Mark scheme

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
1	States $20 \times \left(\frac{1}{2}\right)(21)(22) = 3 \times \left(\frac{1}{2}\right)(k)(k+1)$	M1	1.1b	ТВС
	Makes an attempt to simplify the expression, for example $4620 = \frac{3}{2}k(k+1) \text{ or } 3080 = k^2 + k$	M1	1.1b	
	Correctly solves the quadratic.	M1	1.1b	
	As <i>k</i> must be a positive integer, states $k = 55$	A1	2.3	
		(4)		
(4 marks)				
Notes				

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
2	States or implies that $\sum_{r=n-1}^{2n+1} r = \sum_{r=1}^{2n+1} r - \sum_{r=1}^{n-2} r$	M1	2.1	ТВС
	Correctly substitutes into the standard formulae: $\sum_{r=n-1}^{2n+1} r = \frac{1}{2} (2n+1)(2n+2) - \frac{1}{2} (n-2)(n-1)$	M1	1.1b	
	Makes an attempt to simplify, for example $2n^2 + 3n + 1 - \frac{1}{2}(n^2 - 3n + 2)$ is seen	M1	1.1b	
	Follows a logical progression to obtain $\frac{3}{2}(n^2+3n)$ cso.	A1	1.1b	
		(4)		
				(4 marks)
	Notes			

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
3 a	States or implies that $\sum_{r=1}^{k} (6r-3) = 6 \sum_{r=1}^{k} r - 3 \sum_{r=1}^{k} 1$	M1	2.1	TBC
	Correctly substitutes into the standard formulae: $\sum_{r=1}^{k} (6r-3) = 6 \times \left(\frac{1}{2}\right) (k) (k+1) - 3k$	M1	1.1b	
	Follows a logical progression to obtain $3k^2$ cso.	A1	1.1b	
		(3)		
b	States $3k^2 > 4800$ or $k^2 > 1600$	M1	2.2a	
	Solves the inequality and states that as k must be a positive integer, $k = 41$.	A1	2.3	
		(2)		
				(5 marks)
Notes				

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor	
4	States or implies that $\sum_{r=1}^{n} (ar+b) = a \sum_{r=1}^{n} r + b \sum_{r=1}^{n} 1$	M1	2.1	TBC	
	Correctly substitutes into the standard formulae: $\sum_{r=1}^{n} (ar+b) = a \times \left(\frac{1}{2}\right) (n)(n+1) + bn$	M1	1.1b		
	Simplifies to state: $\sum_{r=1}^{n} (ar+b) = \frac{a}{2}n^{2} + \left(\frac{a}{2}+b\right)n$ oe.	M1	1.1b		
	Equates n^2 coefficients and states that $a = 7$ and equates n coefficients and states that $b = -4$	A1	1.1b		
		(4)			
				(4 marks)	
	Notes				
Alternative Method					
M1: Sta	tes that when $r = 1$, $a + b = 3$.				
M1: Sta	tes that when $r = 2$, $(a + b) + (2a + b) = 13$.				
M1: Ma	kes attempt to solve simultaneous equations.				

A1: States that a = 7 and equates *n* coefficients and states that b = -4

Pearson **Progression Step** Q AOs Scheme Marks and Progress descriptor **M1** TBC 5a 2.1 Attempts to use the fact that $\sum_{r=1}^{5} f(r) = 125$ to substitute into the standard formulae: $a \times \left(\frac{1}{2}\right)(5)(6) + 5b = 125$ A1 1.1b Simplifies to state 3a + b = 25 oe. **M1** 2.1 Attempts to use the fact that $\sum_{r=1}^{10} f(r) = 4755$ to substitute into the standard formulae: $a \times \left(\frac{1}{2}\right) (10)(11) + 10b = 475$ 1.1b Simplifies to state 11a + 2b = 95 oe. A1 Correctly finds a = 9 and b = -2A1 1.1b **M1** 1.1b Attempts to find an expression for $\sum_{r=1}^{n} f(r)$, for example $\frac{9n(n+1)}{2} - 2n$ is seen. Simplifies to obtain $\sum_{r=1}^{n} f(r) = \frac{n(9n+5)}{2}$ oe. **A1** 1.1b (7) b **M1** 2.1 States or implies $\sum_{n=8}^{18} f(r) = \frac{18(9(18)+5)}{2} - \frac{7(9(7)+5)}{2}$ Simplfies to obtain 1265 A1 1.1b (4) (9 marks) Notes

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor	
6a	Substitutes at least one of the standard formulae into their expanded expression.	M1	2.1	ТВС	
	Correctly finds: $\sum_{r=1}^{n} (r+4)(r+1) = \sum_{r=1}^{n} (r^2 + 5r + 4)$ $= \frac{1}{6}n(n+1)(2n+1) + 5 \times \frac{1}{2}n(n+1) + 4n$	A1	1.1b		
	Factorises the <i>n</i> : $\frac{1}{6}n[(n+1)(2n+1)+15(n+1)+24]$	M1*	1.1b		
	Obtains $\frac{1}{3}n(n^2+9n+20)$, showing all work clearly. cso.	A1	1.1b		
		(4)			
b	States or implies that $\sum_{r=6}^{14} (r+4)(r+1) = \sum_{r=1}^{14} (r+4)(r+1) - \sum_{r=1}^{5} (r+4)(r+1)$	M1	2.1		
	Substitutes into the standard formulae: $\sum_{r=6}^{14} (r+4)(r+1)$ $= \frac{1}{3}(14) \Big[14^2 + 9(14) + 20 \Big] - \frac{1}{3}(5) \Big[5^2 + 9(5) + 20 \Big]$	A1	1.1b		
	Solves to obtain 1446	M1	1.1b	•	
		(3)			
	·		<u>.</u>	(7 marks)	
6a: Aw	Solution Notes 6a: Award second method mark providing <i>n</i> is factored out of the expression. Student does not need to factor the				

 $\frac{1}{6}$ at this point in order to achieve the method mark.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
7a	States or implies that $\sum_{r=n+1}^{2n} r^3 = \sum_{r=1}^{2n} r^3 - \sum_{r=1}^{n} r^3$	M1	2.1	TBC
	Correctly substitutes into the standard formulae: $\sum_{r=n+1}^{2n} r^3 = \frac{1}{4} (2n)^2 (2n+1)^2 - \frac{1}{4} (n)^2 (n+1)^2$	M1	1.1b	
	Factors out the n^2 term. For example, $\frac{1}{4}n^2 \left[4(2n+1)^2 - (n+1)^2 \right]$ is seen.	M1	1.1b	
	Follows a logical progression to obtain $\frac{1}{4}n^2(5n+3)(3n+1)$ cso.	A1	1.1b	
		(4)		
b	Makes an attempt to substitute $n = 20$ into $\frac{1}{4}n^{2}(5n+3)(3n+1)$. For example, $\frac{1}{4}(20)^{2}(5(20)+3)(3(20)+1)$ is seen.	M1	2.2a	
	Correctly finds 628 300	A1	1.1b	
		(2)		

(6 marks)

Notes

7a: Award third method mark providing n^2 is factored out of the expression. Student does not need to factor the $\frac{1}{4}$ at this point in order to achieve the method mark.

For the 3rd method mark, it is acceptable to use the method of difference of squares:

$$\frac{1}{4}n^{2} \Big[2(2n+1) - (n+1) \Big] \Big[2(2n+1) + (n+1) \Big]$$
 and then simplify.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
8	Substitutes at least one of the standard formulae into their expanded expression.	M1	2.1	TBC
	Correctly finds: $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ and finds $\sum_{r=1}^{n+1} r + 8\sum_{r=1}^{n+1} 1 = 6 \times \frac{1}{2}(n+1)(n+2) + 8(n+1)$	A1	1.1b	
	Equates both expressions and cancels all terms by $(n+1)$, obtaining $\frac{1}{6}n(2n+1) = 3(n+2)+8$	M1	1.1b	
	Simplifies to obtain $2n^2 - 17n - 84 = 0$	M1	1.1b	
	Solves to find $n = 12$, recognising that n must be a positive integer.	A1	2.3	
		(5)		
				(5 marks)
Notes				

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor	
9a	Correctly finds: $\sum_{r=1}^{n} r^{3} + 2\sum_{r=1}^{n} r^{2} = \frac{1}{4}n^{2}(n+1)^{2} + 2 \times \frac{1}{6}n(n+1)(2n+1)$	M1	2.2a	ТВС	
	Factorises $n(n+1): \frac{1}{6}n(n+1)[3n(n+1)+4(2n+1)]$	M1*	1.1b		
	Obtains $\frac{1}{3}n(n+1)(3n^2+11n+4)$, showing all work clearly. cso.	A1	1.1b		
		(3)			
b	Correctly substitutes into the standard formulae: $\sum_{r=1}^{2n+1} r^2 (r+2)$ $= \frac{1}{12} (2n+1)(2n+2) \Big[3(2n+1)^2 + 11(2n+1) + 4 \Big]$	M1	2.2a		
	Makes an attempt to simplify, for example $\frac{1}{12}(2n+1)(2n+2)[12n^2+34n+18]$ is seen	M1	1.1b		
	Follows a logical progression to obtain $\frac{1}{3}(2n+1)(n+1)(6n^2+17n+9)$ cso.	A1	1.1b		
		(3)			
(6 marks)					
0 1	Notes				
9a: Award second method mark providing n is factored out of the expression. Student does not need to factor the					

 $\frac{1}{12}$ at this point in order to achieve the method mark.