IYGB – MP1 PAPER N – QUESTION 1

a) BY THE REMAINDER/FACTOR THEOREM

$$\boxed{f(x) = x^3 - 3x^2 + 6x - 40}$$

$\Rightarrow f(5) = 5^3 - 3\times5^2 + 6\times5 - 40$

$\Rightarrow f(5) = 125 - 75 + 30 - 40$

$\Rightarrow f(5) = 40 \neq 0$

$\therefore$ $(x-5)$ IS NOT A FACTOR OF $f(x)$

b) USING THE SAME METHOD AND TRYING WITH THE FACTORS OF 40

$\Rightarrow f(1) = 1 - 3 + 6 - 40 \neq 0$

$\Rightarrow f(-1) = -1 - 3 - 1 - 40 \neq 0$

$\Rightarrow f(2) = 8 - 12 + 12 - 40 \neq 0$

$\Rightarrow f(-2) = -8 - 12 - 12 - 40 \neq 0$

$\Rightarrow f(4) = 64 - 48 + 24 - 40 = 0$

$\therefore$ $(x-4)$ IS A FACTOR OF $f(x)$

IYGB - MP1 PAPER N - QUESTION 2

a) COMPLETING THE SQUARE IN x AND IN y GIVES

$$\Rightarrow x^2+y^2-10x+6y-15=0$$
$$\Rightarrow x^2-10x+y^2+6y-15=0$$
$$\Rightarrow (x-5)^2-25+(y+3)^2-9-15=0$$
$$\Rightarrow (x-5)^2+(y+3)^2=49$$

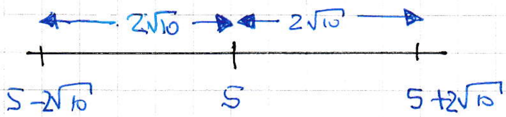
CENTRE AT $(5,-3)$, RADIUS OF $\sqrt{49}=7$

b) SETTING $y=0$ AND SOLVING THE RESULTING EQUATION

$$\Rightarrow (x-5)^2+(0+3)^2=49$$
$$\Rightarrow (x-5)^2=40$$
$$\Rightarrow x-5=\begin{cases}\sqrt{40}\\ -\sqrt{40}\end{cases}$$
$$\Rightarrow x=\begin{cases}5+2\sqrt{10}\\ 5-2\sqrt{10}\end{cases}$$

HENCE $|AB|=4\sqrt{10}$

SINCE



OR $\quad (\cancel{5}+2\sqrt{10})-(\cancel{5}-2\sqrt{10}) = 4\sqrt{10}$

LARGER $\qquad$ SMALLER

IYGB - MP1 PAPER N - QUESTION 3

$$x^2 + (2k+3)x + (k^2+3k+1) = 0$$

USING THE DISCRIMINANT OF THE QUADRATIC

$$\Rightarrow b^2 - 4ac = (2k+3)^2 - 4 \times 1 \times (k^2+3k+1)$$

$$= (2k+3)^2 - 4(k^2+3k+1)$$

$$= \cancel{4k^2} + \cancel{12k} + 9 - \cancel{4k^2} - \cancel{12k} - 4$$

$$= 5$$

AS THE DISCRIMINANT IS POSITIVE, THE EQUATION WILL ALWAYS HAVE 2 DISTINCT REAL ROOTS

IYGB - MP1 PAPER N - QUESTION 4

a) EXPAND USING THE STANDARD FORMULA BELOW

$$\Rightarrow (1+ax)^n = 1 + \frac{n}{1}(ax)^1 + \frac{n(n-1)}{1\times 2}(ax)^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}(ax)^3 + \dots$$

$$\Rightarrow (1+2x)^7 = 1 + \frac{7}{1}(2x)^1 + \frac{7\times 6}{1\times 2}(2x)^2 + \frac{7\times 6\times 5}{1\times 2\times 3}(2x)^3 + \dots$$

$$\Rightarrow (1+2x)^7 = 1 + 14x + 84x^2 + 280x^3 + \dots$$

b) NEXT EXPAND $(3+2x)^4$ IN ASCENDING POWERS OF x, UP TO x (I.E "CONSTANT + x")

$$(3+2x)^4 = \binom{4}{0}(3)^4(2x)^0 + \binom{4}{1}(3)^3(2x)^1 + \dots$$

$$= (1\times 81\times 1) + (4\times 27\times 2x) + \dots$$

$$= 81 + 216x + \dots$$

HENCE WE HAVE

$$(1+2x)^7(3+2x)^4 = (1 + 14x + \dots)(81 + 216x + \dots)$$

$1134x$

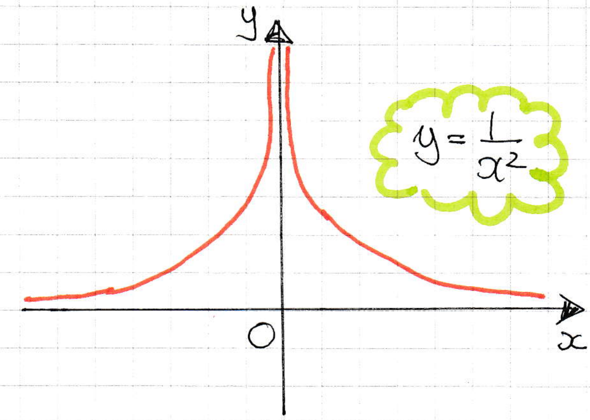
$216x$

$$1134x + 216x = 1350x$$

THE REQUIRED COEFFICIENT IS 1350

IYGB - MP1 - PAPER N - QUESTION 5

a)

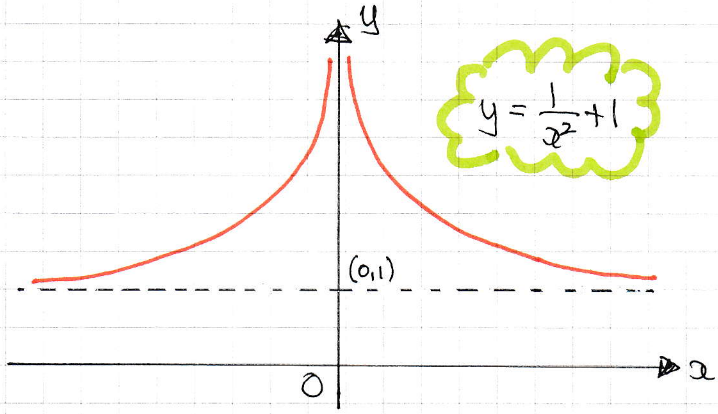


THIS IS A "STANDARD" CURVE

ASYMPTOTES

$x = 0$ (y AXIS)
$y = 0$ (x AXIS)

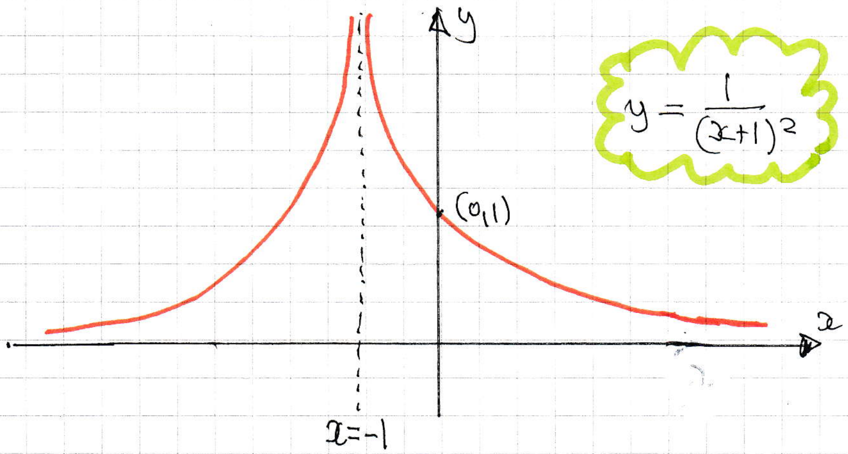
b) I) THIS IS A TRANSLATION "UP" BY 1 UNIT



ASYMPTOTES

$y = 1$
$x = 0$ (y AXIS)

II) THIS IS A TRANSLATION BY 1 UNIT TO THE "LEFT"



ASYMPTOTES

$x = -1$
$y = 0$ (x AXIS)

IYGB, MP1 PAPER N, QUESTION 6

IF $x=\frac{3}{2}$ IS A SOLUTION OF THE EQUATION, THEN IT MUST BALANCE IT

$$\Rightarrow 2x^2+x+k=0$$
$$\Rightarrow 2\left(\frac{3}{2}\right)^2+\frac{3}{2}+k=0$$
$$\Rightarrow 2\times\frac{9}{4}+\frac{3}{2}+k=0$$
$$\Rightarrow \frac{9}{2}+\frac{3}{2}+k=0$$
$$\Rightarrow k=-6$$

SUBSTITUTING $k=-6$ INTO THE EQUATION AND SOLVING

$$\Rightarrow 2x^2+x-6=0$$
$$\Rightarrow (2x-3)(x+2)=0$$
$$\Rightarrow x = \begin{cases} 3/2 & \text{(ALREADY KNOWN)} \\ -2 & (x_0) \end{cases}$$

IYGB - MP1 PAPER N - QUESTION 7

$C: \; y = x^2 + bx + c \qquad\qquad L: \; y = mx + 4$

USING $Q(3,-2)$ INTO THE LINE L

$\Rightarrow y = mx + 4$
$\Rightarrow -2 = 3m + 4$
$\Rightarrow -6 = 3m$
$\Rightarrow m = -2$

USING $P(k,6)$ WITH THE LINE L

$y = mx + 4$
$y = -2x + 4$
$6 = -2k + 4$
$2k = -2$
$k = -1$

FINALLY USING THE TWO POINTS $P(-1,6)$ & $Q(3,-2)$ WITH THE QUADRATIC CURVE C

$\Rightarrow y = x^2 + bx + c$
$\Rightarrow 6 = (-1)^2 + b(-1) + c$
$\Rightarrow 6 = 1 - b + c$
$\Rightarrow b - c = -5$

&

$\Rightarrow y = x^2 + bx + c$
$\Rightarrow -2 = 3^2 + b(3) + c$
$\Rightarrow -2 = 9 + 3b + c$
$\Rightarrow 3b + c = -11$

ADDING THE LAST TWO EXPRESSIONS GIVES

$\Rightarrow 4b = -16$
$\Rightarrow b = -4$

FINALLY USING $3b + c = -11$

$\Rightarrow 3(-4) + c = -11$
$\Rightarrow -12 + c = -11$
$\Rightarrow c = 1$

IYGB - MP1 PAPER N - QUESTION 8

START BY WRITING THE $\tan x$ IN TERMS OF SINES & COSINES

$$\Longrightarrow 2\cos x = 3\tan x$$

$$\Longrightarrow 2\cos x = \frac{3\sin x}{\cos x}$$

$$\Longrightarrow 2\cos^2 x = 3\sin x$$

USING THE IDENTITY $\cos^2 x + \sin^2 x \equiv 1$

$$\Longrightarrow 2(1-\sin^2 x) = 3\sin x$$

$$\Longrightarrow 2-2\sin^2 x = 3\sin x$$

$$\Longrightarrow 0 = 2\sin^2 x + 3\sin x - 2$$

$$\Longrightarrow (2\sin x - 1)(\sin x + 2) = 0$$

$$\Longrightarrow \sin x = \begin{cases} \cancel{-2} \\ \frac{1}{2} \end{cases}$$

$$\arcsin\frac{1}{2} = 30^\circ$$

$$\left(\begin{array}{l} x = 30^\circ \pm 360n \\ x = 150^\circ \pm 360n \end{array} \right. \qquad n = 0,1,2,3,\ldots$$

THUS IN THE RANGE GIVEN

$$x_1 = 30^\circ$$

$$x_2 = 150^\circ$$

IYGB - MP1 PAPER N - QUESTION 9

FROM ELEMENTARY GEOMETRY, THE STRAIGHT LINE THROUGH B & D IS THE PERPENDICULAR BISECTOR OF AC

MIDPOINT OF AC WHERE A(4,3) & C(8,-7) IS

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{4+8}{2}, \frac{3-7}{2}\right) = M(6,-2)$$

GRADIENT AC IS

$$\frac{y_2-y_1}{x_2-x_1} = \frac{-7-3}{8-4} = \frac{-10}{4} = -\frac{5}{2}$$

GRADIENT OF THE LINE THROUGH B & D MUST BE $+\frac{2}{5}$

FINALLY THE EQUATION OF THE REQUIRED LINE IS

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y + 2 = \frac{2}{5}(x - 6)$$

$$\Rightarrow 5y + 10 = 2x - 12$$

$$\Rightarrow 5y = 2x - 22$$

IYGB - MP1 PAPER N - QUESTION 10

a) $\log_2 45 = \log_2(5\times 9)$

$= \log_2 5 + \log_2 9$

$= \log_2 5 + \log_2 3^2$

$= \log_2 5 + 2\log_2 3$

$= q + 2p$

b) $\log_2(0.3) = \log_2 \frac{3}{10}$

$= \log_2 3 - \log_2 10$

$= \log_2 3 - \log_2(5\times 2)$

$= \log_2 3 - [\log_2 5 + \log_2]$

$= p - (q+1)$

$= p - q - 1$

IYGB - MP1 PAPER N - QUESTION 11

a) REWRITE THE EQUATION IN SURD FORM

$$\Rightarrow x\left(x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}\right)^2 = 0$$

$$\Rightarrow x\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)^2 = 0$$

EVIDENTLY $x > 0$, HENCE WE MAY WRITE

$$\Rightarrow \left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)^2 = 0$$

$$\Rightarrow \sqrt{x} - \frac{2}{\sqrt{x}} = 0$$

$$\Rightarrow \sqrt{x}\sqrt{x} - 2 = 0$$

$$\Rightarrow x = 2$$

b) SIMPLIFY THE NUMERATOR BEFORE RATIONALIZING

$$\Rightarrow \frac{\sqrt{98} - \sqrt{8}}{1+\sqrt{2}} = \frac{\sqrt{49}\sqrt{2} - \sqrt{4}\sqrt{2}}{1+\sqrt{2}}$$

$$= \frac{7\sqrt{2} - 2\sqrt{2}}{1+\sqrt{2}}$$

$$= \frac{5\sqrt{2}}{1+\sqrt{2}}$$

$$= \frac{5\sqrt{2}(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})}$$

$$= \frac{5\sqrt{2} - 5\sqrt{2}\sqrt{2}}{1 - \cancel{\sqrt{2}} + \cancel{\sqrt{2}} - 2}$$

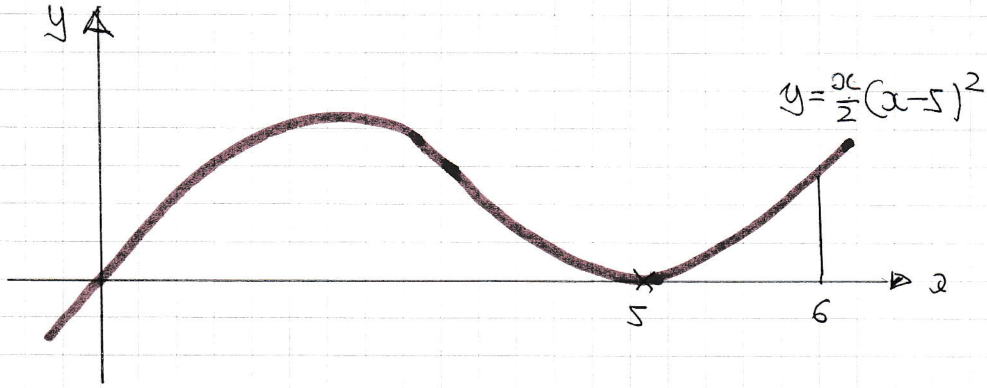
$$= \frac{5\sqrt{2} - 10}{-1}$$

$$= 10 - 5\sqrt{2}$$

$a = 10$
$b = -5$

IYGB – MP1 PAPER N – QUESTION 12

FIND THE AREA "ABOVE" THE x AXIS AND DOUBLE IT



EXPAND THE CUBIC

$$y = \frac{1}{2}x(x-5)^2 = \frac{1}{2}x(x^2-10x+25) = \frac{1}{2}x^3 - 5x^2 + \frac{25}{2}x$$

INTEGRATE FROM 0 to 6 (NO NEED TO SPLIT THE RANGE)

$$\int_0^6 \frac{1}{2}x^3 - 5x^2 + \frac{25}{2}x \, dx = \left[\frac{1}{8}x^4 - \frac{5}{3}x^3 + \frac{25}{4}x^2\right]_0^6$$

$$= (162 - 360 + 225) - (0)$$

$$= 27$$

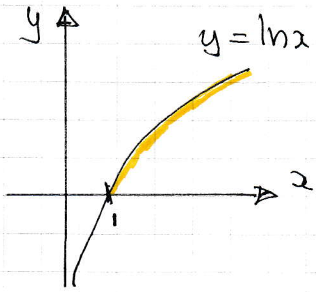
THE REQUIRED AREA IS $27 \times 2 = 54$

AS REQUIRED

IYGB - MP1 PAPER N - QUESTION 13

- RATHER THAN LOOKING FOR NUMBERS TO TRY IT IS BEST TO "SOLVE" AN INEQUALITY

IF $\ln A > 0$ THEN $A > 1$



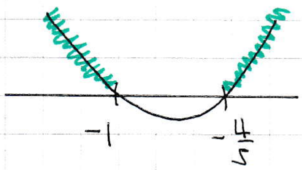
- THUS WE SOLVE A SIMPLE QUADRATIC INEQUALITY (SAY FOR POSITIVE)

$$5x^2 + 9x + 5 > 1$$

$$5x^2 + 9x + 4 > 0$$

$$(5x+4)(x+1) > 0$$

$$\text{C.V} = \begin{cases} -1 \\ -\frac{4}{5} \end{cases}$$



- $f(x) > 0$ IF $x < -1$ OR $x > -\frac{4}{5}$
- $f(x) \leqslant 0$ IF $-1 \leqslant x < -\frac{4}{5}$

- HENCE $f(-0.9) = \ln\left[5\times(-0.9)^2 + 9(-0.9) + 5\right]$

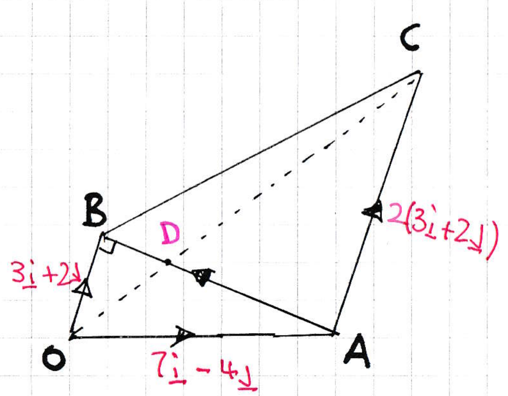
$$= \ln(0.95)$$

$$= -0.051293\ldots$$

AND THE STATEMENT IS DISPROVED

IYGB - MP1 - PAPER N - QUESTION 14

a) LOOKING AT THE DIAGRAM



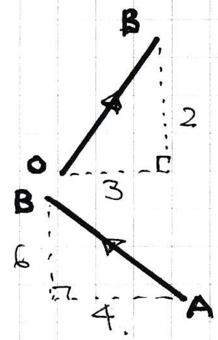
- $\vec{OC} = \vec{OA} + \vec{AC}$
  $= (7\underline{i} - 4\underline{j}) + 2(3\underline{i} + 2\underline{j})$
  $= 13\underline{i}$
- $\vec{AB} = \vec{AO} + \vec{OB}$
  $= -(7\underline{i} - 4\underline{j}) + (3\underline{i} + 2\underline{j})$
  $= -4\underline{i} + 6\underline{j}$
- $\vec{AD} = \frac{2}{3}(\vec{AB}) = \frac{2}{3}(-4\underline{i} + 6\underline{j}) = -\frac{8}{3}\underline{i} + 4\underline{j}$
- $\vec{OD} = \vec{OA} + \vec{AD} = (7\underline{i} - 4\underline{j}) + (-\frac{8}{3}\underline{i} + 4\underline{j}) = \frac{13}{3}\underline{i}$

b) $\vec{OC} = 13\underline{i}$
$\vec{OD} = \frac{13}{3}\underline{i}$

$\therefore$ PARALLEL, SO O, C, D ARE COLLINEAR

$|OC| : |OD|$
$3 : 1$

c) LOOKING AT THE COLUMN VECTORS AS GRADIENT REPRESENTATIONS

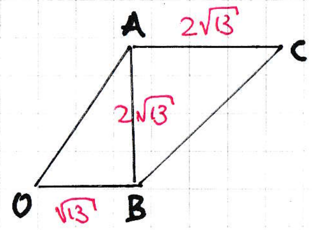


GRADIENT $= \frac{2}{3}$

GRADIENT $= -\frac{6}{4} = -\frac{3}{2}$

NEGATIVE RECIPROCAL GRADIENTS, SO PERPENDICULAR INDEED

$|\vec{OB}| = |3\underline{i} + 2\underline{j}| = \sqrt{3^2+2^2} = \sqrt{13}$
$|\vec{AC}| = 2|\vec{OB}| = 2\sqrt{13}$
$|\vec{AB}| = |-4\underline{i} + 6\underline{j}| = \sqrt{(-4)^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$



AREA $= \frac{2\sqrt{13} + \sqrt{13}}{2} \times 2\sqrt{13}$

$= \frac{3}{2}\sqrt{13} \times 2\sqrt{13}$

$= 39$

IYGB - MP1 - PRACTICE PAPER N - QUESTION 14

d) LOOKING AT THE DIAGRAM

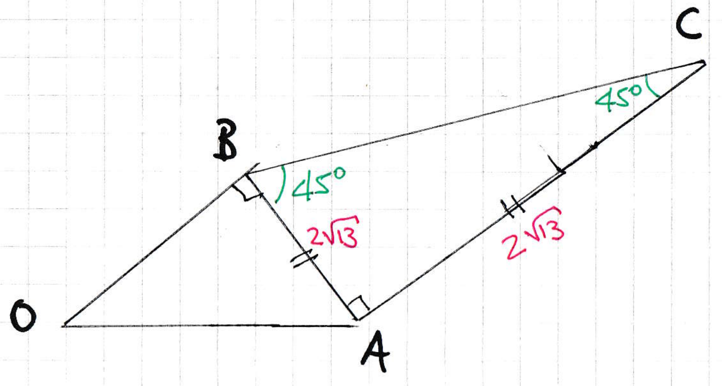
$|AC| = 2\sqrt{13}$

$|AB| = 2\sqrt{13}$

& $B\hat{A}C = 90$
(ALTERNATE TO $O\hat{B}A$)

$\therefore$ $\overset{\triangle}{BAC}$ IS ISOSCELES AND RIGHT ANGLED

SO $A\hat{B}C = 45°$



NOT TO SCALE

IYGB - MP1 PAPER N - QUESTION 15

- FIRSTLY LET US NOTE THAT ALL POLYNOMIALS ARE CONTINUOUS, IE THE GRAPHS HAVE NO ASYMPTOTES OR OTHER DISCONTINUITIES – WRITE THE EQUATION AS A FUNCTION & LOOK FOR INTERSECTIONS WITH THE x AXIS

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 15$$
$$f'(x) = 12x^3 - 12x^2 - 24x$$
$$f'(x) = 12x(x^2 - x - 2)$$
$$f'(x) = 12x(x+1)(x-2)$$

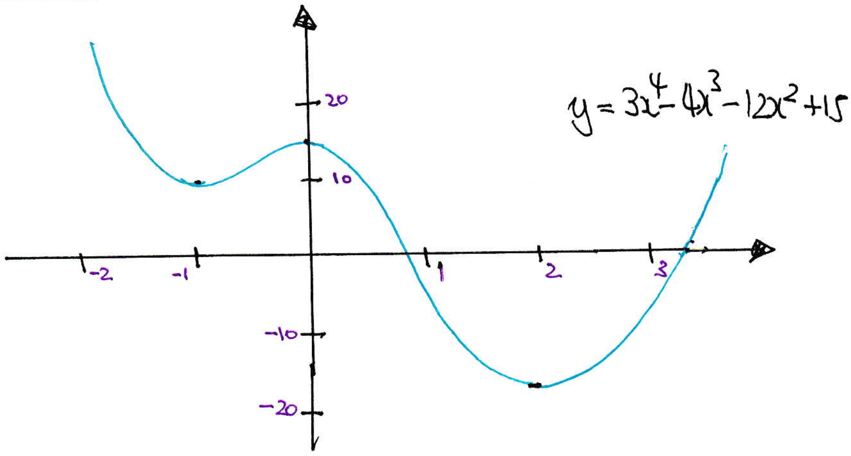
- STATIONARY VALUES $x = 0, -1, 2$

$f(0) = 15$ IE $(0, 15)$

$f(-1) = 3 + 4 - 12 + 15 = 10$ I.E. $(-1, 10)$

$f(2) = 48 - 32 - 48 + 15 = -17$ IE. $(2, -17)$

- A QUICK SKETCH FOLLOWS



$\therefore$ 2 SOLUTIONS

- NOTE – CHANGE OF SIGN WILL YIELD THE INTERVAL