Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Bronze Level B1

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 11 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
74	66	61	55	50	45

1. Simplify

$$\frac{7+\sqrt{5}}{\sqrt{5}-1},$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are integers.

(4)

May 2013

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2. Find $\int (8x^3 + 4) \;\; \mathrm{d}x$, giving each term in its simplest form.

(3)

May 2014

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3. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 4a_n - 3, \qquad n \geq 1$$

$$a_1 = k, \qquad \text{where } k \text{ is a positive integer.}$$

(*a*) Write down an expression for a_2 in terms of k.

(1)

Given that $\sum_{r=1}^{3} a_r = 66$

(*b*) find the value of k.

(4)

May 2014 (R)

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4. Given that $y = 2x^5 + \frac{6}{\sqrt{x}}$, $x > 0$, find in their simplest form

(*a*) $\frac{\mathrm{d}y}{\mathrm{d}x}$

(3)

(*b*) $\int y \mathrm{d}x$

(3)

May 2014 (R)

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5. Solve the simultaneous equations

$$y + 4x + 1 = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(6)

May 2016

6.

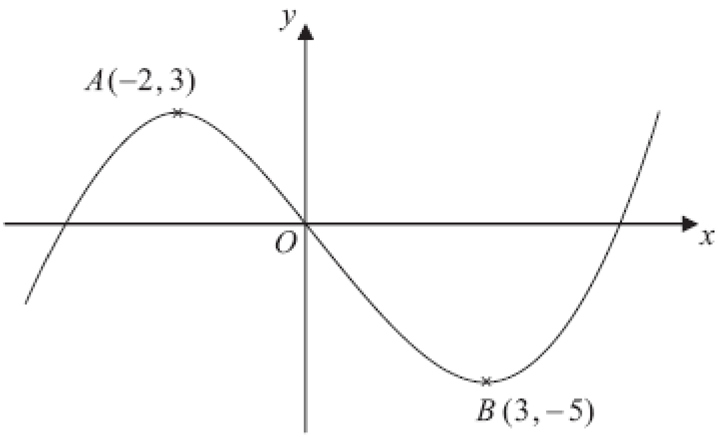

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve has a maximum point A at (–2, 3) and a minimum point B at (3, – 5).

On separate diagrams sketch the curve with equation

(*a*) $y = \mathrm{f}\,(x + 3)$,

(3)

(*b*) $y = 2\mathrm{f}(x)$.

(3)

On each diagram show clearly the coordinates of the maximum and minimum points.

The graph of $y = \mathrm{f}(x) + a$ has a minimum at (3, 0), where a is a constant.

(*c*) Write down the value of a.

(1)

May 2010

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7. Each year, Abbie pays into a savings scheme. In the first year she pays in £500. Her payments then increase by £200 each year so that she pays £700 in the second year, £900 in the third year and so on.

(*a*) Find out how much Abbie pays into the savings scheme in the tenth year.

(2)

Abbie pays into the scheme for n years until she has paid a total of £67 200.

(*b*) Show that $n^2 + 4n - 24 \times 28 = 0$.

(5)

(*b*) Hence find the number of years that Abbie pays into the savings scheme.

(2)

May 2013 (R)

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8. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \qquad x > 0,$$

find $\frac{dy}{dx}$.

(6)

June 2010

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9. (*a*) Factorise completely $9x - 4x^3$.

(3)

(*b*) Sketch the curve C with equation

$$y = 9x - 4x^3.$$

Show on your sketch the coordinates at which the curve meets the x-axis.

(3)

The points A and B lie on C and have x coordinates of -2 and 1 respectively.

(*c*) Show that the length of AB is $k\sqrt{10}$, where k is a constant to be found.

(4)

May 2015

10.

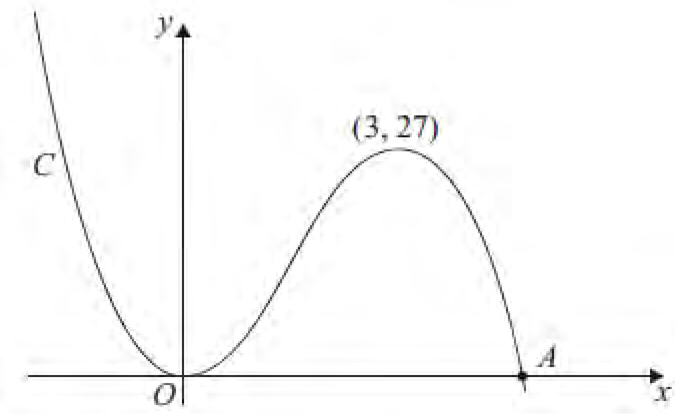


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x^2(9 - 2x).$$

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

(*a*) Write down the coordinates of the point A.

(1)

(*b*) On separate diagrams sketch the curve with equation

(i) $y = \mathrm{f}(x + 3)$,

(ii) $y = \mathrm{f}(3x)$.

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

(6)

The curve with equation $y = \mathrm{f}(x) + k$, where k is a constant, has a maximum point at (3, 10).

(*c*) Write down the value of k.

(1)

May 2012

11.

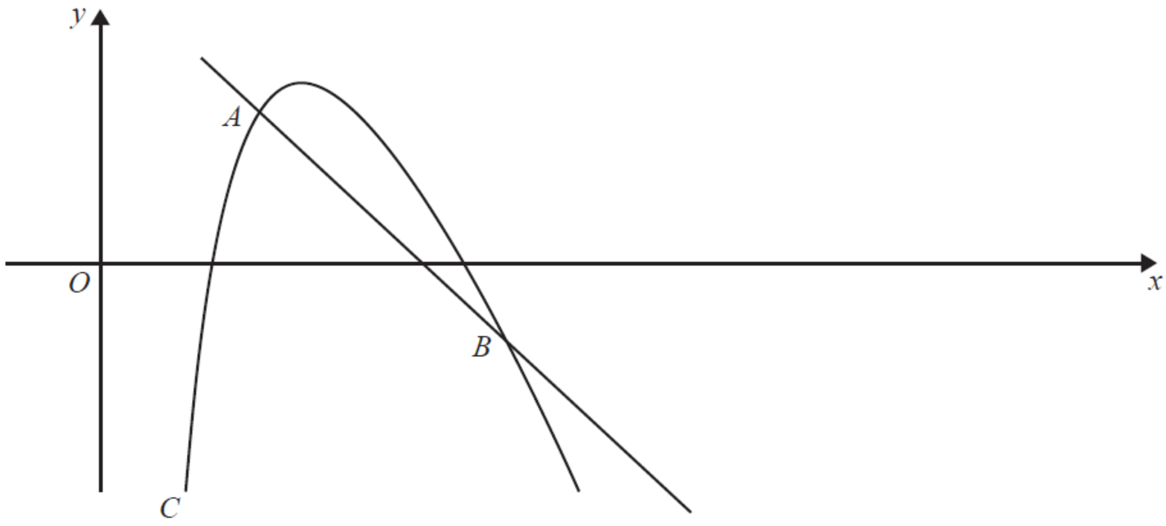


Figure 3

A sketch of part of the curve C with equation

$$y = 20 - 4x - \frac{18}{x}, \qquad x > 0$$

is shown in Figure 3.

Point A lies on C and has an x coordinate equal to 2.

(a) Show that the equation of the normal to C at A is $y = -2x + 7$.

(6)

The normal to C at A meets C again at the point B, as shown in Figure 3.

(b) Use algebra to find the coordinates of B.

(5)

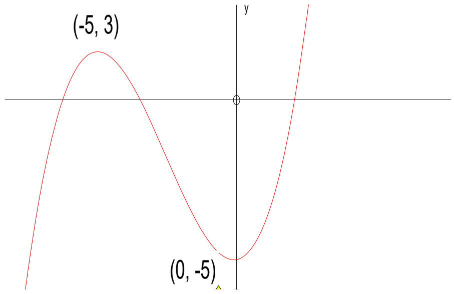
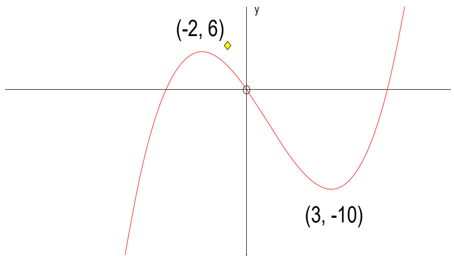
May 2014 (R)

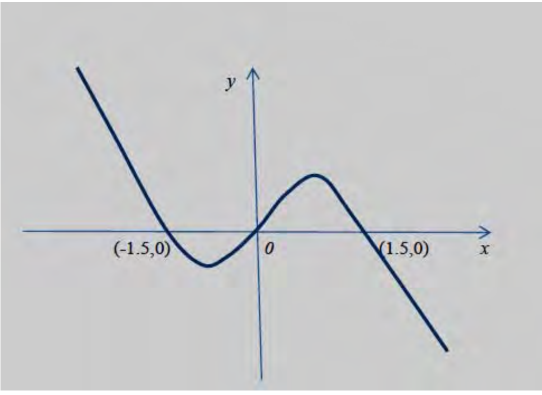
TOTAL FOR PAPER: 75 MARKS

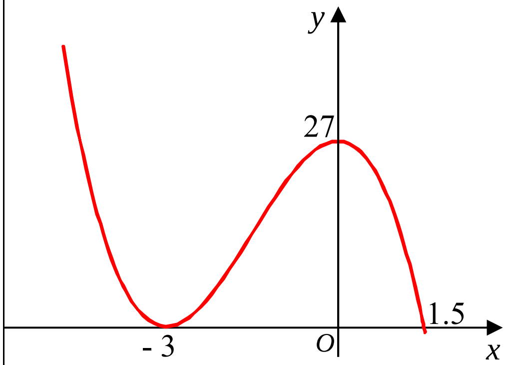
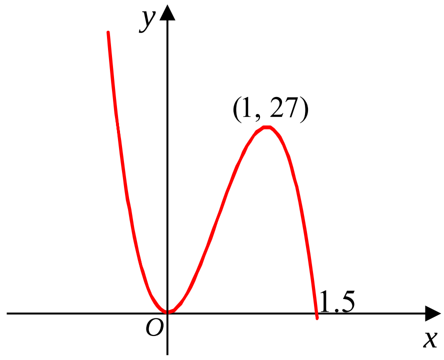
END

Question Number	Scheme	Marks
1	$\dfrac{7+\sqrt{5}}{\sqrt{5}-1}\times\dfrac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$	M1
	$=\dfrac{\dots}{4}$	A1 cso
	$(7+\sqrt{5})(\sqrt{5}+1)=7\sqrt{5}+5+7+\sqrt{5}$	M1
	$3+2\sqrt{5}$	A1 cso **[4]**
2	$\int(8x^3+4)\,\mathrm{d}x \;=\dfrac{8x^4}{4}+4x$	M1 A1
	$=2x^4+4x+c$	A1 **[3]**
3(a)	$(a_2=)\quad 4k-3$	B1 **(1)**
(b)	$a_3=4(4k-3)-3$	M1
	$\sum_{r=1}^{3}a_r=k+4k-3+4(4k-3)-3=..k\pm\dots$	M1
	$21k-18=66\Rightarrow k=\dots$	dM1
	$k=4$	A1 **(4)** **[5]**

Question Number	Scheme	Marks
4(a)	$y = 2x^5 + \dfrac{6}{\sqrt{x}}$	
	$x^n \to x^{n-1}$	M1
	$\dfrac{dy}{dx} = 10x^4 - 3x^{-\frac{3}{2}}$ oe	A1A1
		(3)
(b)	$\int 2x^5 + \dfrac{6}{\sqrt{x}} dx$	
	$x^n \to x^{n+1}$	M1
	$= \dfrac{x^6}{3} + 12x^{\frac{1}{2}} + c$	A1 A1
		(3) **[6]**
5	$y = -4x - 1$ $\Rightarrow (-4x-1)^2 + 5x^2 + 2x = 0$	M1
	$21x^2 + 10x + 1 = 0$	A1
	$(7x+1)(3x+1) = 0 \Rightarrow (x =) -\frac{1}{7}, -\frac{1}{3}$	dM1 A1
	$y = -\dfrac{3}{7}, \dfrac{1}{3}$	M1 A1
		[6]

Question Number	Scheme		Marks
6 (a)	 	Horizontal translation of ± 3	M1
		$(-5, 3)$ marked on sketch **or in text**	B1
		$(0, -5)$ and min intentionally on y-axis Condone $(-5, 0)$ if correctly placed on negative y-axis	A1
			(3)
(b)	 	Correct shape and intentionally through (0,0) between the max and min	B1
		$(-2, 6)$ marked on graph **or in text**	B1
		$(3, -10)$ marked on graph **or in text**	B1
			(3)
(c)	$(a =)\ \underline{5}$		B1
			(1)
			[7]
7 (a)	$U_{10} = 500 + (10-1) \times 200$		M1
	$=(£)2300$		A1
			(2)
(b)	**Mark parts (b) and (c) together**		
	$\frac{n}{2}\{2 \times 500 + (n-1) \times 200\} = 67200$		M1A1
	$n^2 + 4n - 672 = 0$		dM1A1
	$n^2 + 4n - 24 \times 28 = 0$ *		A1
			(5)
(c)	$(n-24)(n+28) = 0 \Rightarrow n = ..$ or $n(n+4) = 24 \times 28 \Rightarrow n = ..$		M1
	24		A1
			(2)
			[9]

Question Number	Scheme	Marks
8	$\dfrac{3x^2+2}{x}=3x+2x^{-1}$	M1 A1
	$(y'=)\,24x^2, -2x^{-\frac{1}{2}}, +3-2x^{-2}$	M1 A1 A1 A1
	$\left[24x^2-2x^{-\frac{1}{2}}+3-2x^{-2}\right]$	**[6]**
9 (a)	$9x-4x^3=x(9-4x^2)$ or $-x(4x^2-9)$	B1
	$9-4x^2=(3+2x)(3-2x)$ or $4x^2-9=(2x-3)(2x+3)$	M1
	$9x-4x^3=x(3+2x)(3-2x)$	A1 **(3)**
(b)		M1 B1 A1 **(3)**
(c)	$A=(-2,\ 14),\quad B=(1,\ 5)$	B1 B1
	$(AB=)\sqrt{(-2-1)^2+(14-5)^2}\ \left(=\sqrt{90}\right)$	M1
	$(AB=)\,3\sqrt{10}$ cao	A1 **(4)** **[10]**

Question Number	Scheme		Marks
10 (a)	{Coordinates of A are} $(4.5, 0)$	See notes below	B1
			(1)
		Horizontal translation -3 and their ft 1.5 on postive x-axis	M1 A1 ft
		Maximum at 27 marked on the y-axis	B1
			(3)
(b)		Correct shape, minimum at $(0, 0)$ and a maximum within the first quadrant.	M1
		1.5 on x-axis	A1 ft
		Maximum at $(1, 27)$	B1
			(3)
(c)	$\{k =\}\ -17$		B1
			(1) **[8]**
11 (a)	Substitutes $x = 2$ into $y = 20 - 4\times 2 - \frac{18}{2}$ and gets 3		B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -4 + \frac{18}{x^2}$		M1 A1
	Substitute $x = 2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{1}{2}\right)$ then finds negative reciprocal (-2)		dM1
	States or uses $y - 3 = -2(x - 2)$ or $y = -2x + c$ with their (2, 3)		dM1
	to deduce that $y = -2x + 7$ *		A1*
			(6)
(b)	Put $20 - 4x - \frac{18}{x} = -2x + 7$ and simplify to give $2x^2 - 13x + 18 = 0$		M1 A1
	Or put $y = 20 - 4\left(\frac{7-y}{2}\right) - \frac{18}{\left(\frac{7-y}{2}\right)}$ to give $y^2 - y - 6 = 0$		
	$(2x - 9)(x - 2) = 0$ so $x =$ or $(y - 3)(y + 2) = 0$ so $y =$		dM1
	$x = \frac{9}{2}, y = -2$		A1 A1
			(5) **[11]**

Examiner reports

Question 1

Full marks were scored by the majority of candidates. Wrong methods involved the use of an incorrect multiplier; for example $(\sqrt{5} - 1)/(\sqrt{5} - 1)$, $(\sqrt{5} - 1)/(\sqrt{5} + 1)$ and $(7 - \sqrt{5})/\sqrt{5} + 1)$ were all seen. There were also problems in calculating the denominator (6 was a common answer). Some candidates failed to understand how to cancel through the 4 from the denominator, cancelling only one term in the numerator; e.g. $(12 + 8\sqrt{5})/4$ became $3 + 8\sqrt{5}$ or $12 + 2\sqrt{5}$. Errors were also seen in multiplying out the numerator and not all candidates found four terms. Arithmetical errors led to $7 + 5 = 11$ or 13 and $7\sqrt{5} + \sqrt{5} = 6\sqrt{5}$.

Question 2

Most candidates (87.5%) achieved full marks on this question. A very few differentiated instead of integrating. Common errors were not simplifying the coefficient of x^2, omitting the $+C$ or leaving the integral sign in the final answer, so that 11% lost just one mark. Very few responses were left blank.

Question 3

This question was generally very well done. Part (a) was usually correct. In part (b) there were some fairly common errors. A minority used the formula for the sum of an AP. Some put a_3 equal to 66 and another group of students did not put their sum equal to 66. There was also some weak algebra solving the linear equation in k.

Question 4

Part (a) was usually correct. Any errors were usually in dealing with the second term, expressing $\frac{6}{\sqrt{x}}$ as a multiple of an index and then differentiating.

In part (b) the main difficulty was in the integration of $\frac{6}{\sqrt{x}}$. There were a few but not many, who integrated their answer to part (a).

Question 5

A large number of fully correct answers were seen.

The majority of candidates obtained the correct answers for x. Mistakes were made from not rearranging the linear equation correctly. Quite a few candidates made an error in making y the subject, e.g. using $y = 4x + 1$. There were sometimes errors in expanding $(-4x - 1)^2$. Some struggled to solve the quadratic equation by factorising and attempted to use the formula often making arithmetic errors with its use. Some followed correct factorisation with $x = +\frac{1}{3}$ and $x = +\frac{1}{7}$. Common slips were sign errors when substituting x back into y and there were some issues multiplying $\frac{1}{7}$ or $\frac{1}{3}$ by 4. A number of candidates substituted their values of x into an incorrect equation for y, usually $y = -4x + 1$, even though they had rearranged correctly at the start.

Very few rearranged to make x the subject before solving. A few candidates used both methods to check their answers and some candidates substituted their values back into both equations to check their answers. Most candidates paired their solutions and wrote their fractions in the simplest form.

A small number of candidates failed to find any y values, having found the x values.

Question 6
In parts (a) and (b) of this question, most candidates were able to quote and accurately use the formulae for length of an arc and area of a sector. Wrong formulae including π were occasionally seen and it was sometimes felt necessary to convert 0.7 radians into degrees.

Despite the right-angled triangle, a very popular method in part (c) was to find the angle at *C* and use the sine rule. For the angle at *C*, many candidates used 0.87 radians (or a similarly rounded version in degrees) rather than a more accurate value. This premature approximation resulted in an answer for *AC* that was not correct to 2 decimal places, so the accuracy mark was lost.

In part (d), although a few candidates thought the region *H* was a segment, most were able to make a fair attempt to find the required area. There was again an unwillingness to use the fact that triangle *OAC* was right-angled, so that $\frac{1}{2}ab\sin C$ appeared frequently. Unnecessary calculations (such as the length of *OC*) were common and again premature approximation often led to the loss of the accuracy mark.

Question 7
Almost all candidates correctly obtained the values –4 and 2 in part (a).
In part (b) most candidates knew they had to integrate to find the area, and most did so correctly, with only a few differentiating and only a few mistakes in the integration. They generally used the limits correctly, though a surprising number split from –4 to 0 and from 0 to 2. It was quite common to leave the final answer as 24. Arithmetic with fractions was invariably well executed. Those who found the rectangle area separately usually did this correctly. Many candidates subtracted the functions before integrating and this often led to the predictable errors of incorrect subtraction, or of obtaining a negative area after subtraction the wrong way round. Some did realise that the area must be positive, but the reason for the sign change was not always explained.

Question 8
In part (a), while some candidates showed little understanding of the theory of logarithms, others produced excellent solutions. The given answer was probably helpful here, giving confidence in a topic that seems to be demanding at this level. It was important for examiners to see full and correct logarithmic working and incorrect statements such as $\log(x-5)^2 - \log(2x-13) = \dfrac{\log(x-5)^2}{\log(2x-13)}$ were penalised, even when there was apparent 'recovery' (helped by the given answer). The most common reason for failure was the inability to deal with the 1 by using $\log_3 3$ or an equivalent approach.

From $\log_3 \dfrac{(x-5)^2}{(2x-13)} = 1$, it was good to see candidates using the base correctly to obtain $\dfrac{(x-5)^2}{(2x-13)} = 3^1$, from which the required equation followed easily.

Even those who were unable to cope with part (a) often managed to understand the link between the parts and solve the quadratic equation correctly in part (b). It was disappointing, however, that some candidates launched into further logarithmic work.

Question 9

This was a discriminating question, especially part (ii)(a), which only very few candidates answered correctly. Perhaps surprisingly, among those who did manage to answer the difficult part (ii)(a), many could not answer (i). However, most candidates did manage to make progress with part (ii)(b), which seemed more familiar.

Knowledge of valid methods for changing or solving trigonometric equations was poor and appears to be a real weakness amongst a sizeable group of the candidates.

Part (i) was the more successfully answered of the two parts overall, though even the task of rearranging the given equation to $\tan\theta$ = "\..\. proved to be problematic for many. Those who did arrive at $\frac{\pi}{3}$ often did not use the most direct method. However, the majority did reach $\frac{\pi}{3}$ somewhere in their solution.

The majority that succeeded used tan, with only a small number squaring the expression and using way 2 on the mark scheme. Such latter attempts often went wrong, leading to incorrect solutions as the algebra in rearranging the trigonometric expressions was not good. For example they often omitted to square root to $\sqrt{3}$. Even successful candidates via this method would often lose the final mark for generating spurious solutions. Completely correct solutions following on from squaring were rare.

Another common method generating extra solutions was to attempt to factor out $\cos 3\theta$, yielding $\cos 3\theta(\tan 3\theta - \sqrt{3}) = 0$ and hence also giving solutions for $\cos 3\theta = 0$.

Most candidates, including those who had not obtained $\frac{\pi}{3}$, succeeded in adding π or 2π (usually the latter) to their previous angle. The omission of one of the solutions, due to only adding 2π to the principal angle, was common. There were a number of candidates that dismissed values outside the limits at the 3θ stage and failed to gain the last marks, and a significant few who did not divide by 3, usually as they had lost the 3 in their equation at an earlier stage.

Many worked in degrees but most of these were usually successful in converting to radians at the end.

In part (ii)(a), the majority of candidates did use the correct identity to gain the first mark, though this was often where they stopped. However, the application of $\sin^2 x = 1 - \cos^2 x$ is well known. Use of $4\sin^2 x = 1 - 4\cos^2 x$ or $4\sin^2 x = 4\cos^2 x - 4$ was sometimes seen.

After achieving a quadratic in $\cos x$, only a very small number of candidates went on to attempt to solve the quadratic by valid means. Those who did generally used the quadratic formula, with attempts at completing the square being very rare.

After proceeding to $4\cos^2 x - \cos x - k = 0$ many simply gave up and proceeded to part (b). Candidates seemed uncertain how to solve the quadratic, the most common error being to write $4\cos^2\theta - \cos\theta = k$ and then factorise the left hand side to $\cos\theta(4\cos\theta - 1) = k$ and deducing $\cos\theta = k$ or $\cos\theta = \frac{1+k}{4}$. Another common response was to see $\cos\theta = \frac{k}{4\cos\theta - 1}$.

For part (ii) (b) most candidates (including those who had correctly solved (a)) picked up from the equation 4 $\cos^2 \theta - \cos \theta - k = 0$ and substituted $k = 3$ and were on more familiar terms with a 3 term quadratic with numerical coefficients.

Having attained 4 $\cos^2 \theta - \cos \theta - k = 0$ solving this equation was generally well done with just the occasional mistake with signs (resulting in values −1 and $\cos \theta = \frac{3}{4}$ for $\cos \theta$). Most solved by factorising the equation, either directly or via an intermediate variable, though quadratic formula or solutions just written (by calculator) were also common. Those who reached $\cos \theta = \cos \theta = -\frac{3}{4}$ and $\cos \theta = 1$ usually went on to earn full marks. However there were also those who tried to use their incorrect work from (a) which usually led nowhere.

Question 10

This question was both well answered and discriminating with about two-thirds of the candidature gaining at least 6 of the 8 marks available and about one-third achieving full marks.

In part (a), most candidates solved $f(x) = 0$ to find the correct x-coordinate of $\frac{9}{2}$ for A. Some candidates, however, found $f(0)$ and arrived at an incorrect value of 9. Other common incorrect values for x were 6 or 27.

In part (b), most candidates were able to give the correct shape for each of the transformed curves.

In part (i), most translated the graph of $y = f(x)$ in the correct direction. Very few candidates translated $y = f(x)$ to the right, and even fewer translated $y = f(x)$ in a vertical direction. Some labelled the y-intercept correctly as $(0, 27)$ but erroneously drew their maximum point slightly to the right of the y-axis in the first quadrant. Most realised that the transformed curve would cut the positive x-axis at "their x in part (a) – 3". Other candidates, who gave no answer to part (a), labelled this x-intercept as $A - 3$ or some left it unlabelled. Occasionally the point $(-3, 0)$ was incorrectly labelled as $(3, 0)$ although it appeared on the negative x-axis.

In part (ii), most graphs had their minimum at the origin and their maximum within the first quadrant. Many realised that the transformed curve would cut the x-axis at $\frac{\text{their } x \text{ in part } (a)}{3}$. Other candidates, who gave no answer to part (a), labelled this intercept as $\frac{A}{3}$, whilst some left it unlabelled. Some misunderstood the given function notation and stretched $y = f(x)$ in the x-direction with scale factor 3 resulting in a maximum of $(9, 27)$ and an x-intercept at $(13.5, 0)$. Very occasionally a stretch of the y-direction; or a two way stretch; or even a reflection of $y = f(x)$ was seen. In a few cases there was an attempt by some candidates to make the graph pass through both $(0, 0)$ and $(0, 27)$.

A significant number of candidates wrote down $k = -17$ whilst some left this part unanswered. Some then wrote down the equation $y = f(x) + k = x^2(9 - 2x) + k$, and substituted in the point $(3, 10)$ to find the correct value of k. Common incorrect answers included $k = 17$ (from $27 - 10$) or $k = 7$ (following $3 + k = 10$).

Question 11
This was a well answered question.

In part (a) most students understood the method and used the curve equation to find the value for y, then differentiated to find an expression for the gradient of the curve. They found a numerical gradient at $x = 2$, then used the negative reciprocal to obtain a numerical gradient for the normal. A few students found $y = 3$ by an incorrect method, using the line equation which they were trying to find, hence producing a circular argument. Differentiation of $\frac{18}{x}$ was a challenge for some, and others made errors calculating the numerical gradients. The printed answer gave them an opportunity to check for errors.

Part (b) was particularly well answered, and most showed good algebraic skills on this question. Very few students attempted the quadratic in y, mostly using the most concise method of solution, involving x. Some students forgot to find the second coordinate.

Statistics for C1 Practice Paper Bronze Level B1

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade: ALL	A*	A	B	C	D	E	U
1	4	4	89	3.54	3.95	3.95	3.84	3.76	3.66	3.51	2.77
2	3		95	2.84	2.99	2.97	2.94	2.92	2.89	2.82	2.52
3	5		93.2	4.66	4.91	4.85	4.75	4.73	4.59	4.33	3.42
4	6		92.2	5.53	5.76	5.82	5.69	5.66	5.55	5.19	4.28
5	6	6	87	5.22	5.82	5.79	5.57	5.41	5.21	4.94	3.84
6	7		82	5.77	6.89	6.75	6.43	6.14	5.80	5.43	4.08
7	9		91	8.22	8.92	8.84	8.67	8.49	8.10	7.50	5.29
8	6		85	5.12	5.94	5.90	5.76	5.62	5.35	4.87	3.34
9	10	10	82	8.17	9.87	9.76	9.40	8.98	8.24	7.15	4.23
10	8		74	5.89	7.82	7.59	7.15	6.58	5.89	5.07	3.03
11	11		76.9	8.46	10.88	10.66	9.73	8.31	6.06	4.62	1.43
	75		**84.56**	**63.42**	**73.75**	**72.88**	**69.93**	**66.60**	**61.34**	**55.43**	**38.23**