**Composite Core Paper June 2013**

**1.**

**M** = 

Given that the matrix **M** is singular, find the possible values of *x*.

**(4)**

**2.** Given that *x* =  is a root of the equation

2*x*3 – 9*x*2 + *kx* – 13 = 0, 

find

(*a*) the value of *k*,

**(3)**

(*b*) the other 2 roots of the equation.

**(4)**

**3*.***(*a*) Use the standard results for  and  to show that



for all positive integers *n*.

**(6)**

(*b*) Hence show that



where *a*, *b* and *c* are integers to be found.

**(4)**

**4.** *z*1 = 2 + 3i, *z*2 = 3 + 2i, *z*3 = *a + b*i, *a*, *b* **

(*a*) Find the exact value of |*z*1 + *z*2|.

**(2)**

Given that *w* = ,

(*b*)find *w* in terms of *a* and *b*, giving your answer in the form *x +* i*y*, *x*, *y* **.

**(4)**

Given also that *w* = ,

(*c*) find the value of *a* and the value of *b*,

**(3)**

(*d*) find arg *w*, giving your answer in radians to 3 decimal places.

**(2)**

**5.**

**A** = 

and **I** is the 2 × 2 identity matrix.

(*a*) Prove that

**A**2 = 7**A** + 2**I**

**(2)**

(*b*)Hence show that

**A**–1 = (**A** – 7**I**)

**(2)**

The transformation represented by **A** maps the point *P* onto the point *Q*.

Given that *Q* has coordinates (2*k* + 8, –2*k* – 5), where *k* is a constant,

(*c*)find, in terms of *k*, the coordinates of *P*.

**(4)**

**6.** (*a*) A sequence of numbers is defined by

*u*1 = 8

*un* + 1 = 4*un* – 9*n*, *n* ≥ 1

Prove by induction that, for *n* **,

*un* = 4*n* + 3*n* +1

**(5)**

(*b*) Prove by induction that, for *m* **,



**(5)**

**7.** (*a*) Express  in partial fractions.

**(2)**

(*b*) Using your answer to (*a*), find, in terms of *n*,



Give your answer as a single fraction in its simplest form.

**(3)**

**8.** (*a*) Given that

*z* = *r*(cos *θ* + i sin *θ*), *r* **

prove, by induction, that *zn* = *rn*(cos *nθ* + i sin *nθ*), *n* **.

**(5)**



(*b*) Find the exact value of *w*5, giving your answer in the form *a* + i*b*, where *a*, *b* **.

**(2)**

**9.** *z* = 5√3 – 5i

Find

(*a*) |*z*|,

**(1)**

(*b*) arg (*z*), in terms of *π*.

**(2)**



Find

(*c*) ,

**(1)**

(*d*) arg , in terms of *π*.

**(2)**

**10*.***(*a*) Find the general solution of the differential equation



**(5)**

(*b*) Find the particular solution for which *y* = 5 at *x* = 1, giving your answer in the form   
*y* = f(*x*).

**(2)**

(*c*) (i) Find the exact values of the coordinates of the turning points of the curve with equation *y* = f(*x*), making your method clear.

(ii) Sketch the curve with equation *y* = f(*x*), showing the coordinates of the turning points.

**(5)**

**11.**



**Figure 1**

Figure 1 shows a curve *C* with polar equation , 0 ≤ *θ* ≤ , and a half-line *l*.

The half-line *l* meets *C* at the pole *O* and at the point *P*. The tangent to *C* at *P* is parallel to the initial line. The polar coordinates of *P* are (*R*, φ).

(*a*) Show that cos φ= .

**(6)**

(*b*)Find the exact value of *R*.

**(2)**

The region *S*, shown shaded in Figure 1, is bounded by *C* and *l*.

(*c*)Use calculus to show that the exact area of *S* is



**(7)**

**12.** (*a*) Find

 d*x*

**(2)**

(*b*) Use your answer to part (*a*) to find the exact value of

 d*x*

giving your answer in the form *k* ln(*a + b* √5), where *a* and *b* are integers and *k* is a constant.

**(3)**

**13.**



**Figure 1**

Figure 1 shows part of the curve with equation

*y* = 40 arcosh *x* – 9*x*, *x* ≥ 1

Use calculus to find the exact coordinates of the turning point of the curve, giving your answer in the form  where *p*, *q*, *r* and *s* are integers.

**(7)**

**14*.***The matrix **P** is given by

, where *d* is constant, *d* ≠ –1

Find

(i) the determinant of **P** in terms of *d*,

(ii) the matrix **P**–1 in terms of *d*.

**(5)**

**15*.***The plane *Π*1 has vector equation

**r**.(3**i** – 4**j** + 2**k**) = 5

(*a*) Find the perpendicular distance from the point (6, 2, 12) to the plane *Π*1.

**(3)**

The plane *Π*2 has vector equation

**r =** *λ*(2**i** + **j** + 5**k**) + *μ*(**i** – **j** – 2**k**), where *λ* and *μ* are scalar parameters.

(*b*) Find the acute angle between *Π*1 and *Π*2 giving your answer to the nearest degree.

**(5)**

(*c*) Find an equation of the line of intersection of the two planes in the form **r** = **a** + *s***b**,   
where **a** and **b** are constant vectors.

**(6)**