**Composite paper June 2017**

**1.**

** **

(*a*)Find **A**–1

**(2)**

The transformation represented by the matrix **B** followed by the transformation represented

by the matrix **A** is equivalent to the transformation represented by the matrix **P**.

(*b*)Find **B**, giving your answer in its simplest form.

**(3)**

**(Total 5 marks)**

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**2.** (i) The complex number *w* is given by

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 where *p* is a real constant.

 (*a*)Express *w* in the form *a* + *b*i, where *a* and *b* are real constants.

Give your answer in its simplest form in terms of *p*.

**(3)**

Given that arg *w* = 

 (*b*)find the value of *p*.

**(2)**

(ii) The complex number *z* is given by

*z* = (1 – 𝜆 i)(4 + 3i)

 where 𝜆is a real constant.

 Given that

|*z*|= 45

 find the possible values of 𝜆.

 Give your answers as exact values in their simplest form.

**(3)**

**(Total 8 marks)**

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**3.** (i)

** **

 where *p* is a constant.

 (*a*)Find, in terms of *p*, the matrix **AB**

**(2)**

 Given that

**AB +** 2**A =** *k***I**

 where *k* is a constant and **I** is the 2 × 2 identity matrix,

 (*b*)find the value of *p* and the value of *k*.

**(4)**

(ii)

, where *a* is a real constant

 Triangle *T* has an area of 15 square units.

 Triangle *T* is transformed to the triangle *T'* by the transformation represented by the

 matrix **M**.

 Given that the area of triangle *T'* is 270 square units, find the possible values

 of *a*.

**(5)**

**(Total 11 marks)**

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**4.** Given that 4 and 2i – 3 are roots of the equation

*x*3 + *ax*2 + *bx* – 52 = 0

where *a* and *b* are real constants,

(*a*)write down the third root of the equation,

**(1)**

(*b*)find the value of *a* and the value of *b*.

**(5)**

**(Total 6 marks)**

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**5.** (*a*)Use the standard results for ** and  to show that



 for all positive integers *n*.

**(5)**

Given that



(*b*)find the exact value of the constant *k*.

**(4)**

**(Total 9 marks)**

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**6.** (i) A sequence of numbers is defined by

*u*1 = 6, *u*2 = 27

*un*+2 = 6*un*+1 – 9*un n* ⩾ 1

 Prove by induction that, for *n* ∈ ℤ+

*un* = 3*n*(*n* + 1)

**(6)**

(ii) Prove by induction that, for *n* ∈ ℤ+

f (*n*) = 33*n*–2 + 23*n*+1 is divisible by 19

**(6)**

**(Total 12 marks)**

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**7.** (*a*)Show that, for *r* > 0

****

**(1)**

(*b*)Hence prove that, for *n* ∈ ℕ



**(3)**

(*c*)Show that, for *n* ∈ ℕ, *n* > 1



 where *a*, *b* and *c* are constants to be found.

**(3)**

**(Total 7 marks)**

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**8.** Solve the equation

*z*3 + 32 + 32i = 0

giving your answers in the form *r*ei𝜃where *r* > 0 and – *π* < 𝜃 ⩽ *π*

**(Total 6 marks)**

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**9.** (*a*)Find the general solution of the differential equation



**(8)**

(*b*)Find the particular solution of this differential equation for which *y* = 0 and = 0

 when *x* = 0

**(5)**

**(Total 13 marks)**

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**10.**

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Figure 1 shows a sketch of a curve with polar equation

*r* = 6 + *a* sin 𝜃

where 0 < *a* < 6 and 0 ⩽ 𝜃< 2*π*

The area enclosed by the curve is 

Find the value of the constant *a*.

**(Total 8 marks)**

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**11.** (*a*)Find, in the form *y* = f(*x*), the general solution of the equation

cos *x* + *y* sin *x* = 2cos3 *x* sin *x* + 1, 0 < *x* < 

**(8)**

Given that  when *x* = 

(*b*)find the value of *y* when *x* =**, giving your answer in the form *a* +**, where *a*

 and *b* are rational numbers to be found.

**(3)**

**(Total 11 marks)**

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**12.** Given that *y* = arsinh(tanh *x*), show that



**(Total 5 marks)**

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**13.** (*a*)Using the definition for cosh *x* in terms of exponentials, show that

cosh 2*x* 2 cosh2 *x* – 1

**(3)**

(*b*)Find the exact values of *x* for which

29cosh *x* – 3cosh 2*x* = 38

 giving your answers in terms of natural logarithms.

**(6)**

**(Total 9 marks)**

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**14.** Use the substitution *x* + 2 = *u*2, where *u* > 0, to show that

 = *a* + *bπ*

where *a* and *b* are rational numbers to be found.

**(Total 9 marks)**

**15.** The plane ** has equation *x* – 2*y* – 3*z* = 5 and the plane ** has equation 6*x* + *y* – 4*z* = 7

(*a*)Find, to the nearest degree, the acute angle between ** and **

**(3)**

The point *P* has coordinates (2, 3, –1). The line *l* is perpendicular to ** and passes through

the point *P*. The line *l* intersects ** at the point *Q*.

(*b*)Find the coordinates of *Q*.

**(4)**

The plane ** passes through the point *Q* and is perpendicular to ** and **

(*c*)Find an equation of the plane ** in the form **r.n** = *p*

**(4)**

**(Total 11 marks)**

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**16.** The matrix **M** is given by

 *k* ∈ ℝ, *k* ≠ 

(*a*)Show that det **M** = 1 – 2*k*.

**(2)**

(*b*)Find **M**–1 in terms of *k*.

**(4)**

The straight line *l*1 is mapped onto the straight line *l*2 by the transformation represented by

the matrix



Given that *l*2 has cartesian equation



(*c*)find a cartesian equation of the line *l*1

**(6)**

**(Total 12 marks)**

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