)}$  | M1A1 | 2.11.1b |
|  | $G\_{X}^{'}\left(1\right)=λe^{0}=λ $ \* | A1\*cso | 1.1b |
|  |  |  | **(3)** |
| **(b)** | Need $G\_{Y}\left(1\right)=1$, so $\left(\frac{1}{a}+b\right)^{n}=1$ | M1 | 3.1a |
|  | $∴ \frac{1}{a}+b=1$ so $b=1-\frac{1}{a}$ \* | A1\*cso | 1.1b |
|  |  |  | **(2)** |
| **(c)** | $b>0$, $∴ 1-\frac{1}{a}>0$  | M1 | 3.1a |
|  | $$a>1$$ | A1 | 1.1b |
|  |  |  | **(2)** |
| **(d)** | $\left[G\_{Y}\left(t\right)=\left(\frac{t^{2}}{a}+b\right)^{4}\right]$ $∴G\_{Y}^{'}\left(t\right)=\frac{8t}{a}\left(\frac{t^{2}}{a}+b\right)^{3}$ | M1 | 2.1 |
|  | $\left[E\left(Y\right)=\right]G\_{Y}^{'}\left(1\right)=\frac{8}{a}\left(\frac{1}{a}+b\right)^{3}$ $\left[=\frac{8}{a}\right]$ | M1 | 1.1b |
|  | $$G\_{Y}^{''}\left(t\right)=\frac{48t^{2}}{a^{2}}\left(\frac{t^{2}}{a}+b\right)^{2}+\frac{8}{a}\left(\frac{t^{2}}{a}+b\right)^{3}$$ | M1 | 2.1 |
|  | $E\left(Y^{2}\right)-E\left(Y\right)=G\_{Y}^{''}\left(1\right)=\frac{48}{a^{2}}\left(\frac{1}{a}+b\right)^{2}+\frac{8}{a}\left(\frac{1}{a}+b\right)^{3}$ $\left[=\frac{48}{a^{2}}+\frac{8}{a}\right]$ | M1A1 | 1.1b1.1b |
|  | $$Var\left(Y\right)=G\_{Y}^{''}\left(1\right)+G\_{Y}^{'}\left(1\right)-\left(G\_{Y}^{'}\left(1\right)\right)^{2}$$$$=\frac{48}{a^{2}}\left(\frac{1}{a}+b\right)^{2}+\frac{8}{a}\left(\frac{1}{a}+b\right)^{3}+\frac{8}{a}\left(\frac{1}{a}+b\right)^{3}-\left(\frac{8}{a}\left(\frac{1}{a}+b\right)^{3}\right)^{2}$$ | M1 | 1.2 |
|  | Use of $ \frac{1}{a}+b=1$ $Var\left(Y\right)=\frac{48}{a^{2}}+\frac{8}{a}+\frac{8}{a}-\left(\frac{8}{a}\right)^{2}$ oe | M1 | 2.1 |
|  | $Var\left(Y\right)=\frac{16}{a}-\frac{16}{a^{2}}=\frac{16}{a^{2}}\left(a-1\right)$ \*  | A1\*cso | 1.1b |
|  |  |  | **(8)** |
|  |  |  | **(15)** |

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| **Notes** |
| a)1st M1 attempt to $G\_{X}^{'}\left(t\right)$2nd A1 \* cso must see convincing use of *t*=1 |
| b)1st M1 Use of $G\_{Y}\left(1\right)=1$A1 \* cso need $\frac{1}{a}+b=1$ oe |
| c)M1 use of $b>0$ |
| d)1st M1 attempt to find $G\_{Y}^{'}\left(t\right)$2nd M1 use of $E\left(Y\right)=G\_{Y}^{'}\left(1\right)$3rd M1 attempt to find $G\_{Y}^{''}\left(t\right)$4th M1 use of $E\left(Y^{2}\right)-E\left(Y\right)=G\_{Y}^{''}\left(1\right)$1st A1 $G\_{Y}^{'}\left(1\right)$ and $G\_{Y}^{''}\left(1 \right)$ both correct5th M1 use of $Var\left(Y\right)=G\_{Y}^{''}\left(1\right)+G\_{Y}^{'}\left(1\right)-\left(G\_{Y}^{'}\left(1\right)\right)^{2}$, may be seen earlier6th M1 Use of $ \frac{1}{a}+b=1$ or substitution for $b$, may be seen earlier e.g. $E\left(Y\right)=\frac{8}{a}$2nd A1 \* cso all correct. |