1 Given that $20\sum_{r=1}^{21} r = 3\sum_{r=1}^{k} r$, find the value of k **(4 marks)**

2 Show that $\sum_{r=n-1}^{2n+1} r = \frac{3}{2}\left(n^2 + 3n\right)$ **(4 marks)**

3 a Show that $\sum_{r=1}^{k}(6r-3) = 3k^2$ **(3 marks)**

   b Find the smallest value of k for which $\sum_{r=1}^{k}(6r-3) > 4800$ **(2 marks)**

4 Given that $f(r) = ar + b$ and $\sum_{r=1}^{n} f(r) = \frac{1}{2}n(7n-1)$, find the values of the
constants a and b **(4 marks)**

5 $f(r) = ar + b$, where a and b are rational constants. Given that $\sum_{r=1}^{5} f(r) = 125$ and
$\sum_{r=1}^{10} f(r) = 475$

   a find an expression for $\sum_{r=1}^{n} f(r)$ **(7 marks)**

   b hence calculate $\sum_{r=8}^{18} f(r)$ **(2 marks)**

6 a Show that $\sum_{r=1}^{n}(r+4)(r+1) = \frac{1}{3}n\left(n^2 + 9n + 20\right)$. **(4 marks)**

   b Hence evaluate $\sum_{r=6}^{14}(r+4)(r+1)$ **(3 marks)**

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7 a Show that $\displaystyle\sum_{r=n+1}^{2n} r^3 = \frac{1}{4}n^2(5n+3)(3n+1)$ **(4 marks)**

b Hence evaluate $\displaystyle\sum_{r=21}^{40} r^3$ **(2 marks)**

8 Find the value of n that satisfies $\displaystyle\sum_{r=1}^{n} r^2 = \sum_{r=1}^{n+1}(6r+8)$ **(5 marks)**

9 a Show that $\displaystyle\sum_{r=1}^{n} r^2(r+2) = \frac{1}{12}n(n+1)(3n^2+11n+4)$ **(3 marks)**

b Hence show that $\displaystyle\sum_{r=1}^{2n+1} r^2(r+2) = \frac{1}{3}(n+1)(2n+1)(6n^2+17n+9)$ **(3 marks)**