**Composite Core Paper June 2014 (R)**

**1*.***The roots of the equation

2*z*3 – 3*z*2 + 8*z* + 5 = 0

are *z*1, *z*2 and *z*3.

Given that *z*1 = 1 + 2i, find *z*2 and *z*3.

**(5)**

**2*.***(i)



 (*a*) Describe fully the single transformation represented by the matrix **A**.

**(2)**

 The matrix **B** represents an enlargement, scale factor –2, with centre the origin.

 (*b*) Write down the matrix **B**.

**(1)**

(ii)

, where *k* is a positive constant.

Triangle *T* has an area of 16 square units.

Triangle *T* is transformed onto the triangle *Tʹ* by the transformation represented by the
matrix **M**.

Given that the area of the triangle *Tʹ* is 224 square units, find the value of *k*.

**(3)**

**3.** The complex number *z* is given by



where *p* is an integer.

(*a*)Express *z* in the form *a* + *b*i where *a* and *b* are real. Give your answer in its simplest form in terms of *p*.

**(4)**

(*b*)Given that arg(*z*) = *θ*, where tan *θ* = 1 find the possible values of *p*.

**(5)**

**4*.***(*a*) Use the standard results for  and  to show that



**(5)**

(*b*) Calculate the value of .

**(3)**

**5*.***

 and 

Given that **M** = (**A** + **B**)(2**A** – **B**),

(*a*) calculate the matrix **M**,

**(6)**

(*b*) find the matrix **C** such that **MC** = **A**.

**(4)**

**6.** (*a*) Prove by induction that, for ,



**(5)**

(*b*)A sequence of numbers is defined by

*u*1 = 0, *u*2 = 32,

*un*+2 = 6*un*+1 – 8*un* *n* ≥ 1

 Prove by induction that, for ,

*un* = 4*n*+1 – 2*n*+3

**(7)**

**7*.***(*a*) Express  in partial fractions.

**(2)**

(*b*) Hence use the method of differences to show that



**(3)**

**8*.***(*a*) Find the general solution of the differential equation

, 

 giving your answer in the form *y* = f(*x*).

**(6)**

(*b*) Find the particular solution for which *y* = 1 at *x* = 0.

**(2)**

**9.**



**Figure 1**

Figure 1 shows the curve *C* with polar equation

*r* = 2cos 2*θ*, 

The line *l* is parallel to the initial line and is a tangent to *C*.

Find an equation of *l*, giving your answer in the form *r* = f(*θ*).

**(9)**

**10*.***(*a*) Use de Moivre’s theorem to show that

sin 5*θ* ≡ 16 sin5 *θ* – 20 sin3 *θ* + 5 sin *θ*

**(5)**

(*b*) Hence find the five distinct solutions of the equation

16*x*5 – 20*x*3 + 5*x* +  = 0

 giving your answers to 3 decimal places where necessary.

**(5)**

(*c*) Use the identity given in (*a*) to find



 expressing your answer in the form *a*√2 + *b*, where *a* and *b* are rational numbers.

**(4)**

**11.** Solve the equation

5 tanh *x* + 7 = 5 sech *x*

Give each answer in the form ln *k* where *k* is a rational number.

**(5)**

**12.**

9*x*2 + 6*x* + 5 ≡ *a*(*x* + *b*)2 + *c*

(*a*) Find the values of the constants *a*, *b* and *c*.

**(3)**

Hence, or otherwise, find

(*b*) 

**(2)**

(*c*) 

**(2)**

**13**The plane *Π*1 has vector equation .

The plane *Π*2 has vector equation .

(*a*) Find a vector equation for the line of intersection of *Π*1 and *Π*2, giving your answer in the form **r** = **a** + *λ***b** where **a** and **b** are constant vectors and *λ* is a scalar parameter.

**(6)**

The plane *Π*3 has cartesian equation

*x* – *y* + 2*z* = 31

(*b*) Using your answer to part (*a*), or otherwise, find the coordinates of the point of intersection of the planes *Π*1, *Π*2 and *Π*3.

**(3)**