**Composite paper MS**

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| Question Number | Scheme | Notes | Marks |
|  | Mark (a) and (b) together – ignore labels | |  |
| **7(a)** |  | Cao No working needed – ignore any shown | B1 |
|  |  |  | (1) |
| **(b)** |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  | First 2 and last term or first and last 2 terms required.  Must start at  (First term complete, 2nd and last may be partial or last term complete 1st and penultimate partial.) | M1 |
|  |  | Cancel terms. | M1A1 |
|  |  |  |  |
|  |  | Find common denominator, dep on second M mark  Cso (All M marks required) | dM1  A1cso |
|  |  | Need not be shown explicitly | (5) |
| **NB: 1** | All marks can be awarded if work done with values 1,2,...*r* and then *r* replaced with *n*; if no replacement made, deduct final A mark. | |  |
| **2** | with **NO** other working gets M0M1A1M1A0 max | |  |
|  |  | |  |
| **(c)** |  | Accept  and only in their answer to (b)  Must be subtracted | M1 |
|  |  | Exact answer  implies method provided no incorrect work seen in (c). | A1 |
|  |  |  | (2) |
| **ALT** | Use the method of differences again, starting at  and ending at | Complete method | M1 |
|  |  | Correct answer | A1 |
|  |  |  | **Total 8** |

Comparison of key skills specifications 2000/2002 with 2004 standardsX015461July 2004Issue 1

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| Question  Number | Scheme | Notes | Marks |
| **8(a)** |  | Correct expansion for sine, including surd values for all 4 trig functions.  accepted | M1 |
| **(i)** | \*\* | Completion to **given answer**:  No errors seen, cso | A1cso |
|  |  | Correct expansion for cosine, including surd values for all 4 trig functions.  accepted  OR other complete method   eg using | M1  NB A1 on e-PEN |
| **(ii)** |  | Completion to **given answer**:  No errors seen, cso | A1cso |
|  |  |  | (4) |
| **(b)** | **Allow all marks using EXACT calculator values for the trig functions. Decimal answers qualify for M marks only.** | |  |
|  |  | Use a valid method to generate at least 2 roots (eg use of  or rotate through , multiply by I, symmetry) | M1 |
|  |  | Application of de Moivre’s theorem resulting in at least 1 root being found.  ( and arg divided by 4)  accepted | M1 |
|  |  | Any correct root (this is the most likely one if only one found)  Can be in any exact form  ( oe)  Can be unsimplified using results from (a) ie  with  oe  Or simplified/calculator values ie | B1 |
|  |  | |  |
|  |  | |  |
|  | Two correct roots in form *a* + i*b* unsimplified or calculator values, must be exact surd form | | A1 |
|  | All 4 correct roots in form *a* + i*b* unsimplified or calculator values must be exact surd form. | | A1 |
|  |  |  | (5) |
|  |  |  | **Total 9** |

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| Question Number | Scheme | Notes | Marks |
| **9.(a)** |  | |  |
|  |  | M1 Forms and solves auxiliary equation  A1 Correct roots | M1A1 |
|  | Complementary Function | CF of the form shown formed using their 2 real roots  Can be awarded if seen in gen solution | B1ft  NB A1 on e-PEN |
|  | Particular Integral | May include higher powers | B1 |
|  |  | Differentiates their PI twice  All powers of *x* to decrease by 1 | M1 |
|  |  |  |  |
|  |  | Substitutes their derivatives into the equation and equates at least one pair of coefficients | M1 |
|  |  | Attempt to solve 3 equations.  Must reach a numerical value for all 3 coefficients | M1 |
|  | General Solution | Must start *y* = ...  cao | A1 |
|  |  |  | (8) |
| **(b)** |  | Differentiates their GS – min 4 terms in their GS | M1 |
|  | , , | Forms 2 simultaneous equations using given boundary values | M1 |
|  | , | Attempt to solve  Must reach *A* = ... or *B* = ... | M1 |
|  |  | Both correct | A1 |
|  |  | Must start *y* = ... | A1 (5) |
|  |  |  | **Total 13** |

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| Question Number | Scheme | Notes | Marks |
| **10. (a)** |  | |  |
| **Way 1** |  | Multiplies *r* by | B1 |
|  |  | M1 Differentiates using product rule  A1 Correct derivative | M1A1 |
|  |  | Use  to form a 3TQ in  and attempt to solve. Reach | M1 |
|  |  | Accept  or +  Any exact equivalent – need not be simplified. | A1 |
|  | \*\* | Must show substitution of correct, exact  in | A1cso |
|  |  |  | (6) |
| **Way 2** |  | Leaves *y* as a product | B1 |
|  |  | M1 Differentiates using product rule  A1 Correct derivative | M1A1 |
|  |  | Use  to form a 3TQ in  and attempt to solve. Reach | M1 |
|  |  | Accept  or +  Any exact equivalent – need not be simplified. | A1 |
|  |  | Must show substitution of correct, exact  in | A1cso |
|  |  |  | (6) |
| **Way 3** |  | Uses a double angle formula | B1 |
|  |  | M1 Differentiates  A1 Correct derivative | M1A1 |
|  |  | Use a double angle identity to form a 3TQ in .  Attempt to solve their 3TQ. Reach | M1 |
|  |  | Accept  or +  Any exact equivalent – need not be simplified. | A1 |
|  | \*\* | Must show substitution of correct, exact  in | A1cso(6) |
| **Way 4** |  |  |  |
|  |  | Correct derivative | B1 |
|  |  | Differentiate using product rule | M1 |
|  |  | Correct derivative as a function  of | A1 |
|  |  | Use  to form a 3TQ in  and attempt to solve. Reach | M1 |
|  |  | Accept  or +  Any exact equivalent – need not be simplified. | A1 |
|  | \*\* | Must show substitution of correct, exact  in | A1cso |
|  |  |  | (6) |
|  |  |  |  |
| **Special Case** |  | **NOT** |  |
|  |  |  | B0 |
|  |  | Differentiates  Cannot obtain correct derivative | M1  A0 |
|  | No further marks available |  |  |
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| **(b)** |  | Attempt to find as a 3 term quadratic and use a double angle formula | M1 |
|  |  | Correct result | A1 |
|  | oe | dM1 Attempts to integrate their  Depends on first M of (b)  A1 Correct integral | dM1A1 |
|  | **Check the integration carefully as the sine terms become 0 when limits substituted.** | |  |
|  |  | |  |
|  |  | Substitutes correct limits in | ddM1 |
|  |  | Correct answer must be exact  Accept  No errors in the working | A1cso |
|  |  |  | (6) |
|  |  |  | **Total 12** |
|  |  |  |  |
| **NB:** |  | Integral evaluated on a calculator.  Correct answer – send to review.  Incorrect answer – 0/6 |  |
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| Question Number | Scheme | | Notes | Marks |
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| **11.** |  | | |  |
| **(a)** | oe | | May be implied by subsequent work | M1 |
|  |  | | Integral in termsof *t* only required. d*t* may be implied Must have attempted to change d*x* to d*t* (ie not just used d*x* = d*t*) | M1 |
|  |  | Use of integration by parts  Reduce the power of *t*.  Sign errors are allowed. | | M1 |
|  |  | | Use of integration by parts again in the same direction | dM1 |
|  |  | | Correct integration, constant not needed | A1 |
|  |  | | Reverse substitution, constant not needed.  **This mark** **cannot be recovered in (b)** | A1 |
|  |  | |  | (6) |
| **ALTs** | Attempts without substitution which may merit part marks – send to review. | | |  |
|  |  | | |  |
| **(b)** | Integrating Factor | | Use ofseen | B1 |
|  |  | | Multiply through by their IF | M1 |
|  |  | | Use their answer for (a), which must be a function of *x*, to integrate RHS | A1ft |
|  |  | | Complete to *y* = ...  Include the constant and deal with it correctly  **Not follow through** | A1 |
|  |  | |  | (4) |
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| **ALT:** | Use the same substitution as in (a)  Following work uses the work shown in (a) rather than just the final answer.  No marks until a first order exact equation in *y* and *t* reached and an attempt is made to solve this. | | |  |
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|  |  | | |  |
|  | Integrating Factor | | Use ofseen | B1 |
|  |  | | Multiply through by IF | M1 |
|  |  | | Use their work in (a) to integrate RHS | A1ft |
|  |  | | Reverse the substitution  Complete to *y* = ...  Include the constant and deal with it correctly  **Not follow through** | A1 |
|  |  | |  | (4) |
| **(c)** |  | | Attempt to substitute  into their *y* provided it includes a constant | M1 |
|  |  | | NB: **Not ft** so must have been obtained using a correct expression for *y* | A1 |
|  |  | | Must start *y* = ...  Follow through their *C* and expression for *y* | A1ft |
|  |  | |  | (3) |
|  |  | |  | Total 13 |

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| Question Number | Scheme | Notes | Marks |
| **12(a)** |  | Substitutes the correct exponential forms. Note that the  may be implied. | M1 |
|  | Correct proof with no errors or omissions or notational errors such as using sin for sinh | A1\* |
| **Note that the question says “starting from the definitions of sinh*x* and cosh*x* in terms of exponentials” so:**  scores M1A1  **BUT**    scores M0A0 as sinh*x* and cosh*x* have not been defined | |  |
|  |  | **(2)** |
| **(b)** | M1 for setting  or any other variables for θ and *y* e.g.  and uses correct processing (allow sign errors only) to make e2”y” or e”y” the subject | | M1 |
|  | Removes e correctly by taking ln’s. **Dependent on the first method mark.** | **d**M1 |
| or  Correct completion with no errors.  Must be in terms of *θ* for this mark but allow “mixed” variables for the M’s. This mark should be withheld if there are any errors such as the appearance of a “tan” instead of “tanh” and/or missing variables. The proof does need to convey that    So if *y* has been defined as artanh*θ* and the proof ends , this is acceptable. So must be in terms of *θ* for the A mark but allow other variables to be used for the M’s.  Allow arctanh, artanh, tanh-1 etc. for the inverse | | A1\* |
|  |  | **(3)** |

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|  |  | **Total 5** |

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| Question Number | Scheme | Notes | Marks |
| **13** |  | |  |
| **(a)** | Substitutes the correct exponential forms but allow the “2’s” to be missing | | M1 |
|  | Correct equation | A1 |
|  | Correct value (oe e.g. ) | A1 |
|  |  |  | **(3)** |

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| **(b)** | Squares to obtain  and attempts to use at least one **correct** “double angle” hyperbolic identity for cosh2*x* or sinh2*x*  e.g. | | | M1 |
|  | Two correct terms in their final expression | | A1 |
| All correct terms in their final expression | | A1 |
|  | |  | **(3)** |

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| **(c)** | **Note that π is not needed for the first 3 marks of (c)** | | |  |
|  | | Uses with their *y*2 where *y*2 is of the form | M1 |
|  | | Correct integration, ft their  *a*, *b* and *c* **or** the letters *a*, *b* and *c* **or** a combination of both **or** “made up” values. | A1ft |
| Note that  Correct use of limits. Must see 0 **and** their value from (a) substituted into all 3 terms (although the “0’s” can be implied) and subtracted the right way round.  **Dependent on the first method mark.** | | | **d**M1 |
| or e.g.  Or e.g. | Correct exact answer in any equivalent **exact** form. | | A1 |
|  | |  | **(4)** |
|  |  | |  | **Total 10** |

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| Question Number | Scheme | | Notes |
| **6** |  | |  |
| **(a)** |  | Shows lines are not parallel. If they say “different direction vectors”, the direction vectors must be identified. | B1 |
| Examples of showing non-parallel:  , | |  |
|  | |  |
| **(1)** and **(2)** yields  **(1)** and **(3)** yields  **(2)** and **(3)** yields | Attempts to solve a pair of equations to find at least one of either  or | M1 |
| Checking **(3)**:  Checking **(2)**:  Checking **(1)**: | Attempts to show a contradiction | M1 |
| So the lines are not parallel and do not intersect so the lines are skew | All complete and with no errors and conclusion. If they have already stated “not parallel” there is no need to repeat this. | A1 |
|  |  | **(4)** |

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|  | **Alternative for the M marks:** | |  |
| **(1)** and **(2)** yields  **(1)** and **(3)** yields  **(2)** and **(3)** yields | Attempts to solve a pair of equations to find at least one of either  or | M1 |
| Shows any two of  **(1)** and **(2)** yielding  **(1)** and **(3)** yielding  **(2)** and **(3)** yielding  **or** shows any two of  **(1)** and **(2)** yielding  **(1)** and **(3)** yielding  **(2)** and **(3)** yielding | Attempts to show a contradiction | M1 |

Note that for (b) the only misinterpretations for Position we are allowing are:

 for  for the position of *l*1 and  for  for the position of *l*2

But allow obvious slips or mis-copies of e.g. signs or elements if the intention is clear.

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| **(b)**  **Way 1** |  | Attempt cross product of direction vectors. If no method is shown, 2 components should be correct. | M1 |
| Correct vector | A1 |
|  | Attempt scalar product between the difference of the position vectors and their normal vector. | M1 |
|  | Correct completion. Divides their scalar product between the difference of the position vectors and their normal vector by the modulus of their vector product. | M1 |
| Any equivalent or awrt 3.84 | A1 |
|  |  |  | **(5)** |

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| **(b)**  **Way 2** |  | Attempt cross product of direction vectors | M1 |
| Correct vector | A1 |
|  | Attempt equation of both planes | M1 |
|  | Correct completion | M1 |
| Any equivalent e.g. or awrt 3.84 but must be positive. | A1 |
|  |  | **(5)** |

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| **(b)**  **Way 3** |  | Finds a general chord between the 2 lines and attempts the scalar product between this and the directions,  sets = 0 to give 2 equations in 2 unknowns | M1 |
|  | Correct values | A1 |
| Or | Uses their values to find the ends of the chord or substitutes into their chord vector | M1 |
|  | Correct completion by finding the distance between their 2 points | M1 |
| Any equivalent e.g. or awrt 3.84 | A1 |
|  |  | **(5)** |

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| **(b)**  **Way 4** |  | Attempt cross product of direction vectors | M1 |
| Correct vector | A1 |
|  | Finds a common chord between the 2 lines and sets equal to a multiple of the normal vector to give 3 equations in 3 unknowns and solves to find a value for *k* | M1 |
|  | Correct completion by finding the length of their vector | M1 |
| Any equivalent e.g. or awrt 3.84 | A1 |
|  |  | **(5)** |

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| **(c)** |  | Attempt another non-parallel vector in  *П* | M1 |
|  | Attempt cross product of two non-parallel vectors in the plane. If the method is not shown, at least 2 components should be correct. **Dependent on the first M mark.** | **d**M1 |
|  | Attempt scalar product with a point in the plane. **Dependent on both previous method marks.** | **dd**M1 |
|  | Any multiple but must be a Cartesian equation. | A1 |
|  |  | **(4)** |
| **(c) Way 2** |  | Attempt another vector in  *П* | M1 |
|  | Forms the vector equation of the plane.  **Dependent on the first M mark.** | **d**M1 |
|  | Eliminates λ or μ.  **Dependent on both previous method marks.** | **dd**M1 |
|  | Any correct equation but must be a correct Cartesian equation. Isw | A1 |
|  |  | **(4)** |
|  |  |  | **Total 13** |