1 **a** Express $(\frac{2}{3})^{-2}$ as an exact fraction in its simplest form. **(2)**

   b Solve the equation

   $$x^{\frac{3}{2}} - 27 = 0.$$ **(3)**

2 Solve the simultaneous equations

   $$x + 3y = 16$$
   $$x^2 - xy + 2y^2 = 46$$ **(7)**

3 Simplify
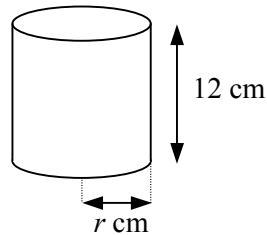
   a $\sqrt{192} - 2\sqrt{12} + \sqrt{75}$ **(4)**

   b $(2 + \sqrt{3})(5 - 2\sqrt{3})$ **(3)**

4 $$f(x) \equiv x^2 - 4\sqrt{2}\,x + 11.$$

   a Express f(x) in the form $a(x + b)^2 + c$ stating the exact values of the
   constants a, b and c. **(4)**

   b Sketch the curve $y = f(x)$, showing the coordinates of the turning point of the
   curve and of any points of intersection of the curve with the coordinate axes. **(3)**

5

12 cm

r cm

A sealed metal container for food is a cylinder of height 12 cm and base radius r cm.

Given that the surface area of the container must be at most 128π cm^2,

   a show that $r^2 + 12r - 64 \leq 0$. **(3)**

   b Hence find the maximum value of r. **(4)**

6 Find the non-zero value of x for which

   $$(2\sqrt{x})^3 = 4x.$$ **(4)**

7 **a** Write down the value of x such that $2^x = 32$. **(1)**

   b Solve the equation

   $$32^{y+1} = 4^y.$$ **(3)**

8 **a** Given that $t = \sqrt{x}$, express $x - 5\sqrt{x}$ in terms of t. **(1)**

   b Hence, or otherwise, solve the equation

   $$x - 5\sqrt{x} + 6 = 0.$$ **(4)**

9 Prove, by completing the square, that there is no real value of the constant k for which the
   equation $x^2 + kx + 3 + k^2 = 0$ has real roots. **(6)**

10 **a** Find the value of x such that
$$8^{2x-1} = 32.$$ **(3)**

b Find the value of y such that
$$\left(\tfrac{1}{3}\right)^{y-2} = 81.$$ **(3)**

11 Solve the inequality
$$x(2x-7) < (x-2)^2.$$ **(5)**

12 Express
$$\frac{2}{3\sqrt{2}-4} - \frac{3-\sqrt{2}}{\sqrt{2}+1}$$
in the form $a + b\sqrt{2}$, where a and b are integers. **(6)**

13 **a** Solve the equation
$$6y^2 + 25y - 9 = 0.$$ **(3)**

b Find the values of the constant k such that the equation
$$x^2 + kx + 16 = 0$$
has equal roots. **(3)**

14 **a** Given that $y = 2^x$,

i show that $4^x = y^2$,

ii express 2^{x-1} in terms of y. **(4)**

b By using your answers to part **a**, or otherwise, find the values of x for which
$$4^x - 9(2^{x-1}) + 2 = 0.$$ **(4)**

15 Find the pairs of values (x, y) which satisfy the simultaneous equations
$$x^2 + 2xy + y^2 = 9$$
$$x - 3y = 1$$ **(7)**

16 **a** Prove, by completing the square, that the roots of the equation $x^2 + ax + b = 0$ are given by
$$x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}.$$ **(6)**

b Hence, find an expression for b in terms of a such that the equation $x^2 + ax + b = 0$ has a repeated root. **(2)**

17 $$f(x) \equiv 2x^2 - 12x + 19.$$

a Prove that $f(x) \geq 1$ for all real values of x. **(5)**

b Find the set of values of x for which $f(x) < 9$. **(4)**

18 **a** Express $(1 - \sqrt{5})^2$ in the form $a + b\sqrt{5}$. **(2)**

b Hence, or otherwise, solve the equation
$$y^2 = 3 - \sqrt{5},$$
giving your answers in the form $c\sqrt{2} + d\sqrt{10}$, where c and d are exact fractions. **(6)**