**Composite Applied paper June 2005**

**1.** Over a long periodof time, accidents happened on a stretch of road at random at a rate of 3 per month.

Find the probability that

(*a*) in a randomly chosen month, more than 4 accidents occurred,

**(3)**

(*b*) in a three-month period, more than 4 accidents occurred.

**(2)**

At a later date, a speed restriction was introduced on this stretch of road. During a randomly chosen month only one accident occurred.

(*c*) Test, at the 5% level of significance, whether or not there is evidence to support the claim that this speed restriction reduced the mean number of road accidents occurring per month.

**(4)**

The speed restriction was kept on this road. Over a two-year period, 55 accidents occurred.

(*d*) Test, at the 5% level of significance, whether or not there is now evidence that this speed restriction reduced the mean number of road accidents occurring per month.

**(7) JAN 05 S2 Q6**

**2.** A sample of size 5 is taken from a population that is normally distributed with mean 10 and standard deviation 3. Find the probability that the sample mean lies between 7 and 10.

**(Total 6 marks) S3 Q2**

**3.** A researcher carried out a survey of three treatments for a fruit tree disease. The contingency table below shows the results of a survey of a random sample of 60 diseased trees.

|  |  |  |  |
| --- | --- | --- | --- |
|  | No action | Remove diseased branches | Spray with chemicals |
| Tree died  within 1 year | 10 | 5 | 6 |
| Tree survived  for 1–4 years | 5 | 9 | 7 |
| Tree survived  beyond 4 years | 5 | 6 | 7 |

Test, at the 5% level of significance, whether or not there is any association between the treatment of the trees and their survival. State your hypotheses and conclusion clearly.

**(Total 11 marks) S3 Q3**

**4.** The number of times per day a computer fails and has to be restarted is recorded for 200 days. The results are summarised in the table.

|  |  |
| --- | --- |
| Number of restarts | Frequency |
| 0 | 99 |
| 1 | 65 |
| 2 | 22 |
| 3 | 12 |
| 4 | 2 |

Test whether or not a Poisson model is suitable to represent the number of restarts per day. Use a 5% level of significance and state your hypothesis clearly.

**(Total 12 marks) S3 Q5**

**5.** Define

(*a*) a Type I error,

**(1)**

(*b*) the size of a test.

**(1)**

Jane claims that she can read Alan’s mind. To test this claim Alan randomly chooses a card with one of 4 symbols on it. He then concentrates on the symbol. Jane then attempts to read Alan’s mind by stating what symbol she thinks is on the card. The experiment is carried out 8 times and the number of times *X* that Jane is correct is recorded.

The probability of Jane stating the correct symbol is denoted by *p*.

To test the hypothesis H0: *p* = 0.25 against H1: *p* > 0.25, a critical region of *X* > 6 is used.

(*c*) Find the size of this test.

**(3)**

(*d*) Show that the power function of this test is 8*p*7 – 7*p*8.

**(3)**

Given that *p* = 0.3, calculate

(*e*) the power of this test,

**(1)**

(*f*) the probability of a Type II error.

**(2)**

(*g*) Suggest two ways in which you might reduce the probability of a Type II error.

**(2)**

**(Total 12 marks) S4 Q5**

**6.** The random variable *X* has probability generating function

G*X* (*t*) = *k*[*t* 3(2 + 3*t*) + (1 + *t*)4],

where *k* is a positive constant.

(*a*) Show that *k* = .

**(2)**

Find

(*b*) E(*X*),

**(3)**

(*c*) Var(*X*),

**(4)**

(*d*) P(*X* = 3).

**(2)**

**(Total 11 marks) S5 Q3**

**7.** A child is repeatedly twisting a coloured spinner which has a probability 0.4 of landing on red. After each twist the child records whether or not the spinner lands on red.

(*a*) Show that the probability that the spinner lands on red for the first time occurs on or before the 7th twist is 0.972, to 3 decimal places.

**(3)**

Find the probability that

(*b*) exactly three reds occur during the first 7 twists,

**(2)**

(*c*) the 3rd red occurs on the 7th twist,

**(3)**

(*d*) the 3rd red occurs on or before the 7th twist.

**(4)**

On another occasion there are 3 children *A*, *B* and *C* playing with the spinner. The children take turns to twist the spinner. Child *A* starts, then *B*, then *C*, then *A* again and so on. The winner is the first child to have the spinner land on red.

(*e*) Find the probability that *A* wins.

**(3)**

Given that the first red occurs on or before the 7th twist,

(*f* ) find the probability that *A* wins.

**(2)**

**(Total 17 marks) S5 Q6**

**8.** A car of mass 1200 kg moves along a straight horizontal road. The resistance to motion of the car from non-gravitational forces is of constant magnitude 600 N. The car moves with constant speed and the engine of the car is working at a rate of 21 kW.

(*a*) Find the speed of the car.

**(3)**

The car moves up a hill inclined at an angle *α* to the horizontal, where sin *α* = .

The car’s engine continues to work at 21 kW and the resistance to motion from non-gravitational forces remains of magnitude 600 N.

(*b*) Find the constant speed at which the car can move up the hill.

**(4) M2 Q1**

**9.** Two small spheres *A* and *B*have mass 3*m* and 2*m* respectively. They are moving towards each other in opposite directions on a smooth horizontal plane, both with speed 2*u*, when they collide directly. As a result of the collision, the direction of motion of *B* is reversed and its speed is unchanged.

(*a*) Find the coefficient of restitution between the spheres.

**(7)**

Subsequently, *B* collides directly with another small sphere *C* of mass 5*m* which is at rest. The coefficient of restitution between *B* and *C* is .

(*b*) Show that, after *B* collides with *C*, there will be no further collisions between the spheres.

**(7) M2 Q5**

**10.** At a demolition site, bricks slide down a straight chute into a container. The chute is rough and is inclined at an angle of 30° to the horizontal. The distance travelled down the chute by each brick is 8 m. A brick of mass 3 kg is released from rest at the top of the chute. When it reaches the bottom of the chute, its speed is 5 m s–1.

(*a*) Find the potential energy lost by the brick in moving down the chute.

**(2)**

(*b*) By using the work-energy principle, or otherwise, find the constant frictional force acting on the brick as it moves down the chute.

**(5)**

(*c*) Hence find the coefficient of friction between the brick and the chute.

**(3)**

Another brick of mass 3 kg slides down the chute. This brick is given an initial speed of 2 m s–1 at the top of the chute.

(*d*) Find the speed of this brick when it reaches the bottom of the chute.

**(5) M2 Q7**

**11. Figure 1**

*α*

1.6 m

*O*

A particle of mass 0.8 kg is attached to one end of a light elastic spring, of natural length 2 m and modulus of elasticity 20 N. The other end of the spring is attached to a fixed point *O* on a smooth plane which is inclined at an angle *α* to the horizontal, where tan *α* = . The particle is held on the plane at a point which is 1.6 m down a line of greatest slope of the plane from *O*, as shown in Figure 1. The particle is then released from rest.

Find the initial acceleration of the particle.

**(Total 6 marks) M3 Q1**

**12.** A small smooth ball of mass kg is falling vertically. The ball strikes a smooth plane which is inclined at an angle *α* to the horizontal, where tan *α* = . Immediately before striking the plane the ball has speed 10 m s–1. The coefficient of restitution between ball and plane is . Find

(*a*) the speed, to 3 significant figures, of the ball immediately after the impact,

**(5)**

(*b*) the magnitude of the impulse received by the ball as it strikes the plane.

**(2) M4 Q1**

**13. Figure 1**

*Q*

*α*

*β*

*P*

A smooth sphere *P* lies at rest on a smooth horizontal plane. A second identical sphere *Q*, moving on the plane, collides with the sphere *P*. Immediately before the collision the direction of motion of *Q* makes an angle *α* with the line joining the centres of the spheres. Immediately after the collision the direction of motion of *Q* makes an angle *β* with the line joining the centres of spheres, as shown in Figure 1. The coefficient of restitution between the spheres is *e*.

Show that (1 – *e*) tan *β* = 2 tan *α*.

**(11) M4 Q3**

**14.** A light elastic string, of natural length *a* and modulus of elasticity 5*maω* 2, lies unstretched along a straight line on a smooth horizontal plane. A particle of mass *m* is attached to one end of the spring. At time *t* = 0, the other end of the spring starts to move with constant speed *U* along the line of the spring and away from the particle. As the particle moves along the plane it is subject to a resistance of magnitude 2*mω v*, where *v* is its speed. At time *t*, the extension of the spring is *x*. Given that

 + 2*ω* + 5*ω* 2 *x* = 2*ω U,*

find *x* in terms of *ω*, *U* and *t*.

**(8) M4 Q7 edited**