**Composite Core Paper June 2014**

**1.** The complex numbers *z*1 and *z*2 are given by

*z*1 = *p* + 2i and *z*2 = 1 – 2i

where *p* is an integer.

(*a*) Find  in the form *a* + *b*i where *a* and *b* are real. Give your answer in its simplest form in terms of *p*.

**(4)**

Given that ,

(*b*) find the possible values of *p*.

**(4)**

**2.** Given that 2 and 1 – 5i are roots of the equation

 *x*3 + *px*2 + 30*x* + *q* = 0, 

(*a*) write down the third root of the equation.

**(1)**

(*b*) Find the value of *p* and the value of *q*.

**(5)**

(*c*) Show the three roots of this equation on a single Argand diagram.

**(2)**

**3.** (i) Given that

 and ,

 (*a*) find **AB**.

 (*b*) Explain why **AB** ≠ **BA**.

**(4)**

(ii) Given that

, where *k* is a real number

 find **C**–1, giving your answer in terms of *k*.

**(3)**

**4*.***(*a*) Use the standard results for  and  to show that



**(6)**

(*b*) Hence show that



where *a* and *b* are constants to be found.

**(3)**

**5.** (i) In each of the following cases, find a 2 × 2 matrix that represents

 (*a*) a reflection in the line *y* = –*x*,

 (*b*) a rotation of 135° anticlockwise about (0, 0),

 (*c*) a reflection in the line *y* = –*x* followed by a rotation of 135° anticlockwise about
(0, 0).

**(4)**

(ii) The triangle *T* has vertices at the points (1, *k*), (3, 0) and (11, 0), where *k* is a constant. Triangle *T* is transformed onto the triangle *T*ʹ by the matrix



 Given that the area of triangle *T*ʹ is 364 square units, find the value of *k*.

**(6)**

**6.** Prove by induction that, for **

f(*n*) = 8*n* – 2*n*

is divisible by 6.

**(6)**

**7.** (*a*) Express  in partial fractions.

**(1)**

(*b*) Hence show that



**(5)**

**8.** , 

Find the series expansion for *y* in ascending powers of *x*, up to and including the term in *x*2, giving each coefficient in its simplest form.

**(8)**

**9.** (*a*) Use de Moivre’s theorem to show that

cos 6*θ* = 32cos6 *θ* – 48cos4 *θ* + 18cos2 *θ* – 1

**(5)**

(*b*) Hence solve for 0 ≤ *θ* ≤ 

64cos6 *θ* – 96cos4 *θ* + 36cos2 *θ* – 3 = 0

 giving your answers as exact multiples of *π*.

**(5)**

**10*.***(*a*) Find the general solution of the differential equation



**(6)**

(*b*) Find the particular solution that satisfies *y* = 0 and  = 0 when *x* = 0.

**(6)**

**11.** The line *l* passes through the point *P*(2, 1, 3) and is perpendicular to the plane *Π* whose
vector equation is

**r**.(**i** – 2**j** – **k**) = 3

Find

(*a*) a vector equation of the line *l*,

**(2)**

(*b*) the position vector of the point where *l* meets *Π*.

**(4)**

(*c*) Hence find the perpendicular distance of *P* from *Π*.

**(2)**

**12.** Using calculus, find the exact value of

(*a*) 

**(4)**

(*b*) 

**(4)**

**13.** Using the definitions of hyperbolic functions in terms of exponentials,

(*a*) show that

sech2 *x* = 1 – tanh2 *x*

**(3)**

(*b*) solve the equation

4sinh *x* – 3cosh *x* = 3

**(4)**

**14*.***Given that 

show that 

**(4)**

**15.**



**Figure 1**

Figure 1 shows a sketch of part of the curve *C* with polar equation

*r* = 1 + tan *θ*, 0 ≤ *θ* < 

The tangent to the curve *C* at the point *P* is perpendicular to the initial line.

(*a*) Find the polar coordinates of the point *P*.

**(5)**

The point *Q* lies on the curve *C*, where *θ* = .

The shaded region *R* is bounded by *OP*, *OQ* and the curve *C*, as shown in Figure 1.

(*b*)Find the exact area of *R*, giving your answer in the form



 where *p*, *q* and *r* are integers to be found.

**(7)**