1 The equation $kx^2 + 6kx + 30 = 0$ has roots of the form *p* and 3*p*. Find the values of *k* and *p*.
(4 marks)

2 The equation $5x^2 + mx + n = 0$, where *m* and *n* are real constants, has roots α and α*.

a Given that Re(α) = 4, find the value of *m*. **(2 marks)**

b Given that Im(α) ≠ 0, find the range of possible values of *n*. **(2 marks)**

3 The cubic equation $5z^3 - 11z^2 + kz - 50 = 0$ has roots α, β and γ.

a Write down the values of $\alpha\beta + \beta\gamma + \gamma\alpha$, $\alpha\beta\gamma$ and $\alpha + \beta + \gamma$. **(1 mark)**

b Given that $\alpha = 1 + 7i$, and $\gamma \in \mathbb{R}$ find the value of γ. **(3 marks)**

c Find the value of *k*. **(2 marks)**

4 $\mathrm{f}(z) = z^3 + mz^2 + nz - 52$

Given that the roots of the cubic equation $\mathrm{f}(z) = 0$ are α, $\frac{1}{\alpha}$ and $\alpha + \frac{13}{\alpha} + 46$ find the

a roots of the equation $\mathrm{f}(z) = 0$ **(5 marks)**

b values of *m* and *n*. **(4 marks)**

5 The equation $4x^4 - 24x^3 + mx^2 - 276x + n = 0$, $x \in \mathbb{C}$, $m, n \in \mathbb{R}$ has roots α, β, γ and δ.
Given that $\delta = 2 + 4i$ and $\gamma = \delta^*$

a show that $\alpha + \beta - 2 = 0$ and $4\alpha\beta + 20(\alpha + \beta) = 69$ **(3 marks)**

b hence find all the roots of the quartic equation **(4 marks)**

c find the value of *n*. **(2 marks)**

6 The roots of the equation $3x^3 - 6x^2 - 10x - 20 = 0$ are α, β and γ.

a Write down the values of $\alpha\beta + \beta\gamma + \lambda\alpha$, $\alpha\beta\lambda$ and $\alpha + \beta + \gamma$. **(1 mark)**

b Hence find the exact value of

i $\alpha^4\beta^4\gamma^4$ **(2 marks)**

ii $\alpha^2 + \beta^2 + \gamma^2$ **(2 marks)**

iii $(2 - \alpha)(2 - \beta)(2 - \gamma)$ **(3 marks)**

7 The cubic equation $2x^3 - 4x^2 + 6x - 9 = 0$ has roots α, β and γ. Without solving the equation, find a cubic equation whose roots are $(2\alpha - 1)$, $(2\beta - 1)$ and $(2\gamma - 1)$, giving your answer in the form $aw^3 + bw^2 + cw + d = 0$ where a, b, c and d are integers to be found.

(5 marks)

8 The quartic equation $2x^4 - 6x^2 + 16x - 1 = 0$ has roots α, β, γ and δ. Without solving the equation, find a quartic equation whose roots are 2α, 2β, 2γ and 2δ, giving your answer in the form $pw^4 + qw^3 + rw^2 + sw + t = 0$ where p, q, r, s and t are integers to be found.

(5 marks)