**Composite Applied Paper June 2016**

**1.** A car of mass 800 kg is moving on a straight road which is inclined at an angle *θ* to the horizontal, where sin *θ* = . The resistance to the motion of the car from non-gravitational forces is modelled as a constant force of magnitude *R* newtons. When the car is moving up the road at a constant speed of 12.5 m s−1, the engine of the car is working at a constant rate of 3*P* watts. When the car is moving down the road at a constant speed of 12.5 m s−1, the engine of the car is working at a constant rate of *P* watts.

(*a*)Find

 (i) the value of *P*,

 (ii) the value of *R*.

**(6)**

When the car is moving up the road at 12.5 m s−1 the engine is switched off and the car comes to rest, without braking, in a distance *d* metres. The resistance to the motion of the car from non-gravitational forces is still modelled as a constant force of magnitude *R* newtons.

(*b*)Use the work-energy principle to find the value of *d*.

**(4)**

**(Total 10 marks)**

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**2.** A particle of mass 0.6 kg is moving with constant velocity (*c***i** + 2*c***j**) m s−1, where *c* is a positive constant. The particle receives an impulse of magnitude 2N s.

Immediately after receiving the impulse the particle has velocity (2*c***i** – *c***j**) m s−1.

Find the value of *c*.

**(6)**

**(Total 6 marks)**

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**3.** Two particles *A* and *B*, of mass 2*m* and 3*m* respectively, are initially at rest on a smooth horizontal surface. Particle *A* is projected with speed 3*u* towards *B*. Particle *A* collides directly with particle *B*. The coefficient of restitution between *A* and *B* is .

(*a*)Find

 (i) the speed of *A* immediately after the collision,

 (ii) the speed of *B* immediately after the collision.

**(7)**

After the collision *B* hits a fixed smooth vertical wall and rebounds. The wall is perpendicular to the direction of motion of *B*. The coefficient of restitution between *B* and the wall is *e*. The magnitude of the impulse received by *B* when it hits the wall is *mu*.

(*b*)Find the value of *e*.

**(3)**

(*c*)Determine whether there is a further collision between *A* and *B* after *B* rebounds from the wall.

**(2)**

**(Total 12 marks)**

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**4.** One end of a light elastic string, of natural length 1.5m and modulus of elasticity 14.7 N, is attached to a fixed point *O* on a ceiling. A particle *P* of mass 0.6 kg is attached to the free end of the string. The particle is held at *O* and released from rest. The particle comes to instantaneous rest for the first time at the point *A*.

Find

(*a*)the distance *OA*,

**(6)**

(*b*)the magnitude of the instantaneous acceleration of *P* at *A*.

**(3)**

**(Total 9 marks)**

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**5.**



A smooth uniform sphere *A* of mass *m* is moving on a smooth horizontal plane when it collides with a second smooth uniform sphere *B*, which is at rest on the plane. The sphere *B* has mass 4*m* and the same radius as *A*. Immediately before the collision the direction of motion of *A* makes an angle *α* with the line of centres of the spheres, as shown in Figure 1. The direction of motion of *A* is turned through an angle of 90° by the collision and the coefficient of restitution between the spheres is  .

Find the value of tan *α.*

 **(Total 8 marks)**

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**6.**



A small spherical ball *P* is at rest at the point *A* on a smooth horizontal floor. The ball is struck and travels along the floor until it hits a fixed smooth vertical wall at the point *X*. The angle between *AX* and this wall is *α*, where *α* is acute. A second fixed smooth vertical wall is perpendicular to the first wall and meets it in a vertical line through the point *C* on the floor. The ball rebounds from the first wall and hits the second wall at the point *Y*. After *P* rebounds from the second wall, *P* is travelling in a direction parallel to *XA*, as shown in Figure 2. The coefficient of restitution between the ball and the first wall is *e*. The coefficient of restitution between the ball and the second wall is *ke*.

Find the value of *k*.

 **(Total 9 marks)**

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**7.** A student is investigating the numbers of cherries in a *Rays* fruit cake. A random sample of *Rays* fruit cakes is taken and the results are shown in the table below.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Number of cherries | 0 | 1 | 2 | 3 | 4 | 5 | ≥ 6 |
| Frequency | 24 | 37 | 21 | 12 | 4 | 2 | 0 |

(*a*)Calculate the mean and the variance of these data.

**(3)**

(*b*)Explain why the results in part (*a*)suggest that a Poisson distribution may be a suitable model for the number of cherries in a *Rays* fruit cake.

**(1)**

The number of cherries in a *Rays* fruit cake follows a Poisson distribution with mean 1.5.

A *Rays* fruit cake is to be selected at random.

Find the probability that it contains

(*c*)(i) exactly 2 cherries,

 (ii) at least 1 cherry.

**(4)**

*Rays* fruit cakes are sold in packets of 5.

(*d*)Show that the probability that there are more than 10 cherries, in total, in a randomly selected packet of *Rays* fruit cakes, is 0.1378 correct to 4 decimal places.

**(3)**

Twelve packets of *Rays* fruit cakes are selected at random.

(*e*)Find the probability that exactly 3 packets contain more than 10 cherries.

**(3)**

**(Total 14 marks)**

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**8.** A new drug to vaccinate against influenza was given to 110 randomly chosen volunteers. The volunteers were given the drug in one of 3 different concentrations, *A*, *B* and *C*, and then were monitored to see if they caught influenza. The results are shown in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | *A* | *B* | *C* |
| Influenza | 12 | 29 | 9 |
| No influenza | 15 | 23 | 22 |

Test, at the 10% level of significance, whether or not there is an association between catching influenza and the concentration of the new drug. State your hypotheses and show your working clearly. You should state your expected frequencies to 2 decimal places.

 **(Total 10 marks)**

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**9.** An airport manager carries out a survey of families and their luggage. Each family is allowed to check in a maximum of 4 suitcases. She observes 50 families at the check-in desk and counts the total number of suitcases each family checks in. The data are summarised in the table below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Number of suitcases | 0 | 1 | 2 | 3 | 4 |
| Frequency | 6 | 25 | 12 | 6 | 1 |

The manager claims that the data can be modelled by a binomial distribution with *p* = 0.3.

(*a*)Test the manager’s claim at the 5% level of significance. State your hypotheses clearly.

 Show your working clearly and give your expected frequencies to 2 decimal places.

**(8)**

The manager also carries out a survey of the time taken by passengers to check in. She records the number of passengers that check in during each of 100 five-minute intervals.

The manager makes a new claim that these data can be modelled by a Poisson distribution. She calculates the expected frequencies given in the table below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Number of passengers | 0 | 1 | 2 | 3 | 4 | 5 or more |
| Observed frequency | 5 | 40 | 31 | 18 | 6 | 0 |
| Expected frequency | 16.53 | 29.75 | *r* | *s* | 7.23 | 3.64 |

(*b*)Find the value of *r* and the value of *s* giving your answers to 2 decimal places.

**(3)**

(*c*)Stating your hypotheses clearly, use a 1% level of significance to test the manager’s new claim.

**(6)**

**(Total 17 marks)**

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**10.** A jar contains a large number of sweets which have either soft centres or hard centres.

The jar is thought to contain equal proportions of sweets with soft centres and sweets with hard centres. A random sample of 20 sweets is taken from the jar and the number of sweets with hard centres is recorded.

(*a*)Using a 5% level of significance, find the critical region for a two-tailed test of the hypothesis that there are equal proportions of sweets with soft centres and sweets with hard centres in the jar.

**(2)**

(*b*)Calculate the probability of a Type I error for this test.

**(2)**

Given that there are 3 times as many sweets with soft centres as there are sweets with hard centres,

(*c*)calculate the probability of a Type II error for this test.

**(2)**

 **(Total 6 marks)**

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**11.** A manufacturer produces boxes of screws containing short screws and long screws. The manufacturer claims that the probability, *p*, of a randomly selected screw being long, is 0.5.

A shopkeeper does not believe the manufacturer’s claim. He designs two tests, *A* and *B*, to test the hypotheses H0 : *p* = 0.5 and H1 : *p* < 0.5.

In test *A*, a random sample of 10 screws is taken from a box of screws and H0 is rejected if there are fewer than 3 long screws.

In test *B*, a random sample of 5 screws is taken from a box of screws and H0 is rejected if there are no long screws, otherwise a second random sample of 5 screws is taken from a box of screws. If there are no long screws in this second sample H0 is rejected, otherwise it is accepted.

(*a*)Find the size of test *A.*

**(1)**

(*b*)Find the size of test *B*.

**(3)**

(*c*)Find an expression for the power function of test *B* in terms of *p*.

**(2)**

Some values, to 2 decimal places, of the power function for test *A* and the power function for test *B* are given in the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *p* | 0.1 | 0.2 | 0.3 | 0.4 |
| Power test *A* | 0.93 | *r* | 0.38 | 0.17 |
| Power test *B* | 0.83 | 0.55 | 0.31 | 0.15 |

(*d*)Find the value of *r*.

**(1)**

The shopkeeper believes that the value of *p* is less than 0.4

(*e*)Suggest which of the tests the shopkeeper should use. Give a reason for your answer.

**(2)**

**(Total 9 marks)**

**TOTAL 110 marks**