

Mark scheme Further Maths Core Pure (AS/Year 1) Unit Test 5: Algebra and Functions

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
1	Uses $\alpha + \beta = -\frac{b}{a}$ to write $4p = -6$	M1	1.1b	TBC
	Solves to find $p = -\frac{3}{2}$	A1	1.1b	
	Uses $\alpha\beta = \frac{c}{a}$ to write $3p^2 = \frac{30}{k}$	M1	1.1b	
	Solves to find $k = \frac{40}{9}$	A1	1.1b	
		(4)		
				(4 marks)
Notes				

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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
2a	Uses $\alpha + \beta = -\frac{b}{a}$ to write $2\operatorname{Re}(\alpha) = -\frac{m}{5}$	M1	2.2a	TBC
	Solves to find $m = -40$	A1	1.1b	
		(2)		
b	Uses $\alpha\beta = \frac{c}{a}$ to write $(\operatorname{Re}(\alpha))^2 + (\operatorname{Im}(\alpha))^2 = \frac{n}{5}$	M1	2.2a	
	Solves to find $n = 80$ and concludes $n > 80$	A1	1.1b	
		(2)		
				(4 marks)
Notes				

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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
3a	States $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{k}{5}$, $\alpha\beta\gamma = 10$ and $\alpha + \beta + \gamma = \frac{11}{5}$	B1	1.1b	TBC
		(1)		
b	Deduces that $\alpha^* = 1 - 7i$ is a root.	M1	2.2a	
	Finds $\alpha\alpha^* = (1 + 7i)(1 - 7i) = 50$	M1	1.1b	
	Uses $\alpha\beta\gamma = 10 \Rightarrow \alpha\alpha^*\gamma = 10$ to state $\gamma = \frac{1}{5}$	A1	1.1b	
		(3)		
c	Uses $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{k}{5}$ to write $50 + \frac{1}{5}(1 - 7i) + \frac{1}{5}(1 + 7i) = \frac{k}{5}$	M1	2.2a	
	Solves to find $k = 252$	A1	1.1b	
		(2)		
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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
4a	Multiplies the three given roots together and sets the result equal to 52 or -52 . For example $(\alpha)\left(\frac{1}{\alpha}\right)\left(\alpha + \frac{13}{\alpha} + 46\right) = -52$ or $(\alpha)\left(\frac{1}{\alpha}\right)\left(\alpha + \frac{13}{\alpha} + 46\right) = 52$ is seen.	M1	1.1b	TBC
	Correctly uses $(\alpha)\left(\frac{1}{\alpha}\right)\left(\alpha + \frac{13}{\alpha} + 46\right) = 52$ to find $\alpha^2 - 6\alpha + 13 = 0$	A1	1.1b	
	Attempts to solve this quadratic using either completing the square or the quadratic formula.	M1	3.1a	
	Correctly finds $\alpha = 3 \pm 2i$	A1	1.1b	
	States that the roots of $f(z) = 0$ are $3 + 2i$, $3 - 2i$, 4	A1	2.2a	
		(5)		
b	Applies the process of finding $-\sum$ (of their three roots found in part (a)) to attempt to find m .	M1	3.1a	
	Correctly finds $m = -10$	A1	1.1b	
	Applies the process of using the pair sums to find the value of n . For example, $13 + 6\gamma = n$ is seen.	M1	3.1a	
	Correctly finds $n = 37$	A1	1.1b	
		(4)		
				(9 marks)
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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
5a	States or implies that $\gamma = 2 - 4i$ is a root.	M1	2.2a	TBC
	Uses $\alpha + \beta + \gamma + \delta = \alpha + \beta + 4 = -\frac{-24}{4}$ to write $\alpha + \beta - 2 = 0$	A1	3.1a	
	Uses $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \alpha\beta(2 + 4i) + \alpha\beta(2 - 4i) + \alpha(20) + \beta(20) = -\frac{-276}{4}$ to write $4\alpha\beta + 20(\alpha + \beta) = 69$	A1	3.1a	
		(3)		
b	Makes an attempt to solve for α and β , for example $\alpha = 2 - \beta$ is substituted into $4\alpha\beta + 20(\alpha + \beta) = 69$	M1	2.2a	
	Forms a quadratic in α or β : $4\alpha^2 - 8\alpha + 29 = 0$ or $4\beta^2 - 8\beta + 29 = 0$ or equivalent is seen and attempts to solve the quadratic.	M1	3.1a	
	States either $\alpha = 1 + \frac{5}{2}i$ or $\beta = 1 + \frac{5}{2}i$	A1	1.1b	
	States the roots of the equation are: $2 + 4i, 2 - 4i, 1 + \frac{5}{2}i, 1 - \frac{5}{2}i$	A1	2.2a	
		(4)		
C	Makes an attempt to use $\alpha\beta\gamma\delta = \frac{n}{a}$ to find n	M1	3.1a	(9 marks)
	Finds $n = 580$	A1	1.1b	
		(2)		
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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
6a	States $\alpha + \beta + \gamma = -\frac{-6}{3} = 2$, $\alpha\beta + \beta\gamma + \gamma\alpha = -\frac{10}{3}$ and $\alpha\beta\gamma = -\frac{-20}{3} = \frac{20}{3}$	B1	1.1b	TBC
		(1)		
bi	Makes an attempt to use $\alpha^4\beta^4\gamma^4 = (\alpha\beta\gamma)^4$	M1	3.1a	
	Finds $\alpha^4\beta^4\gamma^4 = (\alpha\beta\gamma)^4 = \left(\frac{20}{3}\right)^4 = \frac{160\,000}{81}$	A1	1.1b	
		(2)		
bii	Makes an attempt to use $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$	M1	3.1a	
	$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ Finds $= (2)^2 - 2\left(-\frac{10}{3}\right) = \frac{32}{3}$	A1	1.1b	
		(2)		
bi	Makes an attempt to multiply out $(2 - \alpha)(2 - \beta)(2 - \gamma)$	M1	1.1b	
	Finds or states $(2 - \alpha)(2 - \beta)(2 - \gamma)$ $= 8 - 4(\alpha + \beta + \gamma) + 2(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma$	M1	3.1a	
	Finds $(2 - \alpha)(2 - \beta)(2 - \gamma) = 8 - 4(2) + 2\left(-\frac{10}{3}\right) - \frac{20}{3} = -\frac{40}{3}$	A1	1.1b	
		(3)		
				(9 marks)
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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
7	States $w = 2x - 1 \Rightarrow x = \frac{w+1}{2}$	B1	3.1a	TBC
	States $2\left(\frac{w+1}{2}\right)^3 - 4\left(\frac{w+1}{2}\right)^2 + 6\left(\frac{w+1}{2}\right) - 9 = 0$	M1	3.1a	
	Makes an attempt to manipulate the equation into the form $aw^3 + bw^2 + cw + d = 0$	M1	1.1b	
	At least two of a, b, c or d are correct.	A1	1.1b	
	Fully correct final equation: $w^3 - w^2 + 7w - 27 = 0$	A1	1.1b	
		(5)		
				(5 marks)
Notes				
<p>7: Accept an equation that is a multiple of $w^3 - w^2 + 7w - 27 = 0$, most likely $2w^3 - 2w^2 + 14w - 54 = 0$.</p> <p>See also alternative method for first three marks on next page.</p>				

ALTERNATIVE METHOD FOR FIRST THREE MARKS

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
7	States $\alpha + \beta + \gamma = 2$, $\alpha\beta + \beta\gamma + \gamma\alpha = 3$ and $\alpha\beta\gamma = \frac{9}{2}$	B1	3.1a	
	Sum of roots: $(2\alpha - 1) + (2\beta - 1) + (2\gamma - 1)$ $= 2(\alpha + \beta + \gamma) - 3 = 2(2) - 3 = 1$ Pair sum: $(2\alpha - 1)(2\beta - 1) + (2\beta - 1)(2\gamma - 1) + (2\gamma - 1)(2\alpha - 1)$ $= 4(\alpha\beta + \beta\gamma + \gamma\alpha) - 4(\alpha + \beta + \gamma) + 3$ $= 4(3) - 4(2) + 3 = 7$ $(2\alpha - 1)(2\beta - 1)(2\gamma - 1)$ Product: $= 8(\alpha\beta\gamma) - 4(\alpha\beta + \beta\gamma + \gamma\alpha) + 2(\alpha + \beta + \gamma) - 1$ $= 8\left(\frac{9}{2}\right) - 4(3) + 2(2) - 1 = 27$	M1	3.1a	
	Applies: $w^3 - (\text{their sum roots})w^2 + (\text{their pair sum})w - \text{their product} = 0$	M1	1.1b	

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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
8	States $w = 2x \Rightarrow x = \frac{w}{2}$	B1	3.1a	TBC
	States $2\left(\frac{w}{2}\right)^4 - 6\left(\frac{w}{2}\right)^2 + 16\left(\frac{w}{2}\right) - 1 = 0$	M1	3.1a	
	Makes an attempt to manipulate the equation into the form $pw^4 + qw^3 + rw^2 + sw + t = 0$.	M1	1.1b	
	At least two of p, q, r, s and t are correct.	A1	1.1b	
	Fully correct final equation: $w^4 - 12w^2 + 64w - 8 = 0$	A1	1.1b	
		(5)		
				(5 marks)
Notes				
See also alternative method for first three marks on next page.				

ALTERNATIVE METHOD FOR FIRST THREE MARKS

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
8	States $\alpha + \beta + \gamma + \delta = 0$, $\alpha\beta + \beta\gamma + \gamma\alpha + \gamma\delta + \alpha\delta + \beta\delta = -3$, $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -8$ and $\alpha\beta\gamma\delta = -\frac{1}{2}$	B1	3.1a	TBC
	Sum of roots: $2\alpha + 2\beta + 2\gamma + 2\delta = 0$ $(2\alpha)(2\beta) + (2\beta)(2\gamma) + (2\gamma)(2\alpha) + \dots$ $\dots + (2\gamma)(2\delta) + (2\alpha)(2\delta) + (2\beta)(2\delta)$ Pair sum: $= 4(\alpha\beta + \beta\gamma + \gamma\alpha + \gamma\delta + \alpha\delta + \beta\delta)$ $= 4(-3) = -12$ $(2\alpha)(2\beta)(2\gamma) + (2\alpha)(2\beta)(2\delta) + \dots$ $\dots + (2\alpha)(2\gamma)(2\delta) + (2\beta)(2\gamma)(2\delta)$ Triple sum: $= 8(\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta)$ $= 8(-8) = -64$ Product: $(2\alpha)(2\beta)(2\gamma)(2\delta) = 16(\alpha\beta\gamma\delta) = 16\left(-\frac{1}{2}\right) = -8$	M1	3.1a	
	Applies: $w^4 - (\text{their sum roots})w^3 + (\text{their pair sum})w^2$ $- (\text{their triple sum})w + \text{their product} = 0$	M1	1.1b	