IYGB - MP1 PAPER O - QUESTION 1

$$x^2 + 5px + 2p = 0 \quad , \quad p \text{ CONSTANT}$$

"HAS REAL ROOTS" $\Rightarrow$ 2 DISTINCT REAL ROOTS $(b^2-4ac>0)$

$\Rightarrow$ 1 REPEATED REAL ROOT $(b^2-4ac=0)$

THUS WE HAVE

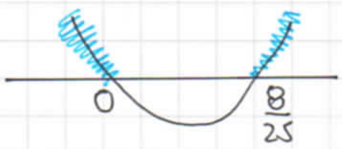
$$b^2-4ac \geqslant 0$$

$$(5p)^2 - 4\times1\times2p \geqslant 0$$

$$25p^2 - 8p \geqslant 0$$

$$p(25p-8) \geqslant 0$$

CRITICAL VALUES ARE $\left\langle \begin{matrix} 0 \\ \frac{8}{25} \end{matrix} \right.$



$\therefore \quad p \leqslant 0 \quad \underline{\text{OR}} \quad p \geqslant \frac{8}{25}$

IYGB – MP1 PAPER O – QUESTION 2

GRADIENT OF L

$$\frac{y_2-y_1}{x_2-x_1} = \frac{3-5}{-2-2} = \frac{-2}{-4} = \frac{1}{2}$$

EQUATION OF L, GRADIENT $\frac{1}{2}$, PASSING THROUGH $(2,5)$

$$\Longrightarrow y-y_0 = m(x-x_0)$$
$$\Longrightarrow y-5 = \tfrac{1}{2}(x-2)$$
$$\Longrightarrow 2y-10 = x-2$$
$$\Longrightarrow 2y = x+8$$

WHEN $x=0$

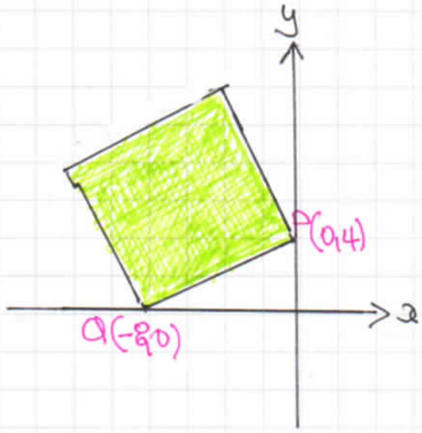
$$2y = 8$$
$$y = 4$$

$\therefore P(0,4)$

WHEN $y=0$

$$0 = x+8$$
$$x = -8$$

$\therefore Q(-8,0)$



LENGTH OF PQ IS GIVEN BY

$$\Longrightarrow d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$
$$\Longrightarrow |PQ| = (0-4)^2+(-8-0)^2$$
$$\Longrightarrow |PQ| = \sqrt{16+64}$$
$$\Longrightarrow |PQ| = \sqrt{80}$$

AREA OF SQUARE IS

$$\sqrt{80} \times \sqrt{80} = 80$$

IYGB - MP1 PAPER O - QUESTION 3

a) $\boxed{f(x) = 4x^3 - 8x^2 - x + k}$

$x-2$ IS A FACTOR $\Rightarrow f(2) = 0$

$\Rightarrow 4\times2^3 - 8\times2^2 - 2 + k$

$\Rightarrow 32 - 32 - 2 + k$

$\Rightarrow k = 2$

$f(x) = 4x^3 - 8x^2 - x + 2$

$f(\tfrac{1}{2}) = 4(\tfrac{1}{2})^3 - 8(\tfrac{1}{2})^2 - \tfrac{1}{2} + 2$

$= \tfrac{1}{2} - 2 - \tfrac{1}{2} + 2$

$= 0$

$\therefore$ $(2x-1)$ IS INDEED ALSO A FACTOR

c) BY INSPECTION WE HAVE

$f(x) = 4x^3 - 8x^2 - x + 2 = (x-2)(2x-1)(2x+1)$

c) $4\sin^3 y - 8\sin^2 y - \sin y + k = 0$

$\Rightarrow (\sin y - 2)(2\sin y - 1)(2\sin y + 1) = 0$

(PART a)

$\Rightarrow \sin y = \begin{cases} \cancel{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{cases}$

IYGB - MP1 PAPER O - QUESTION 3

SOLVING SEPARATELY

- $\arcsin\left(\frac{1}{2}\right) = 30^\circ$

$$\begin{cases} y = 30^\circ \pm 360^\circ n \\ y = 150^\circ \pm 360^\circ n \end{cases}$$

$n = 0, 1, 2, 3, \dots$

- $\arcsin\left(-\frac{1}{2}\right) = -30^\circ$

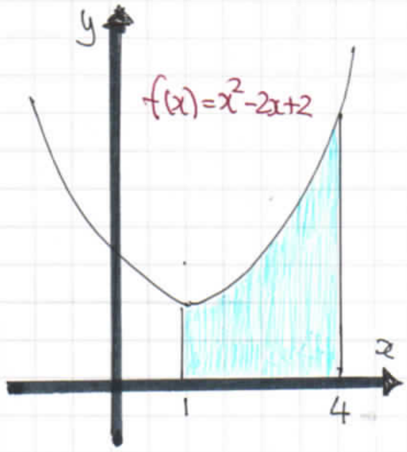
$$\begin{cases} y = -30^\circ \pm 360^\circ n \\ y = 210^\circ \pm 360^\circ n \end{cases}$$

$n = 0, 1, 2, 3, \dots$

$$y = 30^\circ,\ 150^\circ,\ 210^\circ,\ 330^\circ$$

IYGB - MP1 PAPER O - QUESTION 4

a) FINDING THE AREA BY INTEGRATION



$$\text{AREA} = \int_{x_1}^{x_2} f(x)\,dx = \int_1^4 x^2-2x+2\,dx$$

$$= \left[\tfrac{1}{3}x^3 - x^2 + 2x\right]_1^4$$

$$= \left(\tfrac{64}{3} - 16 + 8\right) - \left(\tfrac{1}{3} - 1 + 2\right)$$

$$= \tfrac{40}{3} - \tfrac{4}{3}$$

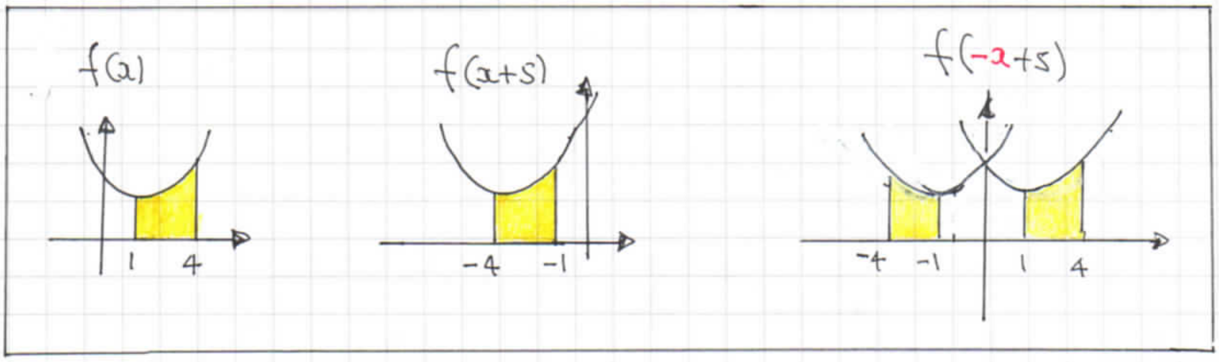
$$= \tfrac{36}{3}$$

$$= 12$$

b) NOW WE HAVE USING THE PROPERTIES OF THE INTEGRAL & TRANSFORMATION

$$\int_1^4 2f(5-x)\,dx = 2\int_1^4 f(5-x)\,dx = 2\int_1^4 f(x)\,dx = 2\times 12 = 24$$

(SEE BELOW)

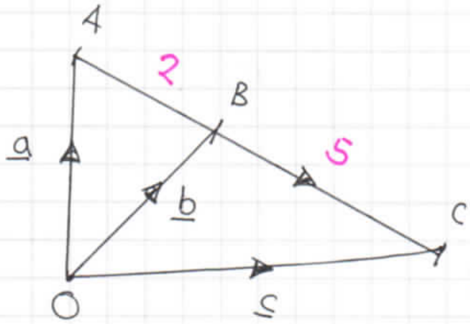


IYGB - MP1 PAPER O - QUESTION 5

a) $\underline{a} = 3\underline{i} - 2\underline{j}$

$\underline{b} = 5\underline{i} + 4\underline{j}$

$AB : BC = 2 : 5$



USING POSITION VECTORS

$\Rightarrow \overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$

$\Rightarrow \overrightarrow{OC} = \overrightarrow{OB} + \frac{5}{2}\overrightarrow{AB}$

$\Rightarrow \underline{c} = \underline{b} + \frac{5}{2}(\underline{b} - \underline{a})$

$\Rightarrow \underline{c} = \underline{b} + \frac{5}{2}\underline{b} - \frac{5}{2}\underline{a}$

$\Rightarrow \underline{c} = \frac{7}{2}\underline{b} - \frac{5}{2}\underline{a}$

$\Rightarrow \underline{c} = \frac{1}{2}(7\underline{b} - 5\underline{a})$

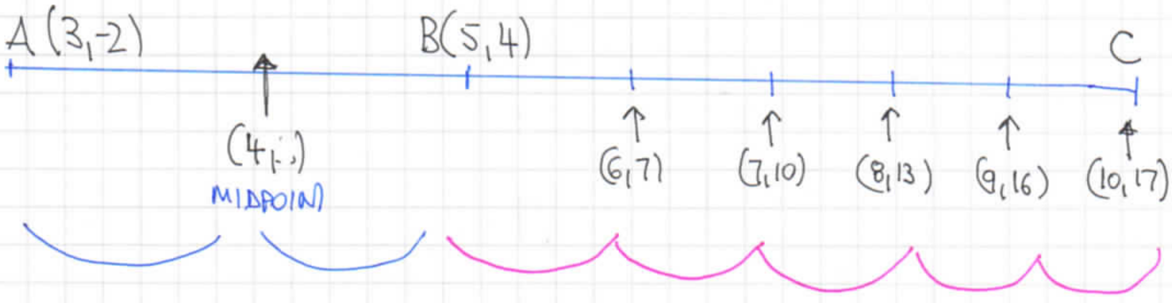
$\Rightarrow \underline{c} = \frac{1}{2}\left[7(5\underline{i} + 4\underline{j}) - 5(3\underline{i} - 2\underline{j})\right]$

$\Rightarrow \underline{c} = \frac{1}{2}\left[35\underline{i} + 28\underline{j} - 15\underline{i} + 10\underline{j}\right]$

$\Rightarrow \underline{c} = \frac{1}{2}\left[20\underline{i} + 38\underline{j}\right]$

$\Rightarrow \underline{c} = 10\underline{i} + 19\underline{j}$

OR SIMPLY BY INSPECTION



$\therefore \underline{c} = 10\underline{i} + 19\underline{j}$

AS BEFORE

IYGB - MP1 PAPER O - QUESTION 5

b)

- $\vec{AB} = \underline{b} - \underline{a} = (5\underline{i} + 4\underline{j}) - (3\underline{i} - 2\underline{j}) = 2\underline{i} + 6\underline{j}$
- DIRECTION CAN BE SCALED TO $\underline{i} + 3\underline{j}$
- HENCE SINCE $|\underline{i} + 3\underline{j}| = \sqrt{1^2 + 3^2} = \sqrt{10}$, WE NEED 6 "VECTOR STEPS" IN EITHER DIRECTION FROM B
- I.E $\underline{d} = \underline{b} + 6(\underline{i} + 3\underline{j}) = 5\underline{i} + 4\underline{j} + 6\underline{i} + 18\underline{j}$

$$\underline{d} = \underline{b} - 6(\underline{i} + 3\underline{j}) = 5\underline{i} + 4\underline{j} - 6\underline{i} - 18\underline{j}$$

$$\therefore \underline{d} = 11\underline{i} + 22\underline{j} \quad \text{OR} \quad \underline{d} = -\underline{i} - 14\underline{j}$$

ALTERNATIVE

- LET $D(a, b)$
- GRADIENT $AB = \dfrac{4-(-2)}{5-3} = \dfrac{6}{2} = 3$
- LINE THROUGH A, B & D IS

$$y - 4 = 3(x - 5)$$
$$y - 4 = 3x - 15$$
$$y = 3x - 11$$

- HENCE $D(a, 3a - 11)$
- NOW THE DISTANCE $|BD| = 6\sqrt{10}$

$$\Rightarrow \sqrt{(3a - 11 - 4)^2 + (a - 5)^2} = 6\sqrt{10}$$

$$\Rightarrow (3a - 15)^2 + (a - 5)^2 = 360$$

IYGB - MP1 PAPER O - QUESTION 5

$$\Rightarrow \left(9a^2 - 90a + 225\right. \atop \left. a^2 - 10a + 25\right. = 360$$

$$\Rightarrow 10a^2 - 100a + 250 = 360$$

$$\Rightarrow a^2 - 10a + 25 = 36$$

$$\Rightarrow a^2 - 10a - 11 = 0$$

$$\Rightarrow (a+1)(a-11) = 0$$

$$\Rightarrow a = \begin{cases} -1 \\ 11 \end{cases} \qquad b = \begin{cases} 3(-1) - 11 = -14 \\ 3(11) - 11 = 22 \end{cases}$$

$\therefore$ $D(-1,-14)$ OR $(11,22)$

$\therefore$ $\underline{d} = -\underline{i} - 14\underline{j}$ OR $\underline{d}' = 11\underline{i} + 22\underline{j}$

IYGB – MP1 PAPER O – QUESTION 6

THE DERIVATIVE IS FORMALLY GIVEN BY

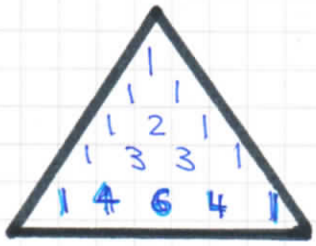
$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

IN THIS CASE WE HAVE

$$f(x) = x^4$$
$$f(x+h) = (x+h)^4$$

EXPANDING BINOMIALLY WE HAVE

$$(x+h)^4 = 1x^4h^0 + 4x^3h^1 + 6x^2h^2 + 4x^1h^3 + 1x^0h^4$$
$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$



TIDYING UP NEXT

$$f(x+h) - f(x) = (x+h)^4 - x^4 = (\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - \cancel{x^4}$$

FINALLY WE HAVE

$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h)-f(x)}{h}\right] = \lim_{h \to 0}\left[\frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}\right]$$

$$= \lim_{h \to 0}\left[4x^3 + \cancel{6x^2h} + \cancel{4xh^2} + \cancel{h^3}\right]$$

$$= 4x^3$$

IYGB - MP1 PAPER O - QUESTION 7

$m = 20e^{0.02t}$, $t \geq 0$

m = MASS IN kg

t = TIME IN HOURS

a) FIRSTLY WITH $t=0$

$m = 20 \times e^{0}$

$m = 20$ kg. ← INITIAL MASS

"THREE TIMES THE INITIAL MASS" ...

$60 = 20e^{0.02t}$

$3 = e^{0.02t}$

$\ln 3 = 0.02t$

$\ln 3 = \frac{1}{50}t$

$t = 50\ln 3 \approx 54.93$

∴ APPROX 55 HOURS

b) RATE OF CHANGE ⟹ DIFFERENTIATION

• $m = 20e^{0.02t}$

$\frac{dm}{dt} = 20 \times 0.02 \times e^{0.02t}$

$\frac{dm}{dt} = 0.4 \times e^{0.02t}$

• $m = 100$

$100 = 20e^{0.02t}$

$5 = e^{0.02t}$

COMBINING WE OBTAIN

$$\left.\frac{dm}{dt}\right|_{m=100} = 0.4 \times e^{0.02t} = 0.4 \times 5 = 2 \text{ kg h}^{-1}$$

IYGB - MP1 PAPER O - QUESTION 8

REWRITE THE EQUATION IN INDICIAL FORM & DIFFERENTIATE

$$\Rightarrow y = x\left(x^2 - 128\sqrt{x}\right)$$

$$\Rightarrow y = x\left(x^2 - 128x^{\frac{1}{2}}\right)$$

$$\Rightarrow y = x^3 - 128x^{\frac{3}{2}}$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 192x^{\frac{1}{2}}$$

FOR STATIONARY POINTS SET $\frac{dy}{dx} = 0$

$$\Rightarrow 3x^2 - 192x^{\frac{1}{2}} = 0$$

$$\Rightarrow x^2 - 64x^{\frac{1}{2}} = 0$$

$$\Rightarrow x^2 = 64x^{\frac{1}{2}}$$

$$\Rightarrow \frac{x^2}{x^{\frac{1}{2}}} = 64$$ (WE ARE NOT CONCERNED WITH $x=0$)

$$\Rightarrow x^{\frac{3}{2}} = 64$$

$$\Rightarrow \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = (64)^{\frac{2}{3}}$$

$$\Rightarrow x^1 = \left(\sqrt[3]{64}\right)^2$$

$$\Rightarrow x = 16$$

CHECK THE NATURE OF THE POINT BY THE SECOND DERIVATIVE TEST

$$\Rightarrow \frac{d^2y}{dx^2} = 6x - 96x^{-\frac{1}{2}}$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=16} = 6\times16 - 96\times16^{-\frac{1}{2}} = 96 - 96\times\frac{1}{4} = 96 - 24 = 72 > 0$$

LOCAL MINIMUM

IYGB - MP1 PAPER O - QUESTION 8

FINALLY TO FIND THE y CO-ORDINATE IN THE REQUIRED FORM

$$y = x\left(x^2 - 128\sqrt{x}\right)$$

$$y = 16\left(16^2 - 128\sqrt{16}\right)$$

$$y = -4096$$

$$y = -2^{12}$$ (TRIAL & ERROR OF POWER OF 2)

$\therefore$ LOCAL MINIMUM AT $\left(16, -2^{12}\right)$

IYGB - MP1 PAPER O - QUESTION 9

a) START BY OBTAINING THE "PARTICULARS" OF THE TWO CIRCLES

- $x^2+y^2-6x-2y=15$

$x^2-6x+y^2-2y=15$

$(x-3)^2-9+(y-1)^2-1=15$

$(x-3)^2+(y-1)^2=25$

CENTRE AT $(3,1)$

RADIUS 5

- $x^2+y^2-18x+14y=95$

$x^2-18x+y^2+14y=95$

$(x-9)^2-81+(y+7)^2-49=95$

$(x-9)^2+(y+7)^2=225$

CENTRE AT $(9,-7)$

RADIUS 15

IF THE DISTANCE BETWEEN THEIR CENTRES IS ....

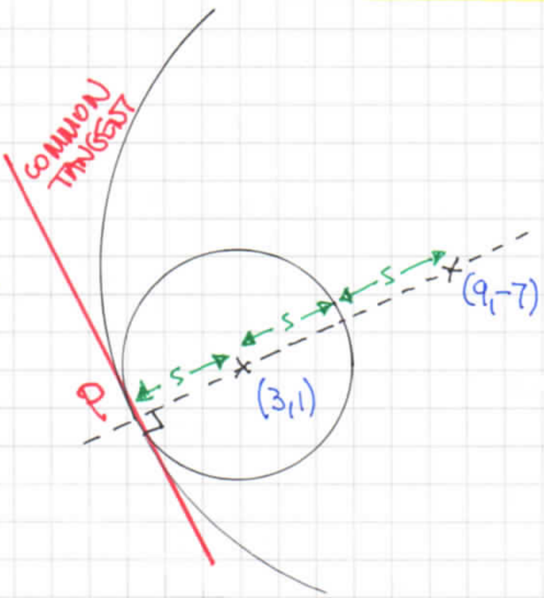
- $15+5=20$, THEY ARE TOUCHING EXTERNALLY
- $15-5=10$, THEY ARE TOUCHING EXTERNALLY

$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$d=\sqrt{(-7-1)^2+(9-3)^2}$

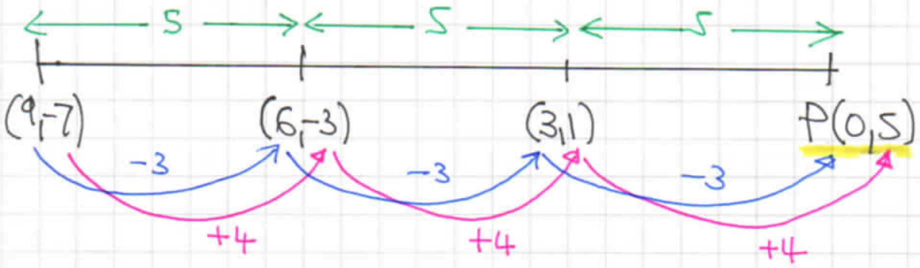
$d=\sqrt{64+36}=10$

INDEED THEY ARE TOUCHING INTERNALLY



IYGB - MP1 PAPER O - QUESTION 9

b) BY INSPECTION WE HAVE



c) GRADIENT OF COMMON RADIUS, USING (9,-7) & (3,1)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-7)}{3-9} = \frac{8}{-6} = -\frac{4}{3}$$

GRADIENT OF THE COMMON TANGENT, LOOKING AT A PREVIOUS DIAGRAM (PART a)

$$m_{(\text{TANGENT})} = +\frac{3}{4}$$

FINALLY WE HAVE, USING P(0,5)

$$y - y_0 = m(x - x_0)$$

OR SIMPLY

$$\Longrightarrow y = mx + c$$

$$\Longrightarrow y = \frac{3}{4}x + 5$$

$$\Longrightarrow 4y = 3x + 20$$

$$\Longrightarrow 0 = 3x - 4y + 20$$

AS REQUIRED

IYGB - MP1 PAPER O - QUESTION 10

a) STARTING BY MANIPULATING THE FORMULA

$$\Rightarrow y = ab^x$$

$$\Rightarrow \log_{10} y = \log_{10}(ab^x)$$

$$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} b^x$$

$$\Rightarrow \log_{10} y = \log_{10} a + x\log_{10} b$$

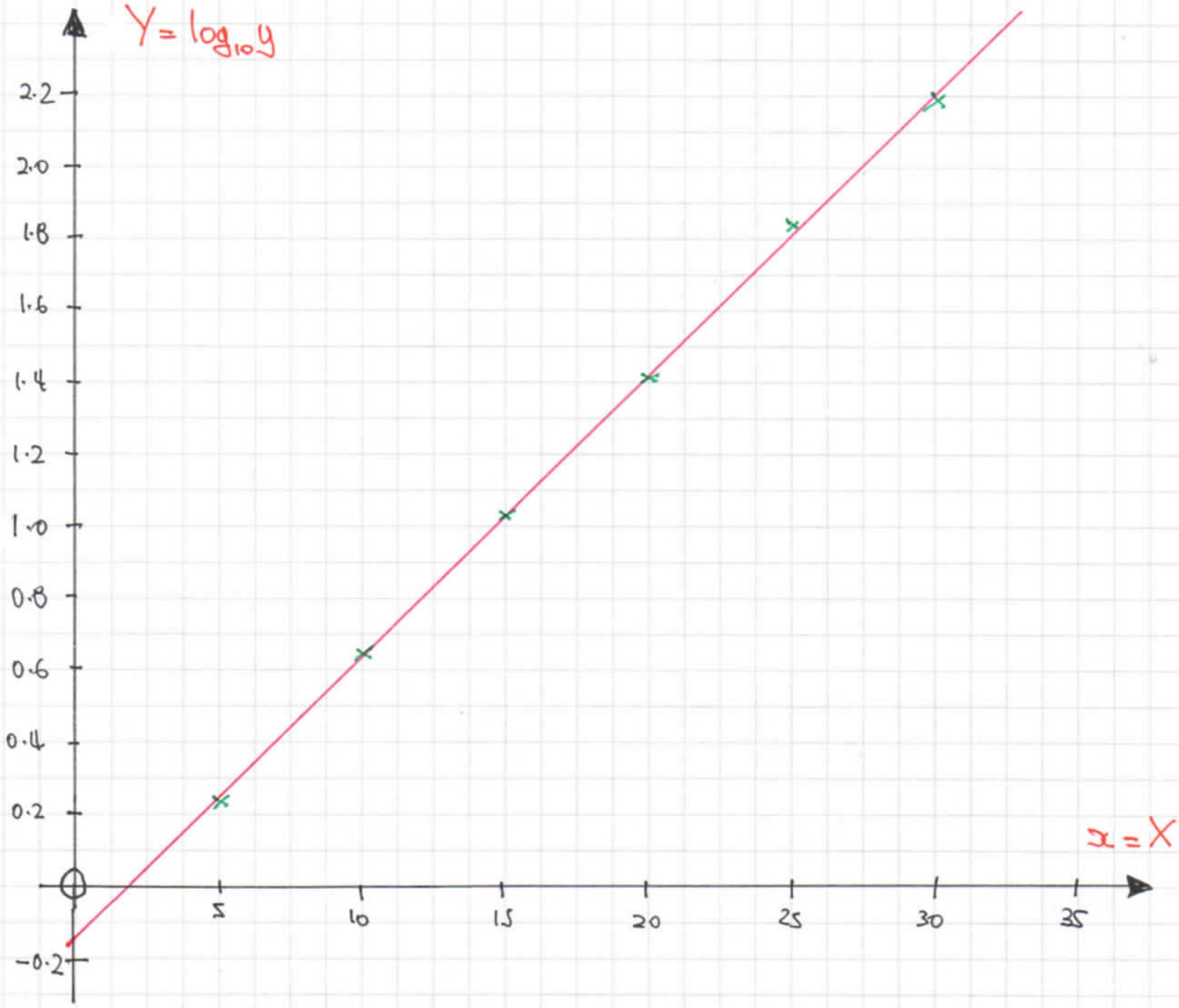
$$\Rightarrow \underbrace{\log_{10} y}_{Y} = \underbrace{(\log_{10} b)}_{m}\underbrace{x}_{X} + \underbrace{\log_{10} a}_{c}$$

PREPARE THE VALUES TO BE PLOTTED

$x = X$	5	10	15	20	25	30
y	1.7	4.5	11.0	26.0	70.0	160.0
$Y = \log_{10} y$	0.23	0.65	1.04	1.41	1.85	2.20

IYGB - MP1 PAPER O - QUESTION 10

PLOTTING THE DATA



AS THE POINTS FORM A STRAIGHT LINE THE RELATIONSHIP IS INDEED OF THE FORM $y = ab^x$

b) NOW WE HAVE BY COMPARING / READING VALUES

- $\log_{10} a = c$
  $\log_{10} a = -0.16$
  $a = 10^{-0.16}$
  $a \approx 0.7$
- $\log_{10} b = m$
  $\log_{10} b = \frac{1.8 - 0.24}{25 - 5}$
  $\log_{10} b = 0.078$
  $b \approx 1.2$

c) USING $y = ab^x = 0.7 \times 1.2^x$ WITH $x = 60$ WE OBTAIN ≈ 39000

IYGB - MP1 PAPER O - QUESTION 11

BY PYTHAGORAS ON THE TRIANGLE ON THE "LEFT"

$$\Rightarrow a^2 + b^2 = c^2$$

$$\Rightarrow a^2 + b^2 - c^2 = 0$$

BY PYTHAGORAS ON THE TRIANGLE ON THE "RIGHT"

$$\Rightarrow (a+1)^2 + (b+1)^2 = (c+1)^2$$

$$\Rightarrow a^2 + 2a + 1 + b^2 + 2b \cancel{+1} = c^2 + 2c \cancel{+1}$$

$$\Rightarrow (a^2 + b^2 - c^2) + 2a + 2b + 1 = 2c$$

$$\Rightarrow 0 + 2(a+b) + 1 = 2c$$

$$\Rightarrow 2(a+b) + 1 = 2c$$

L.H.S. WILL BE ODD IF a & b ARE BOTH INTEGERS

R.H.S WILL BE EVEN IF c IS AN INTEGER

HENCE NOT ALL OF a, b & c ARE INTEGERS

IYGB - MP1 PAPER O - QUESTION 12

a) START BY EXPANDING & COMPARING

$$f(x) \equiv (x-A)^3 - 4 \equiv x^3 - 6x^2 + 12x + B$$

$$\Longrightarrow (x-A)(x-A)^2 - 4 \equiv x^3 - 6x^2 + 12x + B$$

$$\Longrightarrow (x-A)(x^2 - 2Ax + A^2) - 4 \equiv x^3 - 6x^2 + 12x + B$$

$$\Longrightarrow \left.\begin{array}{l} x^3 - 2Ax^2 + A^2x \\ \quad - Ax^2 + 2A^2x - A^3 - 4 \end{array}\right) \equiv x^3 - 6x^2 + 12x + B$$

$$\Longrightarrow x^3 - 3Ax^2 + 2A^2x - (A^3+4) \equiv x^3 - 6x^2 + 12x + B$$

LOOKING AT THE COEFFICIENTS OF x^2 (x DOES NOT QUITE "WORK")

$[x^2]$ $-3A = -6$
$A = 2$ //

$[x]$ $2A^2 = 12$
$A^2 = 4$
NO UNIQUE ANSWER

$[x^0]$ $-A^3 - 4 = B$
$-8 - 4 = B$
$B = -12$ //

b) • $g(x) = x^2 - 4x + 5 = (x-2)^2 - 2^2 + 5 = (x-2)^2 + 1$

• $f(x) = (x-2)^3 - 4$

$f(0) = (-2)^3 - 4 = -12$

$f(x) = 0$

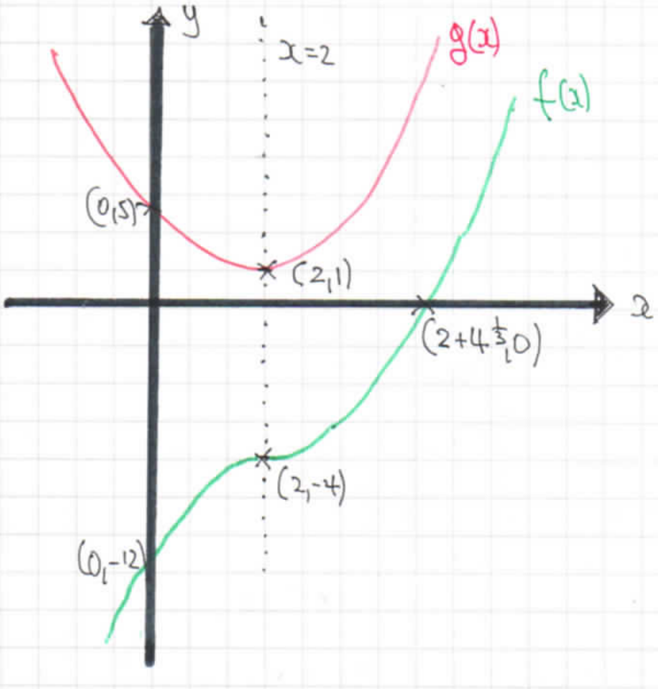
$\Rightarrow (x-2)^3 - 4 = 0$

$\Rightarrow (x-2)^3 = 4$

$\Rightarrow x - 2 = 4^{\frac{1}{3}}$

$\Rightarrow x = 2 + 4^{\frac{1}{3}}$

$g(0) = 5$



IYGB - MP1 PAPER O - QUESTION 12

c) $\Rightarrow x^3 - 7x^2 + 16x + B = 5$

$\Rightarrow x^3 - 7x^2 + 16x - 12 = 5$

$\Rightarrow x^3 - 6x^2 + 12x - 12 = x^2 - 4x + 5$

$\Rightarrow f(x) = g(x)$

ALTHOUGH IT APPEARS FROM THE SKETCH THAT THE TWO GRAPHS DO NOT MEET THE CUBIC WILL EVENTUALLY "OVERTAKE" THE QUADRATIC FOR SUFFICIENTLY LARGE x

$\therefore$ ONLY ONE ROOT