For **Pearson Edexcel**
Level 3 GCE

AS Mathematics

Paper 1: Pure Mathematics

Churchill Paper 1A – Marking Guide

Method marks (M) are awarded for knowing and attempting to apply a valid method

Accuracy marks (A) are awarded for a correct answer, having earned the relevant method marks

(B) marks are unconditional accuracy marks (independent of method marks)

Written by Shaun Armstrong

Churchill AS Paper 1A Marking Guide – Edexcel

1 Grad of $L_1 = \frac{1}{3}$

L_2 perp so grad = $\dfrac{-1}{\left(\frac{1}{3}\right)} = -3$ M1

Grad of $L_2 = \dfrac{(p-1)-10}{p-(p-3)} = \dfrac{p-11}{3}$ M1

So, $\dfrac{p-11}{3} = -3$

$p - 11 = -9$

$p = 2$ M1 A1 Total 4

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2 No real roots so $b^2 - 4ac < 0$

$(-7)^2 - 4 \times a \times 3 < 0$ M1

$49 - 12a < 0$

$12a > 49$

$a > 4\frac{1}{12}$ M1

Smallest integer value = 5 A1 Total 3

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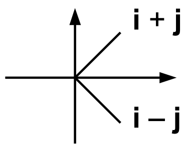
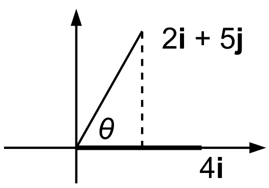
3 **(a)** 90° B1



(b)



$\tan\theta = \frac{5}{2}$ M1

$\theta = \tan^{-1}\frac{5}{2} = 68.2°$ (3sf) A1

(c) $|\mathbf{A}| = \sqrt{14^2 + 2^2} = \sqrt{200}$ M1

$|\mathbf{B}| = \sqrt{2^2 + (-6)^2} = \sqrt{40}$

$|\mathbf{A}| = \sqrt{5 \times 40} = \sqrt{5} \times \sqrt{40} = \sqrt{5} \times |\mathbf{B}|$ M1 A1

So $|\mathbf{A}|$ is $\sqrt{5}$ times $|\mathbf{B}|$ Total 6

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4 **(a)** e.g. *C* is a positive quadratic so "smile" shape. It won't go below minimum value which is above *x*-axis, hence doesn't cross B1

(b) $\dfrac{dy}{dx} = 4x - 3$

At minimum, $\dfrac{dy}{dx} = 0$ so $4x - 3 = 0$ M1

$x = \frac{3}{4}$

When $x = \frac{3}{4}$, $y = \frac{5}{2}$ so $\frac{5}{2} = 2(\frac{3}{4})^2 - 3(\frac{3}{4}) + k$ M1

$\frac{5}{2} = \frac{9}{8} - \frac{9}{4} + k$

$k = \frac{5}{2} + \frac{9}{8} = \frac{29}{8}$ M1

Hence crosses *y*-axis at $(0, \frac{29}{8})$ A1 Total 5

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5 **(a)** (–7, 2) lies on circle so

$(-7)^2 + 2^2 + 2(-7) - 8(2) + c = 0$ — M1

$49 + 4 - 14 - 16 + c = 0$

$c = -23$ — A1

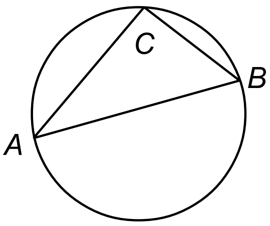
(b) $(x + 1)^2 - 1 + (y - 4)^2 - 16 - 23 = 0$ — M1

$(x + 1)^2 + (y - 4)^2 = 40$

$r^2 = 40$ — M1

radius = $\sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$ — A1

(c)



AB is diameter so angle *ACB* = 90º — B1

Pythagoras': $AB^2 = AC^2 + BC^2$

$(2 \times 2\sqrt{10})^2 = 12^2 + BC^2$ — M1

$BC^2 = 160 - 144 = 16$

$BC = 4$ — A1 — Total 8

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6 **(a)** $x - 4 = 4^2$ — M1

$x = 16 + 4 = 20$ — A1

(b) $2\log_p\left(\frac{3}{p}\right) + \log_p\left(\frac{p^7}{9}\right) = \log_p\left(\frac{3}{p}\right)^2 + \log_p\left(\frac{p^7}{9}\right)$ — M1

$= \log_p\left(\frac{9}{p^2}\right) + \log_p\left(\frac{p^7}{9}\right)$

$= \log_p\left(\frac{9}{p^2} \times \frac{p^7}{9}\right)$ — M1

$= \log_p\left(p^5\right)$

$= 5$ — A1 — Total 5

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7 $\mathrm{f}(x) = 7 - 2x(x^2 - 2x + 1)$

$= 7 - 2x^3 + 4x^2 - 2x$ — M1

$\mathrm{f}'(x) = -6x^2 + 8x - 2$ — M1

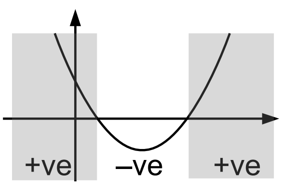
Increasing when $\mathrm{f}'(x) > 0$

$-6x^2 + 8x - 2 > 0$ — A1

$3x^2 - 4x + 1 < 0$

$(3x - 1)(x - 1) < 0$ — M1

c.v. = $\frac{1}{3}$, 1



$\frac{1}{3} < x < 1$ — A1 — Total 5

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8 $(x - 1)(x - 4)(x + c) = (x + c)(x^2 - 5x + 4)$

$= x^3 - 5x^2 + 4x$

$+ cx^2 - 5cx + 4c$ — M1

$= x^3 + (c - 5)x^2 + (4 - 5c)x + 4c$ [can just do *x* coeff] — A1

Equating coeffs of *x* : $4 - 5c = -21$ — M1

$5c = 4 + 21 = 25$

$c = 5$ — A1 — Total 4

9	**(a)**	e.g. $n = 1$ $\quad 3n^2 + n - 1 = 3 + 1 - 1 = 3$ (prime)		
		$n = 2$ $\quad 3n^2 + n - 1 = 12 + 2 - 1 = 13$ (prime)	M1	
		$n = 3$ $\quad 3n^2 + n - 1 = 27 + 3 - 1 = 29$ (prime)		
		$n = 4$ $\quad 3n^2 + n - 1 = 48 + 4 - 1 = 51$		
		When $n = 4$ the value is 51 [= 3 × 17] which is not prime	A1	
		[other examples include n = 7, 10, 13 giving 153, 309, 519]		
	(b)	e.g. $3n^2 + n = n(3n + 1)$		
		when n is even $\quad n(3n + 1)$ will be even × integer which is even	M1	
		when n is odd $\quad 3n$ will be odd × odd which is odd		
		$(3n + 1)$ will be odd + 1 which is even		
		$n(3n + 1)$ will be odd × even which is even	M1	
		n must be even or odd so $3n^2 + n$ will always be even	A1	Total 5
10	**(a)**	1st : Missed a solution (360º) to $\sin x = 0$ in given interval	B1	
		2nd : Replaces $\frac{\cos x}{\sin x}$ with $\tan x$ but they are not equal	B1	
	(b)	$\sin x = 3 \cos x$		
		$\frac{\sin x}{\cos x} = 3$		
		$\tan x = 3$	M1	
		x = 71.6º or 251.6º (1dp)	M1	
		ALL: $\quad x$ = 0º, 71.6º (1dp), 180º, 251.6º (1dp), 360º	A1	Total 5
11	**(a)**	= 15 + 8 = 23	B1	
	(b)	$f(x - 2)$ is a translation of 2 units in positive x direction		
		So region of $f(x)$ between 0 and 2 is translated to between 2 and 4	B1	
		Hence, required area = 15	B1	
	(c)	e.g. Width of "bar" = 2 so average height = 21 ÷ 2 = 10.5		
		From the diagram we can see that $f(x) > f(0)$ for all other values of x $(-2 \le x < 0)$		
		Hence, if $f(0) = 10.5$ or more the area would be greater than 21 so $f(0)$ must be less than 10.5	M1 A1	
	(d)	e.g. The region between $x = 2$ and $x = 4$ is a trapezium		
		Area $= \frac{1}{2} \times 2 \times [f(2) + f(4)] = 8$	M1	
		So $\quad f(2) + f(4) = 8$ $\quad$ **(A)**		
		The region between $x = 4$ and $x = 6$ is also a trapezium		
		Area $= \frac{1}{2} \times 2 \times [f(4) + f(6)] = 7$		
		So $\quad f(4) + f(6) = 7$ $\quad$ **(B)**		
		(A) – **(B)** gives $\quad f(2) - f(6) = 8 - 7$	M1	
		So $\quad f(2) - f(6) = 1$	A1	Total 8

12 Let $x = x + h$, $f(x + h) = 2(x + h)^3$

$= 2(x^3 + 3x^2(h) + 3x(h^2) + h^3)$ B1

$= 2x^3 + 6x^2h + 6xh^2 + 2h^3$

Gradient of chord from x to $x + h$ = $\dfrac{f(x+h) - f(x)}{(x+h) - x}$

$= \dfrac{(2x^3 + 6x^2h + 6xh^2 + 2h^3) - 2x^3}{h}$ M1

$= \dfrac{6x^2h + 6xh^2 + 2h^3}{h}$

$= 6x^2 + 6xh + 2h^2$ A1

As $h \to 0$, the chord becomes the tangent at $(x, f(x))$

As $h \to 0$, terms in $h \to 0$, gradient $\to 6x^2$

Hence, $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = 6x^2$ A1 Total 4

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13 **(a)** **(i)** When $t = 0$, $N = 250$ B1

Number at start of 2005 = 250

(ii) $N < 80$ so $250 - 50\sqrt{t} < 80$

$50\sqrt{t} > 170$

$\sqrt{t} > 3.4$

$t > 11.56$ M1

Hence, the year from 11 to 12 years after start of 2005

2016 A1

(iii) $N = 250 - 50t^{\frac{1}{2}}$

$\dfrac{dN}{dt} = -25t^{-\frac{1}{2}}$ M1

At start of 2014, $t = 9$

$\dfrac{dN}{dt} = -\dfrac{25}{\sqrt{9}} = -\dfrac{25}{3}$

Decreasing by 8.33 (3sf) per year A1

(b) $t = 0$, $N = 250$ so $250 = a\,e^0$

$a = 250$ B1

$t = 4$, first model gives $N = 250 - 50\sqrt{4} = 150$

In second model $150 = 250e^{-4b}$

$e^{-4b} = 150 \div 250 = \dfrac{3}{5}$

$-4b = \ln \dfrac{3}{5}$ M1

$b = -\dfrac{1}{4}\ln \dfrac{3}{5} = 0.127706... = 0.128$ (3sf) A1 Total 8

14	**(a)**	e.g. (8, 0) because it is an odd function [changing the sign of x (–8 to 8) changes the sign of each term and therefore changes the sign of y (0 stays as 0)]	B1	
	(b)	$= \dfrac{4}{\left(\frac{4}{3}\right)} x^{\frac{1}{3}+1} - \frac{1}{2}x^2 + c$	M1 A1	
		$= 3x^{\frac{4}{3}} - \frac{1}{2}x^2 + c$	A1	
	(c)	Area above OB = $\left[3x^{\frac{4}{3}} - \frac{1}{2}x^2\right]_0^8$		
		$= (3 \times 8^{\frac{4}{3}} - \frac{1}{2} \times 8^2) - (0)$	M1	
		$= 48 - 32 = 16$		
		Shaded area = $2 \times 16 = 32$	M1 A1	Total 7
15	**(a)**	$= 4^7 + 7 \times 4^6 \times \left(-\frac{1}{x}\right) + \frac{7 \times 6}{2} \times 4^5 \times \left(-\frac{1}{x}\right)^2 + \ldots$	M1	
		$= 16384 - \dfrac{28672}{x} + \dfrac{21504}{x^2} + \ldots$	A2	
	(b)	$4 - \frac{1}{x} = 3.95$		
		$\frac{1}{x} = 4 - 3.95 = 0.05 = \frac{1}{20}$		
		$x = 20$	B1	
	(c)	$3.95^7 \approx 16384 - \dfrac{28672}{20} + \dfrac{21504}{20^2}$	M1	
		$= 15004.16$		
		$3.95^7 = 15003.053...$		
		Estimate is accurate to 4 significant figures	A1	Total 6

16 **(a)** $P = 2y + x + \frac{1}{2} \times \pi \times x$ B1

$= 2y + x(1 + \frac{1}{2}\pi)$

Area $= xy = 100$

Hence $y = \frac{100}{x}$

Sub into P: $P = 2(\frac{100}{x}) + x(1 + \frac{1}{2}\pi)$ M1

$P = \frac{200}{x} + x(1 + \frac{1}{2}\pi)$ A1

(b) $P = 200x^{-1} + x(1 + \frac{1}{2}\pi)$

$\frac{dP}{dx} = -200x^{-2} + (1 + \frac{1}{2}\pi)$ M1 A1

For minimum, $-200x^{-2} + (1 + \frac{1}{2}\pi) = 0$

$x^{-2} = \frac{1}{200}(1 + \frac{1}{2}\pi)$

$x^2 = \frac{200}{1 + \frac{1}{2}\pi}$

$x = \sqrt{\frac{200}{1 + \frac{1}{2}\pi}} = 8.820...$ M1

Minimum $P = \frac{200}{8.82} + 8.82(1 + \frac{1}{2}\pi)$

$= 45.3501... = 45.4$ cm (3sf) A1

$\frac{d^2P}{dx^2} = 400x^{-3}$

When $x = 8.820...$, $\frac{d^2P}{dx^2} = 0.582...$

As $\frac{d^2P}{dx^2} > 0$, it is a minimum A1 Total 8

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17 **(a)** $(4x + 3)(3x - 1) = 0$ M1

$x = -\frac{3}{4}$ or $\frac{1}{3}$ A2

(b) **(i)** $12 \sin^2 A - 5 \cos A = 9$

$12(1 - \cos^2 A) - 5 \cos A = 9$ M1

$12 - 12 \cos^2 A - 5 \cos A = 9$

$12 \cos^2 A + 5 \cos A - 3 = 0$

From **(a)**, $\cos A = -\frac{3}{4}$ or $\frac{1}{3}$ A1

$\cos A = -\frac{3}{4}$: $A = 180 - 41.41$ or $180 + 41.41$ M1

$A = 138.59$ or 221.41

$\cos A = \frac{1}{3}$: $A = 70.53$ or $360 - 70.53$

$A = 70.53$ or 289.47

$A = 70.5°, 139°, 221°, 289°$ (all 3sf) A1

(ii) Equation is same as **(i)** with A replaced by $3B$

Hence $3B = 70.53, 138.59, 221.41$ (no more in interval)

$B = 23.5°, 46.2°, 73.8°$ (all 3sf) M1 A1 Total 9

TOTAL FOR PAPER: 100 MARKS