**Composite Paper - Mock Applied papers**

**1.**

**1.** A smoothsphere is moving with speed *U* in a straight line on a smooth horizontal plane. It strikes a fixed smooth vertical wall at right angles. The coefficient of restitution between the sphere and the wall is .

 Find the fraction of the kinetic energy of the sphere that is lost as a result of the impact.

 **(5 marks)**

**2.** The resistance to the motion of a cyclist is modelled as *kv*2 N, where *k* is a constant and *v*m s−1 is the speed of the cyclist. The total mass of the cyclist and his bicycle is 100 kg. The cyclist freewheels down a slope inclined at an angle *α* to the horizontal, where sin *α* = , at a constant speed of 3.5 m s−1.

(*a*) Show that *k* = 4. **(3 marks)**

The cyclist ascends a slope inclined at an angle *β* to the horizontal, where sin *β* = , at a constant speed of 2 m s−1.

(*b*) Find the rate at which the cyclist is working. **(6 marks)**

**3.**

A smooth sphere *S* of mass *m* is moving with speed *u* on a smooth horizontal plane. The sphere *S* collides with another smooth sphere *T*, of equal radius to *S* but of mass *km*, moving in the same straight line and in the same direction with speed *λu*, 0 < *λ* < . The coefficient of restitution between *S* and *T* is *e*.

Given that *S* is brought to rest by the impact,

(*a*) show that . **(6 marks)**

(*b*) Deduce that *k* > 1. **(3 marks)**

**4.** At time *t* seconds the acceleration, **a** m s−2, of a particle *P* relative to a fixed origin *O*, is given by **a** = 2**i** + 6*t***j**. Initially the velocity of *P* is (2**i** – 4**j**) m s−1.

(*a*) Find the velocity of *P* at time *t* seconds. **(3 marks)**

At time *t* = 2 seconds the particle *P* is given an impulse (3**i** – 1.5**j**) Ns. Given that the particle *P* has mass 0.5 kg,

(*b*) find the speed of *P* immediately after the impulse has been applied. **(6 marks)**

**5.** A particle *P* is projected up a line of greatest slope of a rough plane which is inclined at an angle *α* to the horizontal, where tan *α* = . The coefficient of friction between *P* and the plane is . The particle is projected from the point *O* with a speed of 10 m s−1 and comes to instantaneous rest at the point *A*.

By Using the Work-Energy principle, or otherwise,

(*a*) find, to 3 significant figures, the length *OA*. **(7 marks)**

(*b*) Show that *P* will slide back down the plane. **(3 marks)**

(*c*) Find, to 3 significant figures, the speed of *P* when it returns to *O*. **(5 marks)**

**6.** A light elastic string *AB* has one end *A* attached to a fixed point on a ceiling. A particle *P* of mass 0.3 kg is attached to *B*. When *P* hangs in equilibrium with *AB* vertical, *AB* = 100 cm. The particle *P* is replaced by another particle *Q* of mass 0.5 kg. When *Q* hangs in equilibrium with *AB* vertical, *AB* = 110 cm. Find

 (*a*) the natural length of the string, **(5 marks)**

 (*b*) the modulus of elasticity of the string. **(2 marks)**

**7.** In a “test your strength” game at an amusement park, competitors hit one end of a small lever with a hammer, causing the other end of the lever to strike a ball which then moves in a vertical tube whose total height is adjustable. The ball is attached to one end of an elastic spring of natural length 3 m and modulus of elasticity 120 N. The mass of the ball is 2 kg. The other end of the spring is attached to the top of the tube. The ball is modelled as a particle, the spring as light and the tube is assumed to be smooth.

 The height of the tube is first set at 3 m. A competitor gives the ball an initial speed of 10 m s−1.

 (*a*) Find the height to which the ball rises before coming to rest. **(6 marks)**

 The tube is now adjusted by reducing its height to 2.5 m. The spring and the ball remain unchanged.

(*b*) Find the initial speed which the ball must now have if it is to rise by the same distance as in part (*a*). **(5 marks)**

**8.** A smooth sphere *S* is moving on a smooth horizontal plane with speed *u* when it collides with a smooth fixed vertical wall. At the instant of collision the direction of motion of *S* makes an angle of 30° with the wall. The coefficient of restitution between *S* and the wall is.

Find the speed of *S* immediately after the collision. **(6 marks)**

**9.** **Figure 1**

 *A B*

 30°

A smooth uniform sphere *A,* moving on a smooth horizontal table, collides with a second identical sphere *B* which is at rest on the table. When the spheres collide the line joining their centres makes an angle of 30° with the direction of motion of *A,* as shown in Fig. 1. The coefficient of restitution between the spheres is *e.* The direction of motion of *A* is deflected through an angle *θ* by the collision.

Show that .**(10 marks)**

**10.** On a typical weekday morning customers arrive at a village post office independently and at a rate of 3 per 10 minute period.

 Find the probability that

 (*a*) at least 4 customers arrive in the next 10 minutes, **(2 marks)**

 (*b*) no more than 7 customers arrive between 11.00 a.m. and 11.30 a.m. **(3 marks)**

 The period from 11.00 a.m. to 11.30 a.m. next Tuesday morning will be divided into 6 periods of 5 minutes each.

 (*c*) Find the probability that no customers arrive in at most one of these periods. **(6 marks)**

 The post office is open for 3 hours on Wednesday mornings.

 (*d*) Using a suitable approximation, estimate the probability that more than 49 customers arrive at the post office next Wednesday morning. **(7 marks)**

1. A random sample *X*1 , *X*2 , . . . , *X*10 is taken from a normal population with mean 100 and standard deviation 14.

**11.**

(*a*) Write down the distribution of *,* the mean of this sample. **(2 marks)**

(*b*) Find P(⏐ − 100⏐ > 5). **(3 marks)**

**12.** A random sample of the invoices, for books purchased by the customers of a large bookshop, was classified by book cover (hardback, paperback) and type of book (novel, textbook, general interest). As part of the analysis of these invoices, an approximate χ2 statistic was calculated and found to be 11.09.

Assuming that there was no need to amalgamate any of the classifications, carry out an appropriate test to determine whether or not there was any association between book cover and type of book. State your hypotheses clearly and use a 5% level of significance.

**(6 marks)**

**13.**

Breakdowns on a certain stretch of motorway were recorded each day for 80 consecutive days. The results are summarised in the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Number of breakdowns | 0 | 1 | 2 | > 2 |
| Frequency | 38 | 32 | 10 | 0 |

It is suggested that the number of breakdowns per day can be modelled by a Poisson distribution.

Using a 5% level of significance, test whether or not the Poisson distribution is a suitable model for these data. State your hypotheses clearly. **(13 marks)**

**14.** The random variable *R* is defined as *R = X +* 4*Y* where *X* ~ N(8, 22 ),  *Y* ~N(14, 32 ) and *X*and *Y* are independent.

Find

(*a*) E(*R*), **(2 marks)**

(*b*) Var (*R*), **(3 marks)**

(*c*) P(*R* < 41) **(3 marks)**

The random variables *Y*1 , *Y*2 and *Y*3 are independent and each has the same distribution as *Y*.The random variable *S* is defined as

*S* = .

(*d*) Find Var (*S*).  **(4 marks)**

**15.** A certain vaccine is known to be only 70% effective against a particular virus; thus 30% of those vaccinated will actually catch the virus. In order to test whether or not a new and more expensive vaccine provides better protection against the same virus, a random sample of 30 people were chosen and given the new vaccine. If fewer than 6 people contracted the virus the new vaccine would be considered more effective than the current one.

(*a*) Write down suitable hypotheses for this test. **(1 mark)**

(*b*)Find the probability of making a Type I error. **(2 marks)**

(*c*) Find the power of this test if the new vaccine is

 (i) 80% effective,

 (ii) 90% effective. **(3 marks)**

An independent research organisation decided to test the new vaccine on a random sample of 50 people to see if it could be considered more than 70% effective. They required the probability of a Type I error to be as close as possible to 0.05.

(*d*)Find the critical region for this test. **(2 marks)**

(*e*) State the size of this critical region. **(1 mark)**

(*f*) Find the power of this test if the new vaccine is

 (i) 80% effective,

 (ii) 90% effective. **(2 marks)**

(*g*) Give one advantage and one disadvantage of the second test. **(2 marks)**

**16.** A biscuit manufacturer has placed a small token inside 3% of the packets of special chocolate biscuits. Jasmine bought 10 packets of these special chocolate biscuits.

(*a*) Find the probability that Jasmine obtains at least 1 token. **(2 marks)**

Graham decides to buy packets of these special chocolate biscuits one at a time and check them for tokens.

(*b*) Find the probability that he obtains his first token with the fourth packet. **(2 marks)**

When 5 tokens are collected a prize can be claimed.

(*c*) Find the probability that Graham collects enough tokens to claim a prize, with the 50th packet. **(3 marks)**

(*d*) Write down the expected number of packets that Graham will have to buy in order to be able to claim a prize.  **(1 mark)**

Given that Graham obtained his fourth token in his 135th packet,

(*e*) find the probability that he will be able to claim a prize before buying his 150th packet.

**(3 marks)**

**17.** The random variable *Y* has probability generating function *GY* (*t*) given by

*GY* (*t*) = (2 + *t*)5(2*t* + 1)5.

(*a*)Find E(*Y* ).  **(3 marks)**

The random variable *X* has a binomial distribution with *n* = 5 and *p* = .

(*b*) Show that the probability generating function of the random variable *W* *=* 5 *– X* is

*GW* (*t*) =   **(5 marks)**

A fair six-sided die is rolled 5 times and the number of times it lands on either a 1 or a 6 is recorded. The experiment is repeated and the random variables *S*1 and *S*2 represent the two outcomes.

(*c*) Show that 5 + *S*1 − *S*2 *= Y.* **(5 marks)**