**Composite paper MS**

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| --- | --- | --- | --- |
| Question Number | Scheme | Notes | Marks |
|  | Mark (a) and (b) together – ignore labels |  |
| **7(a)** |  | Cao No working needed – ignore any shown | B1 |
|  |  |  | (1) |
| **(b)** |   |  |  |
|  |   |  |  |
|  |   |  |  |
|  |   | First 2 and last term or first and last 2 terms required.Must start at (First term complete, 2nd and last may be partial or last term complete 1st and penultimate partial.) | M1 |
|  |  | Cancel terms. | M1A1 |
|  |  |  |  |
|  |  | Find common denominator, dep on second M markCso (All M marks required) | dM1A1cso |
|  |  | Need not be shown explicitly | (5) |
| **NB: 1** | All marks can be awarded if work done with values 1,2,...*r* and then *r* replaced with *n*; if no replacement made, deduct final A mark. |  |
| **2** |  with **NO** other working gets M0M1A1M1A0 max |  |
|  |  |  |
| **(c)** |  | Accept  and only in their answer to (b)Must be subtracted | M1 |
|  |   | Exact answer  implies method provided no incorrect work seen in (c). | A1 |
|  |  |  | (2) |
| **ALT** | Use the method of differences again, starting at  and ending at  | Complete method | M1 |
|  |  | Correct answer | A1 |
|  |  |  | **Total 8** |

Comparison of key skills specifications 2000/2002 with 2004 standardsX015461July 2004Issue 1

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| QuestionNumber | Scheme | Notes | Marks |
| **8(a)**  |  | Correct expansion for sine, including surd values for all 4 trig functions.  accepted | M1 |
| **(i)** | \*\* | Completion to **given answer**: No errors seen, cso | A1cso |
|  |  | Correct expansion for cosine, including surd values for all 4 trig functions.  acceptedOR other complete method  eg using   | M1NB A1 on e-PEN |
| **(ii)** |  | Completion to **given answer**: No errors seen, cso | A1cso |
|  |  |  | (4) |
| **(b)** | **Allow all marks using EXACT calculator values for the trig functions. Decimal answers qualify for M marks only.** |  |
|  |  | Use a valid method to generate at least 2 roots (eg use of  or rotate through , multiply by I, symmetry) | M1 |
|  |  | Application of de Moivre’s theorem resulting in at least 1 root being found.( and arg divided by 4) accepted | M1 |
|  |  | Any correct root (this is the most likely one if only one found)Can be in any exact form ( oe)Can be unsimplified using results from (a) ie  with  oeOr simplified/calculator values ie  | B1 |
|  |  |  |
|  |  |  |
|  | Two correct roots in form *a* + i*b* unsimplified or calculator values, must be exact surd form | A1 |
|  | All 4 correct roots in form *a* + i*b* unsimplified or calculator values must be exact surd form. | A1 |
|  |  |  | (5) |
|  |  |  | **Total 9** |

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| Question Number | Scheme | Notes | Marks |
| **9.(a)** |  |  |
|  |  | M1 Forms and solves auxiliary equationA1 Correct roots | M1A1 |
|  | Complementary Function  | CF of the form shown formed using their 2 real rootsCan be awarded if seen in gen solution | B1ftNB A1 on e-PEN |
|  | Particular Integral   | May include higher powers  | B1 |
|  |  | Differentiates their PI twiceAll powers of *x* to decrease by 1 | M1 |
|  |  |  |  |
|  |  | Substitutes their derivatives into the equation and equates at least one pair of coefficients | M1 |
|  |  | Attempt to solve 3 equations.Must reach a numerical value for all 3 coefficients | M1 |
|  | General Solution | Must start *y* = ...cao | A1 |
|  |  |  | (8) |
| **(b)** |  | Differentiates their GS – min 4 terms in their GS | M1 |
|  | , ,  | Forms 2 simultaneous equations using given boundary values | M1 |
|  | , | Attempt to solveMust reach *A* = ... or *B* = ... | M1 |
|  |  | Both correct | A1 |
|  |  | Must start *y* = ... | A1 (5)  |
|  |  |  | **Total 13** |

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| Question Number | Scheme | Notes | Marks |
| **10. (a)** |  |  |
| **Way 1** |  | Multiplies *r* by  | B1 |
|  |  | M1 Differentiates using product ruleA1 Correct derivative | M1A1 |
|  |  | Use  to form a 3TQ in  and attempt to solve. Reach  | M1 |
|  |  | Accept  or +Any exact equivalent – need not be simplified. | A1 |
|  |  \*\* | Must show substitution of correct, exact  in  | A1cso |
|  |  |  | (6) |
| **Way 2** |  | Leaves *y* as a product | B1 |
|  |  | M1 Differentiates using product ruleA1 Correct derivative | M1A1 |
|  |  | Use  to form a 3TQ in  and attempt to solve. Reach  | M1 |
|  |  | Accept  or +Any exact equivalent – need not be simplified. | A1 |
|  |  | Must show substitution of correct, exact  in  | A1cso |
|  |  |  | (6) |
| **Way 3** |  | Uses a double angle formula | B1 |
|  |  | M1 DifferentiatesA1 Correct derivative | M1A1 |
|  |  | Use a double angle identity to form a 3TQ in .  Attempt to solve their 3TQ. Reach  | M1 |
|  |  | Accept  or +Any exact equivalent – need not be simplified. | A1 |
|  |  \*\* | Must show substitution of correct, exact  in   | A1cso(6) |
| **Way 4** |  |  |  |
|  |  | Correct derivative | B1 |
|  |  | Differentiate using product rule | M1 |
|  |  | Correct derivative as a functionof   | A1 |
|  |  | Use  to form a 3TQ in  and attempt to solve. Reach  | M1 |
|  |  | Accept  or +Any exact equivalent – need not be simplified. | A1 |
|  |  \*\* | Must show substitution of correct, exact  in  | A1cso |
|  |  |  | (6) |
|  |  |  |  |
| **Special Case** |   | **NOT**  |  |
|  |  |  | B0 |
|  |  | DifferentiatesCannot obtain correct derivative | M1A0 |
|  | No further marks available |  |  |
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| **(b)** |  | Attempt to find as a 3 term quadratic and use a double angle formula  | M1 |
|  |  | Correct result | A1 |
|  |  oe | dM1 Attempts to integrate their  Depends on first M of (b)A1 Correct integral  | dM1A1 |
|  | **Check the integration carefully as the sine terms become 0 when limits substituted.** |  |
|  |  |  |
|  |  | Substitutes correct limits in | ddM1 |
|  |  | Correct answer must be exactAccept No errors in the working | A1cso |
|  |  |  | (6) |
|  |  |  | **Total 12** |
|  |  |  |  |
| **NB:** |  | Integral evaluated on a calculator.Correct answer – send to review.Incorrect answer – 0/6 |  |
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| Question Number | Scheme | Notes | Marks |
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| **11.** |  |  |
| **(a)** |  oe | May be implied by subsequent work | M1 |
|  |  | Integral in termsof *t* only required. d*t* may be implied Must have attempted to change d*x* to d*t* (ie not just used d*x* = d*t*) | M1 |
|  |  | Use of integration by parts Reduce the power of *t*. Sign errors are allowed. | M1 |
|  |  | Use of integration by parts again in the same direction | dM1 |
|  |  | Correct integration, constant not needed | A1 |
|  |  | Reverse substitution, constant not needed.**This mark** **cannot be recovered in (b)** | A1 |
|  |  |  | (6) |
| **ALTs** | Attempts without substitution which may merit part marks – send to review. |  |
|  |  |  |
| **(b)** | Integrating Factor  | Use ofseen | B1 |
|  |  | Multiply through by their IF | M1 |
|  |  | Use their answer for (a), which must be a function of *x*, to integrate RHS | A1ft |
|  |  | Complete to *y* = ...Include the constant and deal with it correctly**Not follow through** | A1 |
|  |  |  | (4) |
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| **ALT:** | Use the same substitution as in (a)Following work uses the work shown in (a) rather than just the final answer.No marks until a first order exact equation in *y* and *t* reached and an attempt is made to solve this. |  |
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|  |  |  |
|  | Integrating Factor  | Use ofseen | B1 |
|  |  | Multiply through by IF | M1 |
|  |  | Use their work in (a) to integrate RHS | A1ft |
|  |  | Reverse the substitutionComplete to *y* = ...Include the constant and deal with it correctly**Not follow through** | A1 |
|  |  |  | (4) |
| **(c)** |  | Attempt to substitute  into their *y* provided it includes a constant | M1 |
|  |  | NB: **Not ft** so must have been obtained using a correct expression for *y* | A1 |
|  |  | Must start *y* = ...Follow through their *C* and expression for *y*  | A1ft |
|  |  |  | (3) |
|  |  |  | Total 13 |

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| Question Number | Scheme | Notes | Marks |
| **12(a)** |  | Substitutes the correct exponential forms. Note that the  may be implied. | M1 |
|  | Correct proof with no errors or omissions or notational errors such as using sin for sinh | A1\* |
| **Note that the question says “starting from the definitions of sinh*x* and cosh*x* in terms of exponentials” so:** scores M1A1**BUT** scores M0A0 as sinh*x* and cosh*x* have not been defined |  |
|  |  | **(2)** |
| **(b)** | M1 for setting  or any other variables for θ and *y* e.g.  and uses correct processing (allow sign errors only) to make e2”y” or e”y” the subject  | M1 |
|  | Removes e correctly by taking ln’s. **Dependent on the first method mark.** | **d**M1 |
|  or Correct completion with no errors.Must be in terms of *θ* for this mark but allow “mixed” variables for the M’s. This mark should be withheld if there are any errors such as the appearance of a “tan” instead of “tanh” and/or missing variables. The proof does need to convey thatSo if *y* has been defined as artanh*θ* and the proof ends , this is acceptable. So must be in terms of *θ* for the A mark but allow other variables to be used for the M’s.Allow arctanh, artanh, tanh-1 etc. for the inverse | A1\* |
|  |  | **(3)** |

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|  |  | **Total 5** |

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| Question Number | Scheme | Notes | Marks |
| **13** |  |  |
| **(a)** | Substitutes the correct exponential forms but allow the “2’s” to be missing | M1 |
|  | Correct equation | A1 |
|  | Correct value (oe e.g. ) | A1 |
|  |  |  | **(3)** |

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| **(b)** | Squares to obtain  and attempts to use at least one **correct** “double angle” hyperbolic identity for cosh2*x* or sinh2*x*e.g.  | M1 |
|  | Two correct terms in their final expression | A1 |
| All correct terms in their final expression | A1 |
|  |  | **(3)** |

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| **(c)** | **Note that π is not needed for the first 3 marks of (c)** |  |
|  | Uses with their *y*2 where *y*2 is of the form  | M1 |
|  | Correct integration, ft their  *a*, *b* and *c* **or** the letters *a*, *b* and *c* **or** a combination of both **or** “made up” values. | A1ft |
| Note that Correct use of limits. Must see 0 **and** their value from (a) substituted into all 3 terms (although the “0’s” can be implied) and subtracted the right way round. **Dependent on the first method mark.** | **d**M1 |
|  or e.g. Or e.g.  | Correct exact answer in any equivalent **exact** form.  | A1 |
|  |  | **(4)** |
|  |  |  | **Total 10** |

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| Question Number | Scheme | Notes |
| **6** |   |  |
| **(a)** |   | Shows lines are not parallel. If they say “different direction vectors”, the direction vectors must be identified. | B1 |
| Examples of showing non-parallel:,  |  |
|  |  |
| **(1)** and **(2)** yields **(1)** and **(3)** yields **(2)** and **(3)** yields  | Attempts to solve a pair of equations to find at least one of either  or  | M1 |
| Checking **(3)**: Checking **(2)**: Checking **(1)**:  | Attempts to show a contradiction | M1 |
| So the lines are not parallel and do not intersect so the lines are skew | All complete and with no errors and conclusion. If they have already stated “not parallel” there is no need to repeat this. | A1 |
|  |  | **(4)** |

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|  | **Alternative for the M marks:** |  |
| **(1)** and **(2)** yields **(1)** and **(3)** yields **(2)** and **(3)** yields  | Attempts to solve a pair of equations to find at least one of either  or  | M1 |
| Shows any two of**(1)** and **(2)** yielding **(1)** and **(3)** yielding **(2)** and **(3)** yielding **or** shows any two of**(1)** and **(2)** yielding **(1)** and **(3)** yielding **(2)** and **(3)** yielding  | Attempts to show a contradiction | M1 |

Note that for (b) the only misinterpretations for Position we are allowing are:

 for  for the position of *l*1 and  for  for the position of *l*2

But allow obvious slips or mis-copies of e.g. signs or elements if the intention is clear.

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| **(b)****Way 1** |  | Attempt cross product of direction vectors. If no method is shown, 2 components should be correct. | M1 |
| Correct vector | A1 |
|  | Attempt scalar product between the difference of the position vectors and their normal vector. | M1 |
|  | Correct completion. Divides their scalar product between the difference of the position vectors and their normal vector by the modulus of their vector product. | M1 |
| Any equivalent or awrt 3.84 | A1 |
|  |  |  | **(5)** |

|  |  |  |  |
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| **(b)****Way 2** |  | Attempt cross product of direction vectors | M1 |
| Correct vector | A1 |
|  | Attempt equation of both planes | M1 |
|  | Correct completion | M1 |
| Any equivalent e.g. or awrt 3.84 but must be positive. | A1 |
|  |  | **(5)** |

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| **(b)****Way 3** |  | Finds a general chord between the 2 lines and attempts the scalar product between this and the directions, sets = 0 to give 2 equations in 2 unknowns | M1 |
|  | Correct values | A1 |
| Or | Uses their values to find the ends of the chord or substitutes into their chord vector | M1 |
|  | Correct completion by finding the distance between their 2 points | M1 |
| Any equivalent e.g. or awrt 3.84 | A1 |
|  |  | **(5)** |

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| **(b)****Way 4** |  | Attempt cross product of direction vectors | M1 |
| Correct vector | A1 |
|  | Finds a common chord between the 2 lines and sets equal to a multiple of the normal vector to give 3 equations in 3 unknowns and solves to find a value for *k*  | M1 |
|  | Correct completion by finding the length of their vector | M1 |
| Any equivalent e.g. or awrt 3.84 | A1 |
|  |  | **(5)** |

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| **(c)** |  | Attempt another non-parallel vector in  *П* | M1 |
|  | Attempt cross product of two non-parallel vectors in the plane. If the method is not shown, at least 2 components should be correct. **Dependent on the first M mark.** | **d**M1 |
|  | Attempt scalar product with a point in the plane. **Dependent on both previous method marks.** | **dd**M1 |
|  | Any multiple but must be a Cartesian equation. | A1 |
|  |  | **(4)** |
| **(c) Way 2** |  | Attempt another vector in  *П* | M1 |
|  | Forms the vector equation of the plane.  **Dependent on the first M mark.** | **d**M1 |
|  | Eliminates λ or μ.  **Dependent on both previous method marks.** | **dd**M1 |
|  | Any correct equation but must be a correct Cartesian equation. Isw | A1 |
|  |  | **(4)** |
|  |  |  | **Total 13** |