**June 2010 Composite Paper FP1-3**

**1.** z = 2 – 3i

(*a*) Show that *z*2 = −5 −12i.

**(2)**

Find, showing your working,

(*b*) the value of ⏐*z*2⏐,

**(2)**

(*c*) the value of arg (*z*2), giving your answer in radians to 2 decimal places.

**(2)**

(*d*) Show *z* and *z*2 on a single Argand diagram.

**(1)**

**2.** **M** = , where a is a real constant.

(*a*) Given that *a* = 2, find **M**–1.

**(3)**

(*b*) Find the values of *a* for which **M** is singular.

**(2)**

**3.** f(*x*) = *x*3 + *x*2 + 44*x* + 150.

Given that f (*x*) = (*x* + 3)(*x*2 + *ax* + *b*), where *a* and *b* are real constants,

(*a*) find the value of *a* and the value of *b*.

**(2)**

(*b*) Find the three roots of f(*x*) = 0.

**(4)**

(*c*) Find the sum of the three roots of f (*x*) = 0.

**(1)**

**4.** Write down the 2 × 2 matrix that represents

(*a*) an enlargement with centre (0, 0) and scale factor 8,

**(1)**

(*b*) a reflection in the *x*-axis.

**(1)**

Hence, or otherwise,

(*c*) find the matrix **T** that represents an enlargement with centre (0, 0) and scale factor 8, followed by a reflection in the *x*-axis.

**(2)**

**A** =  and **B** = , where *k* and *c* are constants.

(*d*) Find **AB**.

**(3)**

Given that **AB** represents the same transformation as **T**,

(*e*) find the value of *k* and the value of *c*.

**(2)**

**6.** f(*n*) = 2*n* + 6*n*.

(*a*) Show that f(*k* +1) = 6f(*k*) − 4(2*k* ).

**(3)**

(*b*) Hence, or otherwise, prove by induction that, for *n* ∈ ℤ+, f(*n*) is divisible by 8.

**(4)**

**7.** (*a*) Prove by induction that

 = *n*(*n* + 1)(2*n* + 1).

**(6)**

Using the standard results for  and ,

(*b*) show that

 = *n*(*n*2 + *an* + *b*),

where *a* and *b* are integers to be found.

**(5)**

(*c*) Hence show that

 = *n*(7*n*2 + 27*n* + 26).

**(3)**

**8.** (*a*) Express  in partial fractions.

**(2)**

(*b*) Using your answer to part (a) and the method of differences, show that

 = .

**(3)**

(*c*) Evaluate , giving your answer to 3 significant figures.

**(2)**

**9.** *z* = −8 + (8√3)i

(*a*) Find the modulus of *z* and the argument of *z*.

**(3)**

Using de Moivre’s theorem,

(*b*) find *z*3,

**(2)**

(*c*) find the values of *w* such that *w*4 = *z*, giving your answers in the form *a* + i*b*, where *a*, *b* ∈ℝ.

**(5)**

**10.** A complex number *z* is represented by the point *P* in the Argand diagram.

(*a*) Given that ⏐*z* − 6⏐ = ⏐*z*⏐, sketch the locus of *P.*

**(2)**

(*b*) Find the complex numbers *z* which satisfy both ⏐*z* − 6⏐ = ⏐*z*⏐and⏐ *z* − 3− 4i⏐ = 5.

**(3)**

**11.** (*a*) Show that the transformation *z* =  transforms the differential equation

 – 4*y* tan *x* =  (I)

into the differential equation

 – 2*z* tan *x* = 1 (II)

**(5)**

(*b*) Solve the differential equation (II) to find *z* as a function of *x*.

**(6)**

(*c*) Hence obtain the general solution of the differential equation (I).

**(1)**

**12.**



**Figure 1**

Figure 1 shows the curves given by the polar equations

*r* = 2, 0 ≤ *θ* ≤ ,

and *r* = 1.5 + sin 3*θ*, 0 ≤ *θ* ≤ .

(*a*) Find the coordinates of the points where the curves intersect.

**(3)**

The region *S*, between the curves, for which *r*> 2 and for which *r*< (1.5 + sin 3*θ*), is shown shaded in Figure 1.

(*b*) Find, by integration, the area of the shaded region *S*, giving your answer in the form *aπ* + *b*√3, where *a* and *b* are simplified fractions.

**(7)**

**13.** (*a*) Find the value of *λ* for which *y* = *λx* sin 5*x* is a particular integral of the differential equation

 + 25*y* = 3 cos 5*x*.

**(4)**

(*b*) Using your answer to part (a), find the general solution of the differential equation

 + 25*y* = 3 cos 5*x*.

**(3)**

Given that at *x* = 0, *y* = 0 and  = 5,

(*c*) find the particular solution of this differential equation, giving your solution in the form *y*= f(*x*).

**(5)**

(*d*) Sketch the curve with equation *y* = f(*x*) for 0 ≤ *x* ≤ *π*.

**(2)**

**14.** Use calculus to find the exact value of  d*x*.

**(5)**

**15.** (*a*) Starting from the definitions of sinh *x* and cosh *x* in terms of exponentials, prove that

cosh 2*x* = 1 + 2 sinh2 *x*

**(3)**

(*b*) Solve the equation

cosh 2*x* − 3 sinh *x* = 15,

giving your answers as exact logarithms.

**(1)**

**16.** Given that *y* = (arcosh 3*x*)2, where 3*x* >1, show that

(*a*) (9*x*2 – 1)  = 36*y*,

**(5)**

(*b*) (9*x*2 – 1) + 9*x* = 18.

**(4)**

**17.** **M** = **** , where *k* is a constant.

A transformation *T* : ℝ3 → ℝ3 is represented by **M**.

The transformation *T* maps the line *l*1, with cartesian equations  =  = , onto the line *l*2.

(*d*) Taking *k* = 3, find cartesian equations of *l*2.

**(5)**

**18.** The plane *Π* has vector equation

**r** = 3**i** + **k** + *λ* (–4**i** + **j**) + *μ* (6**i** – 2**j** + **k**)

(*a*) Find an equation of *Π* in the form **r.n** = *p*, where **n** is a vector perpendicular to *Π* and *p* is a constant.

**(5)**

The point *P* has coordinates (6, 13, 5). The line *l* passes through *P* and is perpendicular to *Π*. The line *l* intersects *Π* at the point *N*.

(*b*) Show that the coordinates of *N* are (3, 1, –1).

**(4)**

The point *R* lies on *Π* and has coordinates (1, 0, 2).

(*c*) Find the perpendicular distance from *N* to the line *PR*. Give your answer to 3 significant figures.

**(5)**

**Total Marks 144**