**Composite paper June 2017 (AS Version)**

**1.**

** **

(*a*)Find **A**–1

**(2)**

The transformation represented by the matrix **B** followed by the transformation represented

by the matrix **A** is equivalent to the transformation represented by the matrix **P**.

(*b*)Find **B**, giving your answer in its simplest form.

**(3)**

**(Total 5 marks)**

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**2.** (i) The complex number *w* is given by

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 where *p* is a real constant.

 (*a*)Express *w* in the form *a* + *b*i, where *a* and *b* are real constants.

Give your answer in its simplest form in terms of *p*.

**(3)**

Given that arg *w* = 

 (*b*)find the value of *p*.

**(2)**

(ii) The complex number *z* is given by

*z* = (1 – 𝜆 i)(4 + 3i)

 where 𝜆is a real constant.

 Given that

|*z*|= 45

 find the possible values of 𝜆.

 Give your answers as exact values in their simplest form.

**(3)**

**(Total 8 marks)**

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**3.** (i)

** **

 where *p* is a constant.

 (*a*)Find, in terms of *p*, the matrix **AB**

**(2)**

 Given that

**AB +** 2**A =** *k***I**

 where *k* is a constant and **I** is the 2 × 2 identity matrix,

 (*b*)find the value of *p* and the value of *k*.

**(4)**

(ii)

, where *a* is a real constant

 Triangle *T* has an area of 15 square units.

 Triangle *T* is transformed to the triangle *T'* by the transformation represented by the

 matrix **M**.

 Given that the area of triangle *T'* is 270 square units, find the possible values

 of *a*.

**(5)**

**(Total 11 marks)**

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**4.** Given that 4 and 2i – 3 are roots of the equation

*x*3 + *ax*2 + *bx* – 52 = 0

where *a* and *b* are real constants,

(*a*)write down the third root of the equation,

**(1)**

(*b*)find the value of *a* and the value of *b*.

**(5)**

**(Total 6 marks)**

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**5.** (*a*)Use the standard results for ** and  to show that



 for all positive integers *n*.

**(5)**

Given that



(*b*)find the exact value of the constant *k*.

**(4)**

**(Total 9 marks)**

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**6.** (i) A sequence of numbers is defined by

*u*1 = 6, *u*2 = 27

*un*+2 = 6*un*+1 – 9*un n* ⩾ 1

 Prove by induction that, for *n* ∈ ℤ+

*un* = 3*n*(*n* + 1)

**(6)**

(ii) Prove by induction that, for *n* ∈ ℤ+

f (*n*) = 33*n*–2 + 23*n*+1 is divisible by 19

**(6)**

**(Total 12 marks)**

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**7.** (*a*)Show that, for *r* > 0

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**(1)**

(*b*)Hence prove that, for *n* ∈ ℕ



**(3)**

(*c*)Show that, for *n* ∈ ℕ, *n* > 1



 where *a*, *b* and *c* are constants to be found.

**(3)**

**(Total 7 marks)**

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**8.** The plane ** has equation *x* – 2*y* – 3*z* = 5 and the plane ** has equation 6*x* + *y* – 4*z* = 7

(*a*)Find, to the nearest degree, the acute angle between ** and **

**(3)**

The point *P* has coordinates (2, 3, –1). The line *l* is perpendicular to ** and passes through

the point *P*. The line *l* intersects ** at the point *Q*.

(*b*)Find the coordinates of *Q*.

**(4)**

The plane ** passes through the point *Q* and is perpendicular to ** and **

(*c*)Find an equation of the plane ** in the form **r.n** = *p*

**(4)**

**(Total 11 marks)**

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