

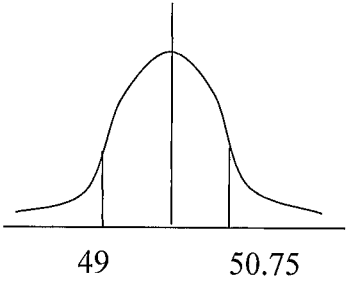
# Specimen A2

## Paper 3: Statistics and Mechanics Mark Scheme

Question	Scheme	Marks	AOs
<b>1(a)</b>	<b>Area</b> = $8 \times 1.5 = 12 \text{ cm}^2$ <b>Frequency</b> = 8 so $1 \text{ cm}^2 = \frac{2}{3} \text{ hour (o.e.)}$	M1	3.1a
	Frequency of 12 corresponds to area of 18 so <b>height</b> = $18 \div 2.5 = 7.2 \text{ (cm)}$	A1	1.1b
	<b>Width</b> = $5 \times 0.5 = 2.5 \text{ (cm)}$	B1cao	1.1b
		<b>(3)</b>	
<b>(b)</b>	$[\bar{y} =] \frac{205.5}{31} = \text{awrt } 6.63$	B1cao	1.1b
	$[\sigma_y =] \sqrt{\frac{1785.25}{31} - \bar{y}^2} = \sqrt{13.644641} = \text{awrt } 3.69$	M1	1.1a
	allow $[s =] \sqrt{\frac{1785.25 - 31\bar{y}^2}{30}} = \text{awrt } 3.75$	A1	1.1b
		<b>(3)</b>	
<b>(c)</b>	Mean of Heathrow is higher than Hurn and standard deviation smaller suggesting Heathrow is more reliable	M1	2.4
	Hurn is South of Heathrow so does <u>not</u> support his belief	A1	2.2b
		<b>(2)</b>	
<b>(d)</b>	$\bar{x} + \sigma \approx 10.3$ so number of days is e.g. $\frac{(11 - "10.3")}{3} \times 8 (+5)$	M1	1.1b
	= 6.86 so <b>7 days</b>	A1	1.1b
		<b>(2)</b>	
<b>(e)</b>	$[H = \text{no. of hours}] \quad P(H > 10.3) \text{ or } P(Z > 1) = [0.15865\dots]$	M1	3.4
	Predict $31 \times 0.15865\dots = \underline{\underline{4.9 \text{ or } 5 \text{ days}}}$	A1	1.1b
		<b>(2)</b>	
<b>(f)</b>	(5 or ) 4.9 days < (7 or ) 6.9 days so model may <b>not</b> be suitable	B1	3.5a
		<b>(1)</b>	
<b>(13 marks)</b>			

<b>Question 1 continued</b>	
<b>Notes:</b>	
<b>(a)</b>	
<b>M1:</b>	for clear attempt to relate the area to frequency. Can also award if their height $\times$ their width = 18
<b>A1:</b>	for height = 7.2 (cm)
<b>(b)</b>	
<b>M1:</b>	for a correct expression for $\sigma$ or $s$ , can ft their value for mean
<b>A1:</b>	awrt 3.69 (allow $s = 3.75$ )
<b>(c)</b>	
<b>M1:</b>	for a suitable comparison of standard deviations to comment on reliability.
<b>A1:</b>	for stating Hurn is south of Heathrow and a correct conclusion
<b>(d)</b>	
<b>M1:</b>	for a correct expression – ft their $\bar{x} + \sigma \approx 10.3$
<b>A1:</b>	for 7 days but accept 6 (rounding down) following a correct expression
<b>(e)</b>	
<b>M1:</b>	for a correct probability attempted
<b>A1:</b>	for a correct prediction
<b>(f)</b>	
<b>B1:</b>	for a suitable comparison and a compatible conclusion

Question	Scheme	Marks	AOs
<b>2(a)</b>	e.g. It requires extrapolation so will be unreliable (o.e.)	B1	1.2
		(1)	
<b>(b)</b>	e.g. Linear association between $w$ and $t$	B1	1.2
		(1)	
<b>(c)</b>	$H_0: \rho = 0$ $H_1: \rho > 0$	B1	2.5
	Critical value 0.5822	M1	1.1a
	Reject $H_0$		
	There is evidence that the product moment correlation coefficient is greater than 0	A1	2.2b
		(3)	
<b>(d)</b>	Higher $\bar{t}$ suggests overseas and not Perth...lower wind speed so perhaps not close to the sea so suggest <b>Beijing</b>	B1	2.4
		(1)	
			<b>(6 marks)</b>
<b>Notes:</b>			
<b>(a)</b>	<b>B1:</b> for a correct statement (unreliable) with a suitable reason		
<b>(b)</b>	<b>B1:</b> for a correct statement		
<b>(c)</b>	<b>B1:</b> for both hypotheses in terms of $\rho$		
	<b>M1:</b> for selecting a suitable 5% critical value compatible with their $H_1$		
	<b>A1:</b> for a correct conclusion stated		
<b>(d)</b>	<b>B1:</b> for suggesting Beijing with some supporting reason based on $t$ or $w$		
	Allow Jacksonville with a reason based just on higher $\bar{t}$		

Question	Scheme	Marks	AOs
<b>Q3(a)</b>			
	$P(L > 50.98) = 0.025$	B1cao	3.4
	$\therefore \frac{50.98 - \mu}{0.5} = 1.96$	M1	1.1b
	$\therefore \mu = 50$	A1cao	1.1b
	$P(49 < L < 50.75)$	M1	3.4
	$= 0.9104\dots$ awrt <b>0.910</b>	A1ft	1.1b
	<b>(5)</b>		
<b>(b)</b>	$S =$ number of strips that cannot be used so $S \sim B(10, 0.090)$	M1	3.3
	$= P(S \leq 3) = 0.991166\dots$ awrt 0.991	A1	1.1b
	<b>(2)</b>		
<b>(c)</b>	$H_0 : \mu = 50.1$ $H_1 : \mu > 50.1$	B1	2.5
	$\bar{X} \sim N\left(50.1, \frac{0.6^2}{15}\right)$ and $\bar{X} > 50.4$	M1	3.3
	$P(\bar{X} > 50.4) = 0.0264$	A1	3.4
	$p = 0.0264 > 0.01$ <u>or</u> $z = 1.936\dots < 2.3263$ and not significant	A1	1.1b
	There is insufficient evidence that the <b>mean length</b> of strips is <b>greater than 50.1</b>	A1	2.2b
	<b>(5)</b>		
<b>(12 marks)</b>			

**Question 3 continued****Notes:****(a)****1<sup>st</sup> M1:** for standardizing with  $\mu$  and 0.5 and setting equal to a  $z$  value ( $|z| > 1$ )**2<sup>nd</sup> M1:** for attempting the correct probability for strips that can be used**2<sup>nd</sup> A1ft:** awrt 0.910 (allow ft of their  $\mu$ )**(b)****M1:** for identifying a suitable binomial distribution**A1:** awrt 0.991 (from calculator)**(c)****B1:** hypotheses stated correctly**M1:** for selecting a correct model (stated or implied)**1<sup>st</sup> A1:** for use of the correct model to find  $p =$  awrt 0.0264 (allow  $z =$  awrt 1.94)**2<sup>nd</sup> A1:** for a correct calculation, comparison and correct statement**3<sup>rd</sup> A1:** for a correct conclusion in context mentioning “mean length” and 50.1

Question	Scheme	Marks	AOs
4(a)	$P(A'   B') = \frac{P(A' \cap B')}{P(B')} \text{ or } \frac{0.33}{0.55}$	M1	3.1a
	$= \frac{3}{5} \text{ or } 0.6$	A1	1.1b
		(2)	
(b)	e.g. $P(A) \times P(B) = \frac{7}{20} \times \frac{9}{20} = \frac{63}{400} \neq P(A \cap B) = 0.13 = \frac{52}{400}$ or $P(A'   B') = 0.6 \neq P(A') = 0.65$	B1	2.4
		(1)	
(c)		B1	2.5
		M1	3.1a
		A1	1.1b
		M1	1.1b
		A1	1.1b
	(5)		
(d)	$P(B \cup C)' = 0.22 + 0.22 \text{ or } 1 - [0.56]$ or $1 - [0.13 + 0.23 + 0.09 + 0.11]$ o.e.	M1	1.1b
	$= 0.44$	A1	1.1b
		(2)	
<b>(10 marks)</b>			
<b>Notes:</b>			
(a)	<b>M1:</b> for a correct ratio of probabilities formula and at least one correct value. <b>A1:</b> a correct answer		
(b)	for a fully correct explanation: correct probabilities and correct comparisons.		
(c)	<b>B1:</b> for box with $B$ intersecting $A$ and $C$ but $C$ not intersecting $A$ . (Or accept three intersecting circles, but with zeros entered for $A \cap C$ and $A \cap B \cap C$ ) No box is B0 <b>M1:</b> for method for finding $P(B \cap C)$ <b>A1:</b> for 0.09 <b>M1:</b> for 0.13 and their 0.09 in correct places and method for their 0.23 <b>A1:</b> fully correct		
(d)	<b>M1:</b> for a correct expression – ft their probabilities from their Venn diagram. <b>A1:</b> cao		

Question	Scheme	Marks	AOs
5 (a)	The seeds would be destroyed in the process so they would have none to sell	B1	2.4
		(1)	
(b)	$[S = \text{no. of seeds out of 24 that germinate, } S \sim B(24, 0.55)]$		
	$T = \text{no. of trays with at least 15 germinating. } T \sim B(10, p)$	M1	3.3
	$p = P(S \geq 15) = 0.299126\dots$	A1	1.1b
	So $P(T \geq 5) = 0.1487\dots$ awrt <b>0.149</b>	A1	1.1b
		(3)	
(c)	$n$ is large and $p$ close to 0.5	B1	1.2
		(1)	
(d)	$X \sim N(132, 59.4)$	B1	3.4
	$P(X \geq 149.5) = P\left(Z \geq \frac{149.5 - 132}{\sqrt{59.4}}\right)$	M1	1.1b
	$= 0.01158\dots$ awrt <b>0.0116</b>	A1cso	1.1b
		(3)	
(e)	e.g The probability is very small therefore there is evidence that the company's claim is incorrect.	B1	2.2b
		(1)	
			<b>(9 marks)</b>
<b>Notes:</b>			
(a)	<b>B1:</b> cao		
(b)	<b>M1:</b> for selection of an appropriate model for $T$ <b>1<sup>st</sup> A1:</b> for a correct value of the parameter $p$ (accept 0.3 or better) <b>2<sup>nd</sup> A1:</b> for awrt 0.149		
(c)	<b>B1:</b> both correct conditions		
(d)	<b>B1:</b> for correct normal distribution <b>M1:</b> for correct use of continuity correction <b>A1:</b> cso		
(e)	<b>B1:</b> correct statement		

Question	Scheme	Marks	AOs
6	Integrate $\mathbf{a}$ w.r.t. time	M1	1.1a
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of $\mathbf{C}$ )	A1	1.1b
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$ , $\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = $100 \text{ m s}^{-1}$	A1ft	1.1b
<b>(6 marks)</b>			
<b>Notes:</b>			
<p><b>1<sup>st</sup> M1:</b> for integrating <math>\mathbf{a}</math> w.r.t. time (powers of <math>t</math> increasing by 1)</p> <p><b>1<sup>st</sup> A1:</b> for a correct <math>\mathbf{v}</math> expression without <math>\mathbf{C}</math></p> <p><b>2<sup>nd</sup> A1:</b> for a correct <math>\mathbf{v}</math> expression including <math>\mathbf{C}</math></p> <p><b>2<sup>nd</sup> M1:</b> for putting <math>t = 4</math> into their <math>\mathbf{v}</math> expression</p> <p><b>3<sup>rd</sup> M1:</b> for finding magnitude of their <math>\mathbf{v}</math></p> <p><b>3<sup>rd</sup> A1:</b> ft for <math>100 \text{ m s}^{-1}</math>, follow through on an incorrect <math>\mathbf{v}</math></p>			



Question	Scheme	Marks	AOs
7(a)	$R = mg\cos\alpha$	B1	3.1b
	Resolve parallel to the plane	M1	3.1b
	$-F - mg\sin\alpha = -0.8mg$	A1	1.1b
	$F = \mu R$	M1	1.2
	Produce an equation in $\mu$ only and solve for $\mu$	M1	2.2a
	$\mu = \frac{1}{4}$	A1	1.1b
		(6)	
(b)	Compare $\mu mg\cos\alpha$ with $mg\sin\alpha$	M1	3.1b
	Deduce an appropriate conclusion	A1 ft	2.2a
		(2)	
			(8 marks)
<b>Notes:</b>			
<p>(a)</p> <p><b>B1:</b> for <math>R = mg\cos\alpha</math></p> <p><b>1<sup>st</sup> M1:</b> for resolving parallel to the plane</p> <p><b>1<sup>st</sup> A1:</b> for a correct equation</p> <p><b>2<sup>nd</sup> M1:</b> for use of <math>F = \mu R</math></p> <p><b>3<sup>rd</sup> M1:</b> for eliminating <math>F</math> and <math>R</math> to give a value for <math>\mu</math></p> <p><b>2<sup>nd</sup> A1:</b> for <math>\mu = \frac{1}{4}</math></p>			
<p>(b)</p> <p><b>M1:</b> comparing size of limiting friction with weight component down the plane</p> <p><b>A1ft:</b> for an appropriate conclusion from their values</p>			

Question	Scheme	Marks	AOs
<b>8(a)</b>	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ : $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j}) \text{ m s}^{-2}$ Given answer	A1	1.1b
		(2)	
<b>(b)</b>	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2} \mathbf{a}t^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j}t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})t^2$	A1	1.1b
		(2)	
<b>(c)</b>	Equating the <b>i</b> and <b>j</b> components of <b>r</b>	M1	3.1b
	$\frac{1}{2} \leftarrow 0.7t^2 = 0.6t - \frac{1}{2} \leftarrow 0.1t^2$	A1ft	1.1b
	$t = 1.5$	A1	1.1b
		(3)	
<b>(d)</b>	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ : $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j})t$	M1	3.1b
	Equating the <b>i</b> and <b>j</b> components of <b>v</b>	M1	3.1b
	$t = 0.75$	A1 ft	1.1b
		(3)	
<b>(10 marks)</b>			
<b>Notes:</b>			
<b>(a)</b>			
<b>M1:</b> for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$			
<b>A1:</b> for given answer correctly obtained			
<b>(b)</b>			
<b>M1:</b> for use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2} \mathbf{a}t^2$			
<b>A1:</b> for a correct expression for <b>r</b> in terms of <i>t</i>			
<b>(c)</b>			
<b>M1:</b> for equating the <b>i</b> and <b>j</b> components of their <b>r</b>			
<b>A1ft:</b> for a correct equation following their <b>r</b>			
<b>A1:</b> for $t = 1.5$			
<b>(d)</b>			
<b>M1:</b> for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for a general <i>t</i>			
<b>M1:</b> for equating the <b>i</b> and <b>j</b> components of their <b>v</b>			
<b>A1ft:</b> for $t = 0.75$ , or a correct follow through answer from an incorrect equation			

Question	Scheme	Marks	AOs
<b>9(a)</b>	Take moments about $A$ (or any other complete method to produce an equation in $S$ , $W$ and $\alpha$ only)	M1	3.3
	$W \cos \alpha + 7W \cos \alpha = S \sin \alpha$	A1 A1	1.1b 1.1b
	Use of $\tan \alpha = \frac{5}{2}$ to obtain $S$	M1	2.1
	$S = 3W$ *	A1*	2.2a
		<b>(5)</b>	
<b>(b)</b>	$R = 8W$	B1	3.4
	$F = \frac{1}{4} R (= 2W)$	M1	3.4
	$P_{\text{MAX}} = 3W + F$ or $P_{\text{MIN}} = 3W - F$	M1	3.4
	$P_{\text{MAX}} = 5W$ or $P_{\text{MIN}} = W$	A1	1.1b
	$W \leq P \leq 5W$	A1	2.5
		<b>(5)</b>	
<b>(c)</b>	M(A) shows that the reaction on the ladder at $B$ is unchanged	M1	2.4
	also $R$ increases (resolving vertically)	M1	2.4
	which increases max $F$ available	M1	2.4
		<b>(3)</b>	
			<b>(13 marks)</b>

**Question 9 continued****Notes:****(a)****1<sup>st</sup> M1:** for producing an equation in  $S$ ,  $W$  and  $\alpha$  only**1<sup>st</sup> A1:** for an equation that is correct, or which has one error or omission**2<sup>nd</sup> A1:** for a fully correct equation**2<sup>nd</sup> M1:** for use of  $\tan \alpha = \frac{S}{2}$  to obtain  $S$  in terms of  $W$  only**3<sup>rd</sup> A1\*:** for given answer  $S = 3W$  correctly obtained**(b)****B1:** for  $R = 8W$ **1<sup>st</sup> M1:** for use of  $F = \frac{1}{4} R$ **2<sup>nd</sup> M1:** for either  $P = (3W + \text{their } F)$  or  $P = (3W - \text{their } F)$ **1<sup>st</sup> A1:** for a correct max or min value for a correct range for  $P$ **2<sup>nd</sup> A1:** for a correct range for  $P$ **(c)****1<sup>st</sup> M1:** for showing, by taking moments about  $A$ , that the reaction at  $B$  is unchanged by the builder's assistant standing on the bottom of the ladder**2<sup>nd</sup> M1:** for showing, by resolving vertically, that  $R$  increases as a result of the builder's assistant standing on the bottom of the ladder**3<sup>rd</sup> M1:** for concluding that this increases the limiting friction at  $A$

Question	Scheme	Marks	AOs
<b>10(a)</b>	Using the model and horizontal motion: $s = ut$	M1	3.4
	$36 = U t \cos \alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2} at^2$	M1	3.4
	$-18 = U t \sin \alpha - \frac{1}{2} g t^2$	A1	1.1b
	Correct strategy for solving the problem by setting up two equations in $t$ and $U$ and solving for $U$	M1	3.1b
	$U = 15$	A1	1.1b
	<b>(6)</b>		
<b>(b)</b>	Using the model and horizontal motion: $U \cos \alpha$ (12)	B1	3.4
	Using the model and vertical motion: $v^2 = (U \sin \alpha)^2 + 2(-10)(-7.2)$	M1	3.4
	$v = 15$	A1	1.1b
	Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = $\sqrt{(12^2 + 15^2)}$	M1	3.1b
	$\sqrt{369} = 19 \text{ m s}^{-1}$ (2sf)	A1 ft	1.1b
		<b>(5)</b>	
<b>(c)</b>	Possible improvement (see below in notes)	B1	3.5c
	Possible improvement (see below in notes)	B1	3.5c
		<b>(2)</b>	
			<b>(13 marks)</b>

**Question 10 continued****Notes:****(a)****1<sup>st</sup> M1:** for use of  $s = ut$  horizontally**1<sup>st</sup> A1:** for a correct equation**2<sup>nd</sup> M1:** for use of  $s = ut + \frac{1}{2}at^2$  vertically**2<sup>nd</sup> A1:** for a correct equation**3<sup>rd</sup> M1:** for correct strategy (need both equations)**2<sup>nd</sup> A1:** for  $U = 15$ **(b)****B1:** for  $U\cos\alpha$  used as horizontal velocity component**1<sup>st</sup> M1:** for attempt to find vertical component**1<sup>st</sup> A1:** for 15**2<sup>nd</sup> M1:** for correct strategy (need both components)**2<sup>nd</sup> A1ft:** for  $19 \text{ m s}^{-1}$  (2sf) following through on incorrect component(s)**(c)****B1, B1:** for any two of

e.g. Include air resistance in the model of the motion

e.g. Use a more accurate value for  $g$  in the model of the motion

e.g. Include wind effects in the model of the motion

e.g. Include the dimensions of the stone in the model of the motion