**Composite Applied paper Jan/Jun 2006**

**1.** Accidents on a particular stretch of motorway occur at an average rate of 1.5 per week.

 (*a*) Write down a suitable model to represent the number of accidents per week on this stretch of motorway.

**(1)**

 Find the probability that

 (*b*) there will be 2 accidents in the same week,

**(2)**

 (*c*) there is at least one accident per week for 3 consecutive weeks,

**(3)**

 (*d*) there are more than 4 accidents in a 2 week period.

 **(2)**

**2.** An area of grass was sampled by placing a 1 m × 1 m square randomly in 100 places. The numbers of daisies in each of the squares were counted. It was decided that the resulting data could be modelled by a Poisson distribution with mean 2. The expected frequencies were calculated using the model.

 The following table shows the observed and expected frequencies.

|  |  |  |
| --- | --- | --- |
| Number of daisies | Observed frequency | Expected frequency |
|  0 |  8 |  13.53 |
|  1 |  32 |  27.07 |
|  2 |  27 |  *r* |
|  3 |  18 |  *s* |
|  4 |  10 |  9.02 |
|  5 |  3 |  3.61 |
|  6 |  1 |  1.20 |
|  7 |  0 |  0.34 |
|  ≥ 8 |  1 |  *t* |

 (*a*) Find values for *r*, *s* and *t*.

**(4)**

 (*b*) Using a 5% significance level, test whether or not this Poisson model is suitable. State your hypotheses clearly.

**(7)**

An alternative test might have been to estimate the population mean by using the data given.

(*c*) Explain how this would have affected the test.

**(2)**

**3.** Rolls of cloth delivered to a factory contain defects at an average rate of *λ* per metre. A quality assurance manager selects a random sample of 15 metres of cloth from each delivery to test whether or not there is evidence that *λ* > 0.3. The criterion that the manager uses for rejecting the hypothesis that *λ* = 0.3 is that there are 9 or more defects in the sample.

 (*a*) Find the size of the test.

 **(2)**

 Table 1 gives some values, to 2 decimal places, of the power function of this test.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *λ* | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| Power | 0.15 | 0.34 | *r* | 0.72 | 0.85 | 0.92 | 0.96 |

Table 1

 (*b*) Find the value of *r*.

**(2)**

 The manager would like to design a test, of whether or not *λ* > 0.3, that uses a smaller length of cloth. He chooses a length of 10 m and requires the probability of a type I error to be less than 10%.

 (*c*) Find the criterion to reject the hypothesis that *λ* = 0.3 which makes the test as powerful as possible.

**(2)**

 (*d*) Hence state the size of this second test.

**(1)**

 Table 2 gives some values, to 2 decimal places, of the power function for the test in part (*c*).

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *λ* | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| Power | 0.21 | 0.38 | 0.55 | 0.70 | *s* | 0.88 | 0.93 |

Table 2

 (*e*) Find the value of *s*.

**(2)**

(*f*) Using the same axes, on graph paper draw the graphs of the power functions of these two tests.

**(4)**

(*g*) (i) State the value of *λ* where the graphs cross.

 (ii) Explain the significance of *λ* being greater than this value.

**(2)**

The cost of wrongly rejecting a delivery of cloth with *λ* = 0.3 is low. Deliveries of cloth with *λ*> 0.7 are unusual.

(*h*) Suggest, giving your reasons, which the test manager should adopt.

**(2)**

1. The probability that Jon wins a coconut in a game at the fair is 0.15.

Jon plays a number of games. Find

 (*a*) the probability of Jon winning his second coconut on his 7th game

**(2)**

 (*b*) the expected number of games Jon would need to play in order to win 3 coconuts.

**(1)**

 *(c)* State two assumptions that you made in part (a).

**(2)**

Sue plays the same game, but has a different probability of winning a coconut. She plays until she has won r coconuts. The random variable G represents the total number of games Sue plays.

 (*d*) Given that the mean and standard deviation of G are 18 and 6 respectively, determine whether Jon or Sue has the greater probability of winning a coconut in a game.

**(5)**

1. The probability generating function of the random variable X is given by

$$G\_{x}\left(t\right)=k(1+2t+2t^{2})^{2}.$$

 (*a*) Show that $k=\frac{1}{25}$

**(2)**

 *(b)* Find $P(X=2)$

**(2)**

 (*c*) Calculate $E(X) $and $Var(X)$

**(8)**

 (*d*) Write down the probability generating function of $2X+1$

**(8)**

**6.** A brick of mass 3 kg slides in a straight line on a horizontal floor. The brick is modelled as a particle and the floor as a rough plane. The initial speed of the brick is 8 m s–1. The brick is brought to rest after moving 12 m by the constant frictional force between the brick and the floor.

 (*a*) Calculate the kinetic energy lost by the brick in coming to rest, stating the units of your answer.

**(2)**

 (*b*) Calculate the coefficient of friction between the brick and the floor.

**(4)**

**7.** A particle *P* of mass 0.4 kg is moving so that its position vector **r** metres at time *t* seconds is given by

**r** = (*t*2 + 4*t*)**i** + (3*t* – *t*3)**j**.

 (*a*) Calculate the speed of *P* when *t* = 3.

**(5)**

 When *t* = 3, the particle *P* is given an impulse (8**i** – 12**j**) N s.

 (*b*) Find the velocity of *P* immediately after the impulse.

 **(3)**

**8.** A car of mass 1000 kg is moving along a straight horizontal road. The resistance to motion is modelled as a constant force of magnitude *R* newtons. The engine of the car is working at a rate of 12 kW. When the car is moving with speed 15 m s–1, the acceleration of the car is 0.2 m s–2.

 (*a*) Show that *R* = 600.

**(4)**

 The car now moves with constant speed *U* m s–1 downhill on a straight road inclined at *θ* to the horizontal, where sin *θ* = . The engine of the car is now working at a rate of 7 kW. The resistance to motion from non-gravitational forces remains of magnitude *R* newtons.

 (*c*) Calculate the value of *U*.

**(5)**

**9.** A particle *A* of mass 2*m* is moving with speed 3*u* in a straight line on a smooth horizontal table. The particle collides directly with a particle *B* of mass *m* moving with speed 2*u* in the opposite direction to *A*. Immediately after the collision the speed of *B* is *u* and the direction of motion of *B* is reversed.

 (*a*) Calculate the coefficient of restitution between *A* and *B*.

**(6)**

(*b*) Show that the kinetic energy lost in the collision is 7*mu*2.

 **(3)**

After the collision *B* strikes a fixed vertical wall that is perpendicular to the direction of motion of *B*. The magnitude of the impulse of the wall on *B* is *mu*.

(*c*) Calculate the coefficient of restitution between *B* and the wall.

**(4)**

**10. Figure 1**

60°

 *F* N

 *A*

 *P*

 A particle *P* of mass 0.8 kg is attached to one end of a light elastic string, of natural length
1.2 m and modulus of elasticity 24 N. The other end of the string is attached to a fixed point *A*. A horizontal force of magnitude *F* newtons is applied to *P*. The particle *P* in in equilibrium with the string making an angle 60° with the downward vertical, as shown in Figure 1.

 Calculate

 (*a*) the value of *F*,

**(3)**

 (*b*) the extension of the string,

**(3)**

 (*c*) the elasticity stored in the string.

**(2)**

**11.** A particle *P* of mass 0.5 kg is released from rest at time *t* = 0 and falls vertically through a liquid. The motion of *P* is resisted by a force of magnitude 2*v*N, where *v* m s–1 is the speed of *v* at time *t* seconds.

 (*a*) Show that 5 = 49 – 20*v*.

**(2)**

 (*b*) Find the speed of *P* when *t* = 1.

**(5)**

**12.** A particle *P* of mass *m* is suspended from a fixed point by a light elastic spring. The spring has natural length *a* and modulus of elasticity 2*mω*2*a*, where *ω* is a positive constant. At time *t* = 0 the particle is projected vertically downwards with speed *U* from its equilibrium position. The motion of the particle is resisted by a force of magnitude 2*mω v*, where *v* is the speed of the particle. At time *t*, the displacement of *P* downwards from its equilibrium position is *x*.

 (*a*) Show that  + 2*ω* + 2*ω*2*x* = 0.

**(5)**

 Given that the solution of this differential equation is *x* = e–*ω t*(*A* cos *ω t* + *B* sin *ω t*), where *A* and *B* are constants,

(*b*) find *A* and *B*.

**(4)**

(*c*) Find an expression for the time at which *P* first comes to rest.

 **(3)**

**13.** Two smooth uniform spheres *A* and *B* have equal radii. Sphere *A* has mass *m* and sphere *B* has mass *km*. The spheres are at rest on a smooth horizontal table. Sphere *A* is then projected along the table with speed *u* and collides with *B*. Immediately before the collision, the direction of motion of *A* makes an angle of 60° with the line joining the centres of the two spheres. The coefficient of restitution between the spheres is .

 (*a*) Show that the speed of *B* immediately after the collision is .

**(6)**

 Immediately after the collision the direction of motion of *A* makes an angle arctan (2√3) with the direction of motion of *B*.

 (*b*) Show that *k* = .

**(6)**

 (*c*) Find the loss of kinetic energy due to the collision.

 **(4)**