**Composite Core Paper June 2015**

**1.** f(*x*) = 9*x*3 – 33*x*2 – 55*x* – 25.

Given that *x* = 5 is a solution of the equation f(*x*) = 0, use an algebraic method to solve f(*x*) = 0 completely.

**(5)**

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**2.** (*a*) Using the formulae for  and , show that



for all positive integers *n*.

**(5)**

(*b*) Hence show that



where *a* and *b* are integers to be found.

**(3)**

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**3.**  = 3i and  = .

(*a*) Express  in the form *a* + i*b*, where *a* and *b* are real numbers.

**(2)**

(*b*) Find the modulus and the argument of , giving the argument in radians in terms of *π*.

**(4)**

(*c*) Show the three points representing ,  and ( + ) respectively, on a single Argand diagram.

**(2)**

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**4.** (i) Prove by induction that, for *n* ∈ ℤ+,

 = .

**(6)**

(ii) Prove by induction that, for *n* ∈ ℤ+,

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**(6)**

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**5.** (i) **A** = , where *k* is a real constant.

Given that **A** is a singular matrix, find the possible values of *k*.

**(4)**

(ii) **B** = 

A triangle *T* is transformed onto a triangle *T'* by the transformation represented by the matrix **B**.

The vertices of triangle *T'* have coordinates (0, 0), (−20, 6) and (10*c*, 6*c*), where *c* is a positive constant.

The area of triangle *T'* is 135 square units.

(*a*) Find the matrix **B**–1.

**(2)**

(*b*) Find the coordinates of the vertices of the triangle *T*, in terms of *c* where necessary.

**(3)**

(*c*) Find the value of *c*.

**(3)**

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**6.** *z* = –2 + (2√3)i

(*a*) Find the modulus and the argument of *z*.

**(3)**

Using de Moivre’s theorem,

(*b*) find *z*6, simplifying your answer,

**(2)**

(*c*) find the values of *w* such that *w*4 = *z*3, giving your answers in the form *a* + i*b*, where *a*, *b*∈ ℝ.

**(4)**

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**7.** Find, in the form *y* = f(*x*), the general solution of the differential equation

tan *x*  + *y* = 3 cos 2*x* tan *x*, 0 < *x* < .

**(6)**

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**8.** (*a*) Show that

*r*2(*r* + 1)2 – (*r* – 1)2 *r*2 ≡ 4*r*3.

**(3)**

Given that  = *n*(*n* + 1),

(*b*) use the identity in (*a*) and the method of differences to show that

(13 + 23 + 33 + … + *n*3) = (1 + 2 + 3 + … + *n*)2.

**(4)**

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**9*.***

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**Figure 1**

The curve *C*, shown in Figure 1, has polar equation

*R* = 3*a*(1 + cos *θ* ), 0 ≤ *θ* < *π*.

The tangent to *C* at the point *A* is parallel to the initial line.

(*a*) Find the polar coordinates of *A*.

**(6)**

The finite region *R*, shown shaded in Figure 1, is bounded by the curve *C*, the initial line and the line *OA*.

(*b*) Use calculus to find the area of the shaded region *R*, giving your answer in the form *a*2(*pπ*+ *q*√3), where *p* and *q* are rational numbers.

**(5)**

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**10*.*** *y* = tan2 *x*, – < *x* < .

(*a*) Show that  = 6 sec4 *x* – 4 sec2 *x*.

**(4)**

(*b*) Hence show that  = 8 sec2 *x* tan *x* (*A* sec2 *x* + *B*), where *A* and *B* are constants to be found.

**(3)**

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**11.** Solve the equation

2 cosh2 *x* – 3 sinh *x* = 1,

giving your answers in terms of natural logarithms.

**(6)**

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**12.** The curve *C* has equation

y = , *x* > 1.

(*a*) Find .

**(3)**

The region *R* is bounded by the curve *C*, the *x*-axis and the lines with equations *x* = 2 and *x* = 3. The region *R* is rotated through 2*π* radians about the *x*-axis.

(*b*) Find the volume of the solid generated. Give your answer in the form *pπ* ln *q*, where *p* and *q* are rational numbers to be found.

**(4)**

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**13.** The points *A*, *B* and *C* have position vectors ,  and  respectively.

(*a*) Find a vector equation of the straight line *AB*.

**(2)**

(*b*) Find a cartesian form of the equation of the straight line *AB*.

**(2)**

The plane *Π* contains the points *A*, *B* and *C*.

(*c*) Find a vector equation of *Π* in the form **r**.**n** = *p*.

**(4)**

(*d*) Find the perpendicular distance from the origin to *Π*.

**(2)**