Name:________________

AS Further Maths Statistics & Mechanics 2017 - 2018

Practice Papers

by Topic

(Graphic) Calculator Allowed

Work through these using the solutions with care, do not just copy them!

Discrete Probability Distributions

1. A discrete random variable X has the probability function

$$\mathrm{P}(X = x) = \begin{cases} k(1-x)^2 & x = -1,\ 0,\ 1 \text{ and } 2 \\ 0 & \text{otherwise.} \end{cases}$$

(*a*) Show that $k = \dfrac{1}{6}$

(3)

(*b*) Find E(X).

(2)

(*c*) Show that $\mathrm{E}(X^2) = \dfrac{4}{3}$

(2)

(*d*) Find Var($1 - 3X$)

(3)

(Total 10 marks)

2. The discrete random variable X has the following probability distribution, where p and q are constants.

x	-2	-1	$\frac{1}{2}$	$\frac{3}{2}$	2
$\mathrm{P}(X = x)$	p	q	0.2	0.3	p

(*a*) Write down an equation in p and q.

(1)

Given that $\mathrm{E}(X) = 0.4$,

(*b*) find the value of q.

(3)

(*c*) Hence find the value of p.

(2)

Given also that $\mathrm{E}(X^2) = 2.275$,

(*d*) find $\mathrm{Var}(X)$.

(2)

Sarah and Rebecca play a game.

A computer selects a single value of X using the probability distribution above.

Sarah's score is given by the random variable $S = X$ and Rebecca's score is given by the random variable $R = \frac{1}{X}$.

(*e*) Find $\mathrm{E}(R)$.

(3)

Sarah and Rebecca work out their scores and the person with the higher score is the winner. If the scores are the same, the game is a draw.

(*f*) Find the probability that

(i) Sarah is the winner,

(ii) Rebecca is the winner.

(4)

(Total 15 marks)

3. In a quiz, a team gains 10 points for every question it answers correctly and loses 5 points for every question it does not answer correctly. The probability of answering a question correctly is 0.6 for each question. One round of the quiz consists of 3 questions.

The discrete random variable X represents the total number of points scored in one round. The table shows the incomplete probability distribution of X.

x	30	15	0	-15
$\mathrm{P}(X = x)$	0.216			0.064

(*a*) Show that the probability of scoring 15 points in a round is 0.432.

(2)

(*b*) Find the probability of scoring 0 points in a round.

(1)

(*c*) Find the probability of scoring a total of 30 points in 2 rounds.

(3)

(*d*) Find $\mathrm{E}(X)$.

(2)

(*e*) Find $\mathrm{Var}(X)$.

(3)

In a bonus round of 3 questions, a team gains 20 points for every question it answers correctly and loses 5 points for every question it does not answer correctly.

(*f*) Find the expected number of points scored in the bonus round.

(3)

(Total 14 marks)

4. The discrete random variable X has the probability distribution

x	1	2	3	4
$\mathrm{P}(X = x)$	k	$2k$	$3k$	$4k$

(a) Show that $k = 0.1$

(1)

Find

(b) $\mathrm{E}(X)$

(2)

(c) $\mathrm{E}(X^2)$

(2)

(d) $\mathrm{Var}(2 - 5X)$

(3)

Two independent observations X_1 and X_2 are made of X.

(e) Show that $\mathrm{P}(X_1 + X_2 = 4) = 0.1$

(2)

(f) Complete the probability distribution table for $X_1 + X_2$.

(2)

y	2	3	4	5	6	7	8
$\mathrm{P}(X_1 + X_2 = y)$	0.01	0.04	0.10		0.25	0.24	

(g) Find $\mathrm{P}(1.5 < X_1 + X_2 \leq 3.5)$

(2)

(Total 14 marks)

5. A fair blue die has faces numbered 1, 1, 3, 3, 5 and 5. The random variable B represents the score when the blue die is rolled.

(*a*) Write down the probability distribution for B.

(2)

(*b*) State the name of this probability distribution.

(1)

(*c*) Write down the value of $\mathrm{E}(B)$.

(1)

A second die is red and the random variable R represents the score when the red die is rolled.

The probability distribution of R is

r	2	4	6
$\mathrm{P}(R = r)$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

(*d*) Find $\mathrm{E}(R)$.

(2)

(*e*) Find $\mathrm{Var}(R)$.

(3)

Tom invites Avisha to play a game with these dice.

Tom spins a fair coin with one side labelled 2 and the other side labelled 5. When Avisha sees the number showing on the coin she then chooses one of the dice and rolls it. If the number showing on the die is **greater** than the number showing on the coin, Avisha wins, otherwise Tom wins.

Avisha chooses the die which gives her the best chance of winning each time Tom spins the coin.

(*f*) Find the probability that Avisha wins the game, stating clearly which die she should use in each case.

(4)

(Total 13 marks)

6. The score S when a spinner is spun has the following probability distribution.

s	0	1	2	4	5
$P(S = s)$	0.2	0.2	0.1	0.3	0.2

(*a*) Find $E(S)$.

(2)

(*b*) Show that $E(S^2) = 10.4$.

(2)

(*c*) Hence find $\text{Var}(S)$.

(2)

(*d*) Find

(i) $E(5S - 3)$,

(ii) $\text{Var}(5S - 3)$.

(4)

(*e*) Find $P(5S - 3 > S + 3)$.

(3)

The spinner is spun twice.

The score from the first spin is S_1 and the score from the second spin is S_2.

The random variables S_1 and S_2 are independent and the random variable $X = S_1 \times S_2$.

(*f*) Show that $P(\{S_1 = 1\} \cap X < 5) = 0.16$.

(2)

(*g*) Find $P(X < 5)$.

(3)

(Total 18 marks)

7. A spinner is designed so that the score S is given by the following probability distribution.

s	0	1	2	4	5
$P(S = s)$	p	0.25	0.25	0.20	0.20

(*a*) Find the value of p.

(2)

(*b*) Find $E(S)$.

(2)

(*c*) Show that $E(S^2) = 9.45$.

(2)

(*d*) Find $\text{Var}(S)$.

(2)

Tom and Jess play a game with this spinner. The spinner is spun repeatedly and S counters are awarded on the outcome of each spin. If S is even then Tom receives the counters and if S is odd then Jess receives them. The first player to collect 10 or more counters is the winner.

(*e*) Find the probability that Jess wins after 2 spins.

(2)

(*f*) Find the probability that Tom wins after exactly 3 spins.

(4)

(*g*) Find the probability that Jess wins after exactly 3 spins.

(3)

(Total 17 marks)

TOTAL FOR PAPER: 101 MARKS

Poisson & Binomial Distributions

1. Patients arrive at a hospital accident and emergency department at random at a rate of 6 per hour.

(*a*) Find the probability that, during any 90 minute period, the number of patients arriving at the hospital accident and emergency department is

(i) exactly 7,

(ii) at least 10.

(5)

A patient arrives at 11.30 a.m.

(*b*) Find the probability that the next patient arrives before 11.45 a.m.

(3)

(Total 8 marks)

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2. (*a*) Write down the conditions under which the Poisson distribution can be used as an approximation to the binomial distribution.

(2)

The probability of any one letter being delivered to the wrong house is 0.01.
On a randomly selected day Peter delivers 1000 letters.

(*b*) Using a Poisson approximation, find the probability that Peter delivers at least 4 letters to the wrong house.

Give your answer to 4 decimal places.

(3)

(Total 5 marks)

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3. A disease occurs in 3% of a population.

(*a*) State any assumptions that are required to model the number of people with the disease in a random sample of size n as a binomial distribution.

(2)

(*b*) Using this model, find the probability of exactly 2 people having the disease in a random sample of 10 people.

(3)

(*c*) Find the mean and variance of the number of people with the disease in a random sample of 100 people.

(2)

A doctor tests a random sample of 100 patients for the disease. He decides to offer all patients a vaccination to protect them from the disease if more than 5 of the sample have the disease.

(*d*) Using a suitable approximation, find the probability that the doctor will offer all patients a vaccination.

(3)

(Total 10 marks)

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4. A student is investigating the numbers of cherries in a *Rays* fruit cake. A random sample of *Rays* fruit cakes is taken and the results are shown in the table below.

Number of cherries	0	1	2	3	4	5	≥ 6
Frequency	24	37	21	12	4	2	0

(*a*) Calculate the mean and the variance of these data.

(3)

(*b*) Explain why the results in part (*a*) suggest that a Poisson distribution may be a suitable model for the number of cherries in a *Rays* fruit cake.

(1)

The number of cherries in a *Rays* fruit cake follows a Poisson distribution with mean 1.5.

A *Rays* fruit cake is to be selected at random.
Find the probability that it contains

(*c*) (i) exactly 2 cherries,

(ii) at least 1 cherry.

(4)

Rays fruit cakes are sold in packets of 5.

(*d*) Show that the probability that there are more than 10 cherries, in total, in a randomly selected packet of *Rays* fruit cakes, is 0.1378 correct to 4 decimal places.

(3)

Twelve packets of *Rays* fruit cakes are selected at random.

(*e*) Find the probability that exactly 3 packets contain more than 10 cherries.

(3)

(Total 14 marks)

5. A company receives telephone calls at random at a mean rate of 2.5 per hour.

(*a*) Find the probability that the company receives

(i) at least 4 telephone calls in the next hour,

(ii) exactly 3 telephone calls in the next 15 minutes.

(5)

(*b*) Find, to the nearest minute, the maximum length of time the telephone can be left unattended so that the probability of missing a telephone call is less than 0.2.

(3)

The company puts an advert in the local newspaper. The number of telephone calls received in a randomly selected 2 hour period after the paper is published is 10.

(*c*) Test at the 5% level of significance whether or not the mean rate of telephone calls has increased. State your hypotheses clearly.

(5)

(Total 13 marks)

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6. A random variable X has the distribution $\mathrm{B}(12, p)$.

(*a*) Given that $p = 0.25$, find

(i) $\mathrm{P}(X < 5)$,

(ii) $\mathrm{P}(X \geq 7)$.

(3)

(*b*) Given that $\mathrm{P}(X = 0) = 0.05$, find the value of p to 3 decimal places.

(3)

(*c*) Given that the variance of X is 1.92, find the possible values of p.

(4)

(Total 10 marks)

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7. The probability of an electrical component being defective is 0.075.

The component is supplied in boxes of 120.

(*a*) Using a suitable approximation, estimate the probability that there are more than 3 defective components in a box.

(5)

A retailer buys 2 boxes of components.

(*b*) Estimate the probability that there are at least 4 defective components in each box.

(2)

(Total 7 marks)

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8. As part of a selection procedure for a company, applicants have to answer all 20 questions of a multiple choice test. If an applicant chooses answers at random the probability of choosing a correct answer is 0.2 and the number of correct answers is represented by the random variable X.

(*a*) Suggest a suitable distribution for X.

(2)

Each applicant gains 4 points for each correct answer but loses 1 point for each incorrect answer. The random variable S represents the final score, in points, for an applicant who chooses answers to this test at random.

(*b*) Show that $S = 5X - 20$.

(2)

(*c*) Find $\mathrm{E}(S)$ and $\mathrm{Var}(S)$.

(4)

An applicant who achieves a score of at least 20 points is invited to take part in the final stage of the selection process.

(*d*) Find $\mathrm{P}(S \geq 20)$.

(4)

Cameron is taking the final stage of the selection process which is a multiple choice test consisting of 100 questions. He has been preparing for this test and believes that his chance of answering each question correctly is 0.4.

(*e*) Using a suitable approximation, estimate the probability that Cameron answers more than half of the questions correctly.

(5)

(Total 17 marks)

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9. A telesales operator is selling a magazine. Each day he chooses a number of people to telephone. The probability that each person he telephones buys the magazine is 0.1.

(*a*) Suggest a suitable distribution to model the number of people who buy the magazine from the telesales operator each day.

(1)

(*b*) On Monday, the telesales operator telephones 10 people. Find the probability that he sells at least 4 magazines.

(3)

(*c*) Calculate the least number of people he needs to telephone on Tuesday, so that the probability of selling at least 1 magazine, on that day, is greater than 0.95.

(3)

A call centre also sells the magazine. The probability that a telephone call made by the call centre sells a magazine is 0.05. The call centre telephones 100 people every hour.

(*d*) Using a suitable approximation, find the probability that more than 10 people telephoned by the call centre buy a magazine in a randomly chosen hour.

(3)

(Total 10 marks)

TOTAL FOR PAPER: 94 MARKS

Chi-Squared

1. A doctor takes a random sample of 100 patients and measures their intake of saturated fats in their food and the level of cholesterol in their blood. The results are summarised in the table below.

Intake of saturated fats \ Cholesterol level	High	Low
High	12	8
Low	26	54

Using a 5% level of significance, test whether or not there is an association between cholesterol level and intake of saturated fats. State your hypotheses and show your working clearly.

(Total 10 marks)

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2. A number of males and females were asked to rate their happiness under the headings "not happy", "fairly happy" and "very happy".

The results are shown in the table below

		Happiness			Total
		Not happy	Fairly happy	Very happy	
Gender	Female	9	43	34	86
	Male	13	25	16	54
Total		22	68	50	140

Stating your hypotheses, test at the 5% level of significance, whether or not there is evidence of an association between happiness and gender. Show your working clearly.

(Total 10 marks)

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3. A factory manufactures batches of an electronic component. Each component is manufactured in one of three shifts. A component may have one of two types of defect, D_1 or D_2, at the end of the manufacturing process. A production manager believes that the type of defect is dependent upon the shift that manufactured the component. He examines 200 randomly selected defective components and classifies them by defect type and shift.

The results are shown in the table below.

Shift \ Defect type	D_1	D_2
First shift	45	18
Second shift	55	20
Third shift	50	12

Stating your hypotheses, test, at the 10% level of significance, whether or not there is evidence to support the manager's belief. Show your working clearly.

(Total 10 marks)

4. A psychologist carries out a survey of the perceived body weight of 150 randomly chosen people. He asks them if they think they are underweight, about right or overweight. His results are summarised in the table below.

	Underweight	About right	Overweight
Male	20	22	30
Female	16	28	34

The psychologist calculates two of the expected frequencies, to 2 decimal places, for a test of independence between perceived body weight and gender. These results are shown in the table below.

	Underweight	About right	Overweight
Male	17.28		
Female	18.72		

(*a*) Complete the table of expected frequencies shown above.

(2)

(*b*) Test, at the 10% level of significance, whether or not perceived body weight is independent of gender. State your hypotheses clearly.

(7)

The psychologist now combines the male and female data to test whether or not body weight types are chosen equally.

(*c*) Find the smallest significance level, from the tables in the formula booklet, for which there is evidence of a preference.

(5)

(Total 14 marks)

5. John thinks that a person's eye colour is related to their hair colour. He takes a random sample of 600 people and records their eye and hair colours. The results are shown in Table 1.

		Hair colour				
		Black	Brown	Red	Blonde	Total
Eye colour	Brown	45	125	15	58	243
	Blue	34	90	10	58	192
	Hazel	20	38	16	26	100
	Green	6	29	7	23	65
	Total	105	282	48	165	600

Table 1

John carries out a χ^2 test in order to test whether eye colour and hair colour are related. He calculates the expected frequencies shown in Table 2.

		Hair colour			
		Black	Brown	Red	Blonde
Eye colour	Brown	42.5	114.2	19.4	66.8
	Blue	33.6	90.2	15.4	52.8
	Hazel	17.5	47	8	27.5
	Green	11.4	30.6	5.2	17.9

Table 2

(*a*) Show how the value 47 in Table 2 has been calculated.

(1)

(*b*) Write down the number of degrees of freedom John should use in this χ^2 test.

(1)

Given that the value of the χ^2 statistic is 20.6, to 3 significant figures,

(*c*) find the smallest value of α for which the null hypothesis will be rejected at the α% level of significance.

(1)

(*d*) Use the data from Table 1 to test at the 5% level of significance whether or not the proportions of people in the population with black, brown, red and blonde hair are in the ratio 2:6:1:3.
State your hypotheses clearly.

(9)

(Total 12 marks)

6. A research station is doing some work on the germination of a new variety of genetically modified wheat.

They planted 120 rows containing 7 seeds in each row.

The number of seeds germinating in each row was recorded. The results are as follows

Number of seeds germinating in each row	0	1	2	3	4	5	6	7
Observed number of rows	2	6	11	19	25	32	16	9

(*a*) Write down two reasons why a binomial distribution may be a suitable model.

(2)

(*b*) Show that the probability of a randomly selected seed from this sample germinating is 0.6.

(2)

The research station used a binomial distribution with probability 0.6 of a seed germinating. The expected frequencies were calculated to 2 decimal places. The results are as follows:

Number of seeds germinating in each row	0	1	2	3	4	5	6	7
Expected number of rows	0.20	2.06	s	23.22	t	31.35	15.68	3.36

(*c*) Find the value of s and the value of t.

(2)

(*d*) Stating your hypotheses clearly, test, at the 1% level of significance, whether or not the data can be modelled by a binomial distribution.

(7)

(Total 13 marks)

7. A total of 100 random samples of 6 items are selected from a production line n a factory and the number of defective items in each sample is recorded. The results are summarised in the table below.

Number of defective items	0	1	2	3	4	5	6
Number of samples	6	16	20	23	17	10	8

(*a*) Show that the mean number of defective items per sample is 2.91.

(2)

A factory manager suggests that the data can be modelled by a binomial distribution with n = 6. He uses the mean from the sample above and calculates expected frequencies as shown in the table below.

Number of defective items	0	1	2	3	4	5	6
Expected frequency	1.87	10.54	24.82	a	22.01	8.29	b

(*b*) Calculate the value of a and the value of b, giving your answers to 2 decimal places.

(4)

(*c*) Test, at the 5% level, whether or not the binomial distribution is a suitable model for the number of defective items in samples of 6 items. State your hypotheses clearly.

(8)

(Total 14 marks)

8. An airport manager carries out a survey of families and their luggage. Each family is allowed to check in a maximum of 4 suitcases. She observes 50 families at the check-in desk and counts the total number of suitcases each family checks in. The data are summarised in the table below.

Number of suitcases	0	1	2	3	4
Frequency	6	25	12	6	1

The manager claims that the data can be modelled by a binomial distribution with $p = 0.3$.

(*a*) Test the manager's claim at the 5% level of significance. State your hypotheses clearly. Show your working clearly and give your expected frequencies to 2 decimal places.

(8)

The manager also carries out a survey of the time taken by passengers to check in. She records the number of passengers that check in during each of 100 five-minute intervals.

The manager makes a new claim that these data can be modelled by a Poisson distribution. She calculates the expected frequencies given in the table below.

Number of passengers	0	1	2	3	4	5 or more
Observed frequency	5	40	31	18	6	0
Expected frequency	16.53	29.75	r	s	7.23	3.64

(*b*) Find the value of r and the value of s giving your answers to 2 decimal places.

(3)

(*c*) Stating your hypotheses clearly, use a 1% level of significance to test the manager's new claim.

(6)

(Total 17 marks)

TOTAL FOR PAPER: 100 MARKS

Collisions

1. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. The particle P collides directly with a particle Q of mass $4m$ moving on the plane with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(*a*) Find the speed of Q immediately after the collision.

(6)

Given that the direction of motion of P is reversed by the collision,

(*b*) find the range of possible values of e.

(5)

(Total 11 marks)

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2. A particle P of mass m is moving in a straight line on a smooth horizontal surface with speed $4u$. The particle P collides directly with a particle Q of mass $3m$ which is at rest on the surface. The coefficient of restitution between P and Q is e. The direction of motion of P is reversed by the collision.

Show that $e > \frac{1}{3}$.

(Total 8 marks)

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3. A particle of mass m kg lies on a smooth horizontal surface. Initially the particle is at rest at a point O midway between a pair of fixed parallel vertical walls. The walls are 2 m apart. At time $t = 0$ the particle is projected from O with speed u m s^{-1} in a direction perpendicular to the walls. The coefficient of restitution between the particle and each wall is $\frac{2}{3}$. The magnitude of the impulse on the particle due to the first impact with a wall is λmu N s.

(*a*) Find the value of λ.

(3)

The particle returns to O, having bounced off each wall once, at time $t = 3$ seconds.

(*b*) Find the value of u.

(6)

(Total 9 marks)

4.

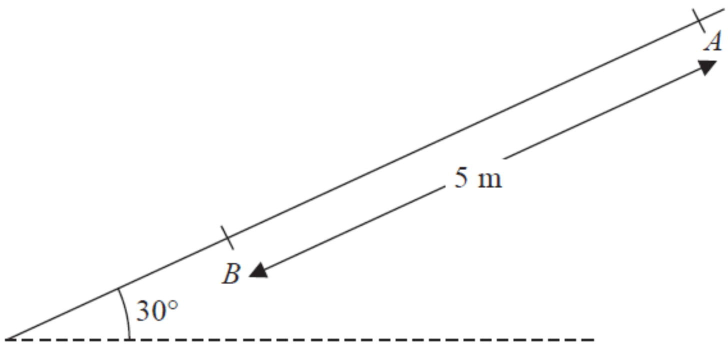

Figure 1

A particle P of mass 2 kg is released from rest at a point A on a rough inclined plane and slides down a line of greatest slope. The plane is inclined at 30° to the horizontal. The point B is 5 m from A on the line of greatest slope through A, as shown in Figure 1.

(*a*) Find the potential energy lost by P as it moves from A to B.

(2)

The speed of P as it reaches B is 4 m s^{-1}.

(*b*) (i) Use the work-energy principle to find the magnitude of the constant frictional force acting on P as it moves from A to B.

(ii) Find the coefficient of friction between P and the plane.

(7)

The particle P is now placed at A and projected down the plane towards B with speed 3 m s^{-1}. Given that the frictional force remains constant,

(*c*) find the speed of P as it reaches B.

(4)

(Total 13 marks)

Turn over

5. Three identical particles, A, B and C, lie at rest in a straight line on a smooth horizontal table with B between A and C. The mass of each particle is m. Particle A is projected towards B with speed u and collides directly with B. The coefficient of restitution between each pair of particles is $\frac{2}{3}$.

(a) Find, in terms of u,

(i) the speed of A after this collision,

(ii) the speed of B after this collision.

(7)

(b) Show that the kinetic energy lost in this collision is $\frac{5}{36}mu^2$.

(4)

After the collision between A and B, particle B collides directly with C.

(c) Find, in terms of u, the speed of C immediately after this collision between B and C.

(4)

(Total 15 marks)

6. Three particles P, Q and R lie at rest in a straight line on a smooth horizontal table with Q between P and R. The particles P, Q and R have masses $2m$, $3m$ and $4m$ respectively. Particle P is projected towards Q with speed u and collides directly with it. The coefficient of restitution between each pair of particles is e.

(*a*) Show that the speed of Q immediately after the collision with P is $\frac{2}{5}(1 + e)u$.

(6)

After the collision between P and Q there is a direct collision between Q and R.

Given that $e = \frac{3}{4}$, find

(*b*) (i) the speed of Q after this collision,

(ii) the speed of R after this collision.

(6)

Immediately after the collision between Q and R, the rate of increase of the distance between P and R is V.

(*c*) Find V in terms of u.

(3)

(Total 15 marks)

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7. A particle A of mass m is moving with speed u on a smooth horizontal floor when it collides directly with another particle B, of mass $3m$, which is at rest on the floor. The coefficient of restitution between the particles is e. The direction of motion of A is reversed by the collision.

(*a*) Find, in terms of e and u,

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

After being struck by A the particle B collides directly with another particle C, of mass $4m$, which is at rest on the floor. The coefficient of restitution between B and C is $2e$. Given that the direction of motion of B is reversed by this collision,

(*b*) find the range of possible values of e,

(6)

(*c*) determine whether there will be a second collision between A and B.

(3)

(Total 16 marks)

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8. Three identical particles P, Q and R, each of mass m, lie in a straight line on a smooth horizontal plane with Q between P and R. Particles P and Q are projected directly towards each other with speeds $4u$ and $2u$ respectively, and at the same time particle R is projected along the line away from Q with speed $3u$. The coefficient of restitution between each pair of particles is e. After the collision between P and Q there is a collision between Q and R.

(*a*) Show that $e > \frac{2}{3}$.

(7)

It is given that $e = \frac{3}{4}$.

(*b*) Show that there will not be a further collision between P and Q.

(6)

(Total 13 marks)

TOTAL FOR PAPER: 100 MARKS

Momentum & Impulse

1. Particle P of mass m and particle Q of mass km are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision the speed of P is $5u$ and the speed of Q is u. Immediately after the collision the speed of each particle is halved and the direction of motion of each particle is reversed.

Find

(*a*) the value of k,

(3)

(*b*) the magnitude of the impulse exerted on P by Q in the collision.

(3)

(Total 6 marks)

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2. Two particles A and B, of mass 2 kg and 3 kg respectively, are moving towards each other in opposite directions along the same straight line on a smooth horizontal surface. The particles collide directly. Immediately before the collision the speed of A is 5 m s^{-1} and the speed of B is 6 m s^{-1}. The magnitude of the impulse exerted on B by A is 14 N s.

Find

(*a*) the speed of A immediately after the collision,

(3)

(*b*) the speed of B immediately after the collision.

(3)

(Total 6 marks)

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3. Particle P has mass 3 kg and particle Q has mass m kg. The particles are moving in opposite directions along a smooth horizontal plane when they collide directly. Immediately before the collision, the speed of P is 4 m s^{-1} and the speed of Q is 3 m s^{-1}. In the collision the direction of motion of P is unchanged and the direction of motion of Q is reversed. Immediately after the collision, the speed of P is 1 m s^{-1} and the speed of Q is 1.5 m s^{-1}.

(*a*) Find the magnitude of the impulse exerted on P in the collision.

(3)

(*b*) Find the value of m.

(3)

(Total 6 marks)

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4. A railway truck P, of mass m kg, is moving along a straight horizontal track with speed 15 m s^{-1}. Truck P collides with a truck Q of mass 3000 kg, which is at rest on the same track. Immediately after the collision the speed of P is 3 m s^{-1} and the speed of Q is 9 m s^{-1}. The direction of motion of P is reversed by the collision.

Modelling the trucks as particles, find

(*a*) the magnitude of the impulse exerted by P on Q,

(2)

(*b*) the value of m.

(3)

(Total 5 marks)

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5. Two particles B and C have mass m kg and 3 kg respectively. They are moving towards each other in opposite directions on a smooth horizontal table. The two particles collide directly. Immediately before the collision, the speed of B is 4 m s^{-1} and the speed of C is 2 m s^{-1}. In the collision the direction of motion of C is reversed and the direction of motion of B is unchanged. Immediately after the collision, the speed of B is 1 m s^{-1} and the speed of C is 3 m s^{-1}.

Find

(*a*) the value of m,

(3)

(*b*) the magnitude of the impulse received by C.

(2)

(Total 5 marks)

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6. Particle P has mass 3 kg and particle Q has mass 2 kg. The particles are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision, P has speed 3 m s^{-1} and Q has speed 2 m s^{-1}. Immediately after the collision, both particles move in the same direction and the difference in their speeds is 1 m s^{-1}.

(*a*) Find the speed of each particle after the collision.

(5)

(*b*) Find the magnitude of the impulse exerted on P by Q.

(3)

(Total 8 marks)

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7. Two particles, P and Q, have masses $2m$ and $3m$ respectively. They are moving towards each other in opposite directions on a smooth horizontal plane when they collide directly. Immediately before they collide the speed of P is $4u$ and the speed of Q is $3u$. As a result of the collision, Q has its direction of motion reversed and is moving with speed u.

(*a*) Find the speed of P immediately after the collision.

(3)

(*b*) State whether or not the direction of motion of P has been reversed by the collision.

(1)

(*c*) Find the magnitude of the impulse exerted on P by Q in the collision.

(3)

(Total 7 marks)

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8. A particle P of mass 0.4 kg is moving on rough horizontal ground when it hits a fixed vertical plane wall. Immediately before hitting the wall, P is moving with speed 4 m s^{-1} in a direction perpendicular to the wall. The particle rebounds from the wall and comes to rest at a distance of 5 m from the wall. The coefficient of friction between P and the ground is $\frac{1}{8}$.

Find the magnitude of the impulse exerted on P by the wall.

(7)

(Total 7 marks)

TOTAL FOR PAPER: 50 MARKS

Work, Energy & Power 1

1. A van of mass 900 kg is moving down a straight road that is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{30}$. The resistance to motion of the van has constant magnitude 570 N. The engine of the van is working at a constant rate of 12.5 kW.

At the instant when the van is moving down the road at 5 m s^{-1}, the acceleration of the van is a m s^{-2}.

Find the value of a.

(Total 5 marks)

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2. A van of mass 600 kg is moving up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{16}$. The resistance to motion of the van from non-gravitational forces has constant magnitude R newtons. When the van is moving at a constant speed of 20 m s^{-1}, the van's engine is working at a constant rate of 25 kW.

(*a*) Find the value of R.

(4)

The power developed by the van's engine is now increased to 30 kW. The resistance to motion from non-gravitational forces is unchanged. At the instant when the van is moving up the road at 20 m s^{-1}, the acceleration of the van is a m s^{-2}.

(*b*) Find the value of a.

(4)

(Total 8 marks)

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3. A caravan of mass 600 kg is towed by a car of mass 900 kg along a straight horizontal road. The towbar joining the car to the caravan is modelled as a light rod parallel to the road. The total resistance to motion of the car is modelled as having magnitude 300 N. The total resistance to motion of the caravan is modelled as having magnitude 150 N. At a given instant the car and the caravan are moving with speed 20 m s^{-1} and acceleration 0.2 m s^{-2}.

(*a*) Find the power being developed by the car's engine at this instant.

(5)

(*b*) Find the tension in the towbar at this instant.

(2)

(Total 7 marks)

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4. A car of mass 1000 kg moves with constant speed V m s^{-1} up a straight road inclined at an angle θ to the horizontal, where $\sin \theta = \frac{1}{30}$. The engine of the car is working at a rate of 12 kW. The resistance to motion from non-gravitational forces has magnitude 500 N.

Find the value of V.

(Total 5 marks)

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5. A truck of mass 900 kg is towing a trailer of mass 150 kg up an inclined straight road with constant speed 15 m s^{-1}. The trailer is attached to the truck by a light inextensible towbar which is parallel to the road. The road is inclined at an angle θ to the horizontal, where $\sin \theta = \frac{1}{9}$. The resistance to motion of the truck from non-gravitational forces has constant magnitude 200 N and the resistance to motion of the trailer from non-gravitational forces has constant magnitude 50 N.

(*a*) Find the rate at which the engine of the truck is working.

(5)

When the truck and trailer are moving up the road at 15 m s^{-1} the towbar breaks, and the trailer is no longer attached to the truck. The rate at which the engine of the truck is working is unchanged. The resistance to motion of the truck from non-gravitational forces and the resistance to motion of the trailer from non-gravitational forces are still forces of constant magnitudes 200 N and 50 N respectively.

(*b*) Find the acceleration of the truck at the instant after the towbar breaks.

(3)

(*c*) Use the work-energy principle to find out how much further up the road the trailer travels before coming to instantaneous rest.

(4)

(Total 12 marks)

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6. A car of mass 800 kg is moving on a straight road which is inclined at an angle θ to the horizontal, where $\sin\theta = \dfrac{1}{20}$. The resistance to the motion of the car from non-gravitational forces is modelled as a constant force of magnitude R newtons. When the car is moving up the road at a constant speed of 12.5 m s^{-1}, the engine of the car is working at a constant rate of $3P$ watts. When the car is moving down the road at a constant speed of 12.5 m s^{-1}, the engine of the car is working at a constant rate of P watts.

(*a*) Find

(i) the value of P,

(ii) the value of R.

(6)

When the car is moving up the road at 12.5 m s^{-1} the engine is switched off and the car comes to rest, without braking, in a distance d metres. The resistance to the motion of the car from non-gravitational forces is still modelled as a constant force of magnitude R newtons.

(*b*) Use the work-energy principle to find the value of d.

(4)

(Total 10 marks)

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7. A particle P of mass 3 kg moves from point A to point B up a line of greatest slope of a fixed rough plane. The plane is inclined at 20° to the horizontal. The coefficient of friction between P and the plane is 0.4.

Given that AB = 15 m and that the speed of P at A is 20 m s^{-1}, find

(*a*) the work done against friction as P moves from A to B,

(3)

(*b*) the speed of P at B.

(4)

(Total 7 marks)

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8. A ball of mass 0.2 kg is projected vertically upwards from a point O with speed 20 m s^{-1}. The non-gravitational resistance acting on the ball is modelled as a force of constant magnitude 1.24 N and the ball is modelled as a particle. Find, using the work-energy principle, the speed of the ball when it first reaches the point which is 8 m vertically above O.

(Total 6 marks)

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9. A truck of mass 1800 kg is towing a trailer of mass 800 kg up a straight road which is inclined to the horizontal at an angle α, where $\sin \alpha = \frac{1}{20}$. The truck is connected to the trailer by a light inextensible rope which is parallel to the direction of motion of the truck. The resistances to motion of the truck and the trailer from non-gravitational forces are modelled as constant forces of magnitudes 300 N and 200 N respectively. The truck is moving at constant speed v m s^{-1} and the engine of the truck is working at a rate of 40 kW.

(*a*) Find the value of v.

(5)

As the truck is moving up the road the rope breaks.

(*b*) Find the acceleration of the truck immediately after the rope breaks.

(4)

(Total 9 marks)

10.

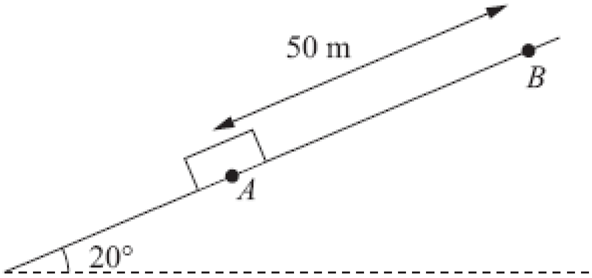

Figure 1

A box of mass 30 kg is held at rest at point A on a rough inclined plane. The plane is inclined at 20° to the horizontal. Point B is 50 m from A up a line of greatest slope of the plane, as shown in Figure 1. The box is dragged from A to B by a force acting parallel to AB and then held at rest at B. The coefficient of friction between the box and the plane is $\frac{1}{4}$. Friction is the only non-gravitational resistive force acting on the box. Modelling the box as a particle,

(*a*) find the work done in dragging the box from A to B.

(6)

The box is released from rest at the point B and slides down the slope. Using the work-energy principle, or otherwise,

(*b*) find the speed of the box as it reaches A.

(5)

(Total 11 marks)

TOTAL FOR PAPER: 80 MARKS

Turn over

Work, Energy & Power 2

1.

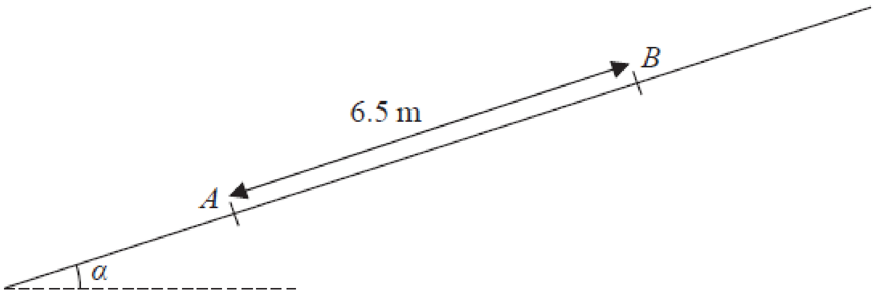

Figure 2

A particle P of mass 10 kg is projected from a point A up a line of greatest slope AB of a fixed rough plane. The plane is inclined at angle α to the horizontal, where $\tan \alpha = \frac{5}{12}$ and $AB = 6.5$ m, as shown in Figure 2. The coefficient of friction between P and the plane is μ. The work done against friction as P moves from A to B is 245 J.

(a) Find the value of μ.

(5)

The particle is projected from A with speed 11.5 m s^{-1}. By using the work-energy principle,

(b) find the speed of the particle as it passes through B.

(4)

(Total 9 marks)

2.

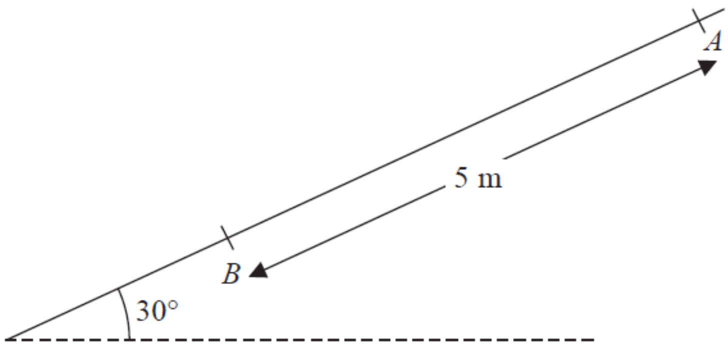

Figure 2

A particle P of mass 2 kg is released from rest at a point A on a rough inclined plane and slides down a line of greatest slope. The plane is inclined at 30° to the horizontal. The point B is 5 m from A on the line of greatest slope through A, as shown in Figure 2.

(*a*) Find the potential energy lost by P as it moves from A to B.

(2)

The speed of P as it reaches B is 4 m s^{-1}.

(*b*) (i) Use the work-energy principle to find the magnitude of the constant frictional force acting on P as it moves from A to B.

(ii) Find the coefficient of friction between P and the plane.

(7)

The particle P is now placed at A and projected down the plane towards B with speed 3 m s^{-1}. Given that the frictional force remains constant,

(*c*) find the speed of P as it reaches B.

(4)

(Total 13 marks)

3.

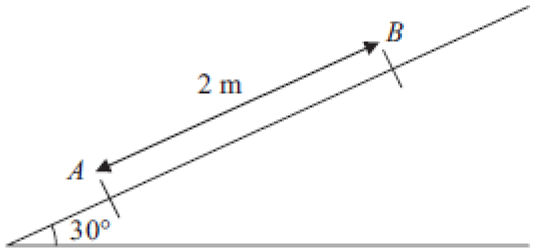

Figure 3

A particle P of mass 0.5 kg is projected from a point A up a line of greatest slope AB of a fixed plane. The plane is inclined at 30° to the horizontal and AB = 2 m with B above A, as shown in Figure 32. The particle P passes through B with speed 5 m s^{-1}. The plane is smooth from A to B.

(*a*) Find the speed of projection.

(4)

The particle P comes to instantaneous rest at the point C on the plane, where C is above B and BC = 1.5 m. From B to C the plane is rough and the coefficient of friction between P and the plane is μ.

By using the work-energy principle,

(*b*) find the value of μ.

(6)

(Total 10 marks)

4. The point A lies on a rough plane inclined at an angle θ to the horizontal, where $\sin\theta = \dfrac{24}{25}$. A particle P is projected from A, up a line of greatest slope of the plane, with speed U m s^{-1}. The mass of P is 2 kg and the coefficient of friction between P and the plane is $\dfrac{5}{12}$. The particle comes to instantaneous rest at the point B on the plane, where AB = 1.5 m. It then moves back down the plane to A.

(*a*) Find the work done against friction as P moves from A to B.

(4)

(*b*) Use the work-energy principle to find the value of U.

(4)

(*c*) Find the speed of P when it returns to A.

(3)

(Total 11 marks)

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5. A car of mass 1200 kg pulls a trailer of mass 400 kg up a straight road which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{14}$. The trailer is attached to the car by a light inextensible towbar which is parallel to the road. The car's engine works at a constant rate of 60 kW. The non-gravitational resistances to motion are constant and of magnitude 1000 N on the car and 200 N on the trailer.

At a given instant, the car is moving at 10 m s^{-1}. Find

(*a*) the acceleration of the car at this instant,

(5)

(*b*) the tension in the towbar at this instant.

(4)

The towbar breaks when the car is moving at 12 m s^{-1}.

(*c*) Find, using the work-energy principle, the further distance that the trailer travels before coming instantaneously to rest.

(5)

(Total 14 marks)

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6. Two particles A and B, of masses $3m$ and $4m$ respectively, lie at rest on a smooth horizontal surface. Particle B lies between A and a smooth vertical wall which is perpendicular to the line joining A and B. Particle B is projected with speed $5u$ in a direction perpendicular to the wall and collides with the wall. The coefficient of restitution between B and the wall is $\frac{3}{5}$.

(*a*) Find the magnitude of the impulse received by B in the collision with the wall.

(3)

After the collision with the wall, B rebounds from the wall and collides directly with A. The coefficient of restitution between A and B is e.

(*b*) Show that, immediately after they collide, A and B are both moving in the same direction.

(7)

The kinetic energy of B immediately after it collides with A is one quarter of the kinetic energy of B immediately before it collides with A.

(*c*) Find the value of e.

(4)

(Total 14 marks)

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7. The points A and B are 10 m apart on a line of greatest slope of a fixed rough inclined plane, with A above B. The plane is inclined at 25° to the horizontal. A particle P of mass 5 kg is released from rest at A and slides down the slope. As P passes B, it is moving with speed 7 m s^{-1}.

(*a*) Find, using the work-energy principle, the work done against friction as P moves from A to B.

(4)

(*b*) Find the coefficient of friction between the particle and the plane.

(5)

(Total 9 marks)

TOTAL FOR PAPER: 80 MARKS