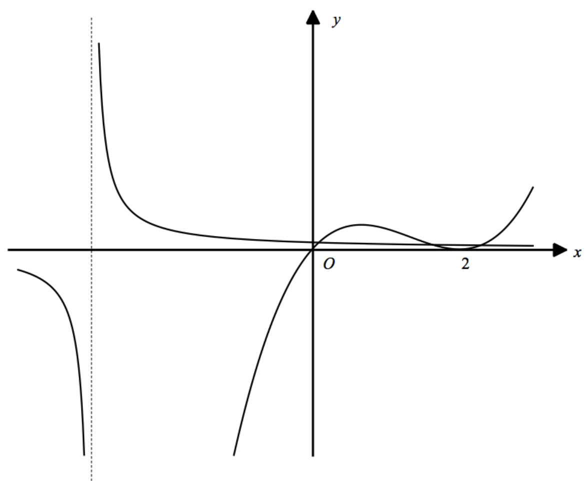
AS Level / Year 1

Paper 1 (Edexcel Version)

Version 1

Question	Scheme		Marks
1			
(a)	$k = 3$	Correct value of k	B1 **(1)**
(b)		Correct sketch Root at $x = 0$ and $x = 2$ Repeated root at $x = 2$ ****MARKS ONLY FOR** $y = x^3 - 4x^2 + 4x$ ******	B1 B1 B1 **(3)**
(c)	two solutions ; graphs intersect twice	Two roots + reason	B1ft B1 **(2)**
			6

Question 1 Notes

(b) Mark only information about the **cubic curve**.

(c) 1st B1ft: two roots (or ft their (b))

2nd B1: correct explanation oe. Accept equivalent phrasing, i.e. 'graphs meet twice'.

Question	Scheme		Marks
2			
	$\int_1^4 \left(\sqrt{x} - 2x^{-3} + 4\right) dx = \int_1^4 \left(x^{\frac{1}{2}} - 2x^{-3} + 4\right) dx$	Attempts to integrate	M1
	$= \left[\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} - \dfrac{2x^{-2}}{-2} + 4x \quad (+c)\right]_1^4$	One term integrated correctly All terms integrated correctly	A1 A1
	$= \left[\dfrac{2}{3}x^{\frac{3}{2}} + x^{-2} + 4x \quad (+c)\right]_1^4$		
	$\Rightarrow \int_1^4 \left(\sqrt{x} - 2x^{-3} + 4\right) dx = \left(\dfrac{2}{3}(4)^{\frac{3}{2}} + 4^{-2} + 4(4)\right) - \left(\dfrac{2}{3}(1)^{\frac{3}{2}} + 1^{-2} + 4(1)\right)$		dM1
	$= \dfrac{755}{48}$		A1
			5
	Question 2 Notes		

1st and 2nd A1: ignore any constant terms

2nd M1: dependent on 1st M1. Limits must be substituted the right way around

3rd A1: must be an exact value

Question	Scheme		Marks
3			
(a)	$\mathbf{F}_R = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} + \begin{pmatrix} -5 \\ -7 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$	Works out the resultant force	M1A1
	$\Rightarrow \lvert \mathbf{F}_R \rvert = 3$	Correct magnitude	A1
			(3)
(b)	$F_{R^*} = (-3+x)\mathbf{i} + y\mathbf{j}$ with $F_4 = x\mathbf{i} + y\mathbf{j}$ $\tan^{-1}\left(\dfrac{y}{-3+x}\right) = 45 \Rightarrow y = -3 + x$	Correct condition for y in terms of x (oe)	M1A1
	So e.g. $F_4 = \mathbf{i} - 2\mathbf{j}$	Any correct force	A1
			(3)
			6

Question 3 Notes

(a) M1 : attempts to combine the forces i.e. two forces added correctly

(b) **Correct answer only scores 3/3. **There are infinitely many correct answers****

An answer is correct provided: **j** component of F_4 = (**i** component of F_4) – 3

Question	Scheme		Marks
4			
(a)	$\dfrac{a+\sqrt{b}}{c+\sqrt{d}}\times\dfrac{c-\sqrt{d}}{c-\sqrt{d}}=\dfrac{\left(a+\sqrt{b}\right)\left(c-\sqrt{d}\right)}{\left(c+\sqrt{d}\right)\left(c-\sqrt{d}\right)}$	Rationalises	M1
	$=\dfrac{ac-a\sqrt{d}+c\sqrt{b}-\sqrt{bd}}{c^2-d}$	Expands the numerator	M1A1
		Correct expansion on denominator	A1
	$=\dfrac{ac}{c^2-d}+\dfrac{-a\sqrt{d}+c\sqrt{b}-\sqrt{bd}}{c^2-d}$ *	Cso	A1
			(5)
(b)	$\dfrac{m+\sqrt{2}}{1+\sqrt{8}}=...+\dfrac{-m\sqrt{8}+1\sqrt{2}}{...}$	Sets irrational part = 0	M1A1
	Rational if $-m\sqrt{8}+\sqrt{2}=0\Rightarrow m=\dfrac{\sqrt{2}}{\sqrt{8}}=\dfrac{1}{2}$	Correct value of *m* cao	
			(2)
			7

Question 4 Notes

(a) M1 : multiplies top and bottom by $c-\sqrt{d}$.

M1 : expands the numerator producing four terms with at least two terms correct

Answer is given. Solution must be presented convincingly and with no errors

Special case: Multiplies by $c+\sqrt{d}$. Can score maximum M0 M1 A0 A0 A0

(b) M1: sets irrational part = 0. **NB:** setting $-m\sqrt{8}+1\sqrt{2}+\sqrt{8\times 2}=0$ scores M0.

Question	Scheme		Marks
5			
(a) **Way 1**	$\lim_{h\to 0}\left(\frac{6(x+h)^2+1-(6x^2+1)}{h}\right)=\lim_{h\to 0}\left(\frac{12xh+h^2}{h}\right)$	Uses limit definition Expands brackets	M1 M1A1
	$=\lim_{h\to 0}(12x+h)$ $=12x$	Correct derivative	A1 **(4)**
(a) **Way 2**	$\lim_{x\to c}\left(\frac{6x^2+1-(6c^2+1)}{x-c}\right)=\lim_{x\to c}\left(\frac{6(x-c)(x+c)}{(x-c)}\right)$	Uses limit definition Uses difference of two squares	M1 M1
	$=\lim_{x\to c}(6x+6c)$ $=6c+6c$ $=12c$	Correct expression	A1
	$\Rightarrow(6x^2+1)'=12x$	Correct derivative	A1 **(4)**
(b)	$f'(x)=3x^2-4x+11$	Differentiates	M1A1
	$\text{Normal}=-\frac{1}{2}\Rightarrow \text{tangent}=2$		
	$3x^2-4x+11=2\Rightarrow 3x^2-4x+9=0$	Forms an equation	dM1A1
	$(-4)^2-4(3)(9)=16-108$ $=-92$ <0 $\therefore$ no real roots / no normal line with gradient -0.5	Attempts to show the equation has no real roots	ddM1 A1 **(6)**
			10

Question 5 Notes

(a) **Way 1**: Accept δx instead of *h*.
1st A1 – sight of $12x + h$ oe

Way 2: Uses $\lim_{x\to c}\left(\frac{f(x)-f(c)}{x-c}\right)$

1st A1: sight of $6x + 6c$.
2nd A1 – Final answer should be in terms of *x*

(b) 1st M1: attempts to differentiate.

2nd M1: sets $f'(x) = 2$ – dependent on 1st M1

2nd A1: forms the correct 3TQ

3rd M1: attempts to show the equation has no real roots – dependent on previous M marks; for example, use of the discriminant. Can also use completing the square.

3rd A1: convincing proof **and** conclusion.

Question	Scheme		Marks
6			
(a)	$90\left(2-e^{-0.05(0)}\right)=90(2-1)=90$	Cao	M1A1 **(2)**
(b)	$\frac{dN}{dt}=-90(-0.05)e^{-0.05t}$ $=4.5e^{-0.05t}$	Differentiates	B1B1 **(2)**
(c)	$4.5e^{-0.05t}>0$ <u>for all *t*</u>, so it is increasing	Explanation	B1 **(1)**
(d/i)	*idea that:* model predicts 180 plants will be infected and there are only 150 plants in the field	Correct idea	B1 **(1)**
(d/ii)	$150=90\left(2-e^{-0.05t}\right)\Rightarrow e^{-0.05t}=...$	Rearranges for $e^{-0.05t}$	M1
	$e^{-0.05t}=\frac{1}{3}\Rightarrow \ln\left(e^{-0.05t}\right)=\ln\frac{1}{3}$	Takes logs	dM1
	$\Rightarrow t=\frac{\ln(1/3)}{-0.05}=21.97...=22$ days	Correct value of *t*	A1 **(3)**
			9

Question 6 Notes

(a) M1 – substitutes 0 into the model

(b) 1st B1 – constant term goes to 0
2nd B1 – correct differentiation of exponential term (need not be simplified)

(c) B1 – convincing explanation. **Must** convey idea that derivative is positive <u>**for all *t***</u>.

(d) (i) B1 – use your judgment and award a mark for the idea that the model predicts <u>180</u> plants will be infected which is more than there are in the field.

Question	Scheme		Marks
7			
(a)	$m=\frac{5--3}{4-12}=\frac{8}{-8}=-1$	Attempts to find gradient	M1A1
	$l:\ y-5=-1(x-4)\Rightarrow x+y-9=0$	Attempts to find equation of l	M1A1
	$\{a=b=1,\ c=-9\}$		**(4)**
(b)	$3x^2+4x+7=9-x$	Attempts to find coordinates of intersection	M1
	$\Rightarrow 3x^2+5x-2=0$		
	$\Rightarrow (3x-1)(x+2)=0$	Attempts to solve their 3TQ	dM1
	$\Rightarrow x=\frac{1}{3}$ or $x=-2$		A1
	$x=\frac{1}{3}\Rightarrow y=9-\frac{1}{3}=\frac{26}{3}$	Attempts to find the y coordinates of intersection	ddM1
	$x=-2\Rightarrow y=9--2=11$		
	$A(-2,11)\quad B\left(\frac{1}{3}.\frac{26}{3}\right)$	Correct coordinates of A and B	A1
			(5)
(c)	$P(0,9)$		
	$\lvert AP\rvert=\sqrt{(-2-0)^2+(11-9)^2}$ $=2\sqrt{2}$	Attempts to find $\lvert AP\rvert$ or $\lvert BP\rvert$	M1
	$\lvert BP\rvert=\sqrt{\left(\frac{1}{3}-0\right)^2-\left(\frac{26}{3}-9\right)^2}$		
	$=\frac{\sqrt{2}}{3}$	Correctly finds $\lvert AP\rvert$ and $\lvert BP\rvert$	A1
	$\Rightarrow \lvert AP\rvert:\lvert BP\rvert=2\sqrt{2}:\frac{\sqrt{2}}{3}$		
	$=6:1$, so $m=6$	Correct ratio	A1
			(3)
			9

	Question 7 Notes

(a) 1st M1 – attempts to find gradient, i.e. use of $m = \frac{y_2 - y_1}{x_2 - x_1}$. Coordinates must be consistent. In other words $m = \frac{y_2 - y_1}{x_1 - x_2}$ scores M0.

2nd M1 – attempts to find equation of line with **their** *m* using $y - y_1 = m(x - x_1)$. If they use $y = mx + c$, method mark should be awarded for an attempt to find the constant.

2nd A1 – equation in the specified form oe. Values of *a*, *b* and *c* need not be quoted.

(b) Final A1 – must see coordinates **<u>attributed correctly to *A* and *B*</u>**. Coordinates alone score A0 and *A* and *B* mixed up is A0.

(c) ***No marks for coordinates of *P****

1st M1 – attempts to find distance between *A* and *P* **or** *P* and *B* using a correct method

1st A1 – correct distance between *A* and *P* **and** *P* and *B*

2nd A1 – correct ratio in the correct form. Value of *m* need not be stated.

Question	Scheme		Marks
8			
(a)	$\binom{15}{6 \text{ or } 9}(1)^{15-6}(-px)^6 = 3648645x^6$	Correct equation	M1A1
	$5005p^6 = 3648645 \Rightarrow p = ...$		
	$p = \sqrt[6]{729} = 3$	Correct value of p	A1
			(3)
(b)	$\binom{15}{8 \text{ or } 7}(1)^{15-8}(-3)^8 = 4220035$		M1
	$= 4220000 \ (4\text{sf})$		A1
			(2)
			5

Question 8 Notes

(a) M1 – attempts to form a correct equation. LHS of equation alone or copying down 3648645 incorrectly with correct LHS can score M1

(b) A1 – answer must be to 4 sf or A0.

Question	Scheme		Marks
9			
(a)	$a=26-b$	Seen or implied	B1
	$\cos\theta=\dfrac{b^2+14^2-a^2}{2(b)(14)}$	Uses cosine rule and subs in values (can be in terms of a here)	M1
	$=\dfrac{b^2+196-(26-b)^2}{28b}$	Replaces a with $26-b$	dM1
	$=\dfrac{52b-480}{28b}$		
	$=\dfrac{13}{7}-\dfrac{120}{7b}$ *	Convincing proof	A1 **(4)**
(b)	$A^2=\dfrac{1}{4}b^2(14)^2\sin^2\theta$	Uses $A=\dfrac{1}{2}ab\sin C$ for this situation	M1
	$=49b^2(1-\cos^2\theta)$		
	$=49b^2\left(1-\left(\dfrac{13}{7}-\dfrac{120}{7b}\right)^2\right)$	Uses $\sin^2\theta=1-\cos^2\theta$	dM1
	$=49b^2\left(-\dfrac{120}{49}+\dfrac{3120}{49b}-\dfrac{14400}{49b^2}\right)$	Expands the brackets and manipulates the terms	ddM1
	$=-120b^2+3120b-14400$ *	Convincing proof	A1 **(4)**
(c/i) **Way 1**	$A^2=-120(b^2-26b+120)$		
	$=-120\left[(b-13)^2-13^2+120\right]$	Attempts to complete the square	M1A1
	$=-120(b-13)^2+5880$		
	$\Rightarrow \max(A^2)=5880$, so max $A\approx 76.7\text{ cm}^2$		A1A1 **(4)**
(c/i) **Way 2**	$\dfrac{d}{db}(A^2)=-240b+3120$	Differentiates A^2 with respect to b	M1
	max occurs when $b=\dfrac{3120}{240}=13$	Value of b for which A^2 is max	A1
	$\therefore \max(A^2)=-120(13)^2+3120(13)-14400=5880$		A1A1
	So max $A\approx 76.7\text{ cm}^2$		**(4)**
(c/ii)	Isosceles	Cao	B1 **(1)**
			13

	Question 9 Notes
(a) and (b) - ***Answers given***. Final A1 – cso, no errors seen (c/i) **Way 1** – 1st M1 – Attempts to complete the square 1st A1 – correctly completes the square oe 2nd A1 – correct max value of A^2 3rd A1 – correct max value of A (c/i) **Way 2** – 1st M1 – differentiation must be correct 2nd A1 – correct max value of A^2 3rd A1 – correct max value of A	

Question	Scheme		Marks
10			
(a)	Counter-example **and** shows it doesn't work e.g. $n = 3$, then $n^2 + 1 = 10$ which is not prime	Counterexample Shows it doesn't work	B1 B1 **(2)**
(b)	$(-1)^{x+y} = (-1)^x(-1)^y$ $= (-1)^x\left(\frac{1}{-1}\right)^y$ $= (-1)^x(-1)^{-y}$ $= (-1)^{x-y}$	Uses multiplication rule for indices Uses $-1 = \frac{1}{-1}$ Convincing proof	M1 dM1 A1 **(3)**
(c)	$\{(-1)^k = 1$ if (and only if) k is even and $(-1)^k = -1$ if (and only if) k is odd.$\}$ {Therefore,} $(-1)^{x+y} = (-1)^{x-y}$ if (and only if) both powers are either odd or even. So $x+y$ is even (if and only) if $x-y$ is even.	Correct explanation containing at least all underlined elements	B1 **(1)**
			6

Question 10 Notes

(a) 1st B1 – any counter-example that works
2nd B1 – **shows** the statement fails for their counter-example

(b) **Proof must involve index manipulation.** Accept other correct manipulations that give the correct answer.

(c) B1 – correct explanation with at least all the underlined elements.

Accept alternate wording provided the idea is conveyed clearly and without ambiguity. Poor mathematical expression should result in B0.

Question	Scheme		Marks
11			
(a)	$\dfrac{1}{\sin x} - \sin x \equiv \dfrac{1-\sin^2 x}{\sin x}$	Common denominator + use of $\cos^2 x = 1 - \sin^2 x$	M1
	$\equiv \dfrac{\cos^2 x}{\sin x}$		
	$\left\{\begin{array}{l} \equiv \dfrac{\cos x}{\sin x}(\cos x) \\ \equiv \dfrac{1}{\tan x}(\cos x) \end{array}\right\}$		
	$\equiv \dfrac{\cos x}{\tan x}$	Complete and convincing proof **containing at least one of the lines from the lines in the braces**	A1
			(2)
(b)	$4 + \tan^2 X = \dfrac{2}{\cos X}\left(-\dfrac{\cos X}{\tan X}\right) + \tan^2 X$	Uses identities	M1M1
	$\Rightarrow 4 = -\dfrac{2}{\tan X}$		
	$\Rightarrow \tan X = -\dfrac{1}{2}$	Correct value of tan$X$	A1
	$X = \tan^{-1}\left(-\dfrac{1}{2}\right) = -26.565...$	Correct principal value of X and attempts to find the other values in range	dM1 ddM1
	$\Rightarrow X = 153.435...,\ 333.435...,\ -26.565...,\ -206.565...$		
	$\{\Rightarrow X = 153.4°,\ 333.4,\ -26.6°,\ -206.6°\}$	Correct values of X	A1
			(6)
			8

Question 11 Notes

(a) For the **A** mark, the proof must be convincing. Since this a show that question, only give the A1 mark to candidates whose working contains at least one line from the lines in braces in the scheme. Accept equivalent writings of these lines.
SC: use of other variable. Accept use of another variable provided this is recovered in the final line.

(b) 1st M1 – writes $\sin^2 X\left(\cos^2 X\right)^{-1} = \tan^2 X$

2nd M1 – uses part (a) to write $\sin X - \dfrac{1}{\sin X} = -\dfrac{\cos X}{\tan X}$

3rd M1 – dependent on **both** previous M marks. Mark for finding principal values of X.

4th M1 – attempts to find other values of *X* in range, i.e. by adding/subtracting 180 degrees from values/clear use of tan graph/cast diagram

Final A1 – correct values of *X* in range. Accept any degree of accuracy as none is specified, but penalise answers given to 1sf unless more accurate values are previously seen. Ignore additional values outside of the given range, but award A0 for any additional values of *X* given inside the range.

Question	Scheme	Marks
12		
(a)	Long division: $x-1\overline{)4x^3+0x^2+6x-10}$ with quotient $4x^2-4x+10$ $\underline{4x^3-4x^2}$ $-4x^2+6x$ $\underline{-4x^2-4x}$ $10x-10$ $\underline{10x-10}$ 0 Since the remainder is 0, x – 1 is a root $4(1)^3+6(1)-10$ **OR** $=4+6-10=0$ $\therefore (x-1)$ is a root Shows x – 1 is a root + conclusion	M1A1
	Other factor is $4x^2-4x+10$	A1
	$(-4)^2-4(4)(10)=-144<0$, so $4x^2-4x+10=0$ has no real roots $\therefore (x-1)$ is the only real root of $4x^3+6x-10$ Attempts to show the other factor has no additional real roots + conclusion	M1A1 **(5)**
(b)	$d^2=(x-5)^2+(y--1)^2$ $=(x-5)^2+(x^2+1)^2$ $=x^2-10x+25+x^4+2x^2+1$ $=x^4+3x^2-10x+26$	M1 dM1 A1 **(3)**
(c)	$\dfrac{d}{dx}(d^2)=4x^3+6x-10$ — Differentiates d^2 with respect to x This is minimised when x = 1 — Correct value of x So coordinates of closest point are (1,1). — Correct coordinates	M1 A1 A1
	$\dfrac{d^2}{dx^2}(d^2)=12x^2+6$ — Differentiates d^2 a second time When x = 1, $12x^2+6\{=18\}>0$, so the point is a minimum bo — Convincing justification + conclusion	M1 A1 **(5)**
		13

	Question 12 Notes
(a) 1st M1 – attempts to use long division or factor theorem to show $x - 1$ is a factor 1st A1 – convincing proof + conclusion 2nd M1 – attempts to show their other factor has no real roots using the discriminant or completing the square or another method. This can be follow through their factor provided their factor has no real roots. (b) 1st M1 – correct expression for d^2 in terms of x and y. 2nd M1 – replaces y with x^2 A1 – convincing proof. ***Answer given*** (c) Final A1 – convincing justification. Must have a conclusion stating that it is a minimum (using symbols or words) since the second derivative is positive when $x = 1$. Need not necessarily evaluate the second derivative when $x = 1$, but must clearly show it is positive.	